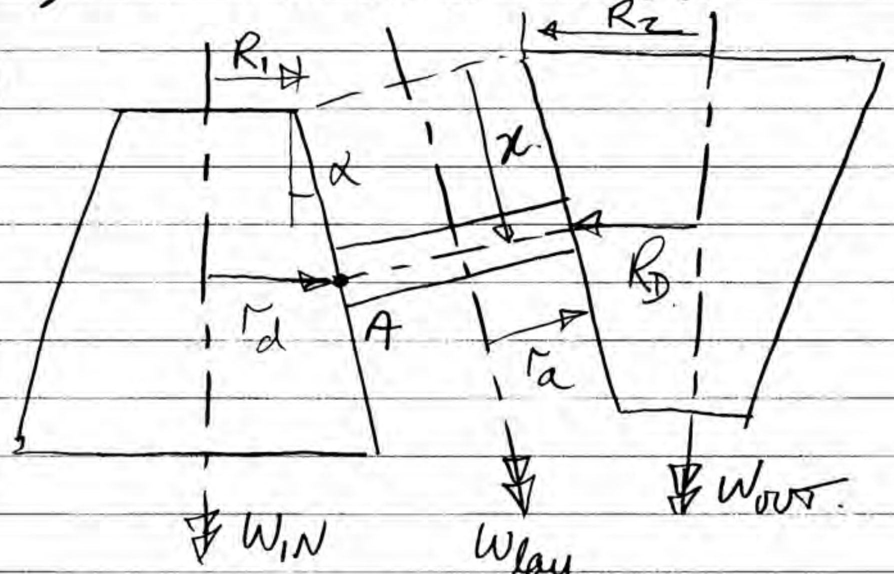
3C8 - 2019

1 a) Cylindrical wheel would have line contact with cone. Spin in contact would lead to significant sliding velocities towards the ends of the line contact, and thus larger friction losses than for a barrelled wheel.

b)



$$r_d = R_1 + x \sin\alpha$$

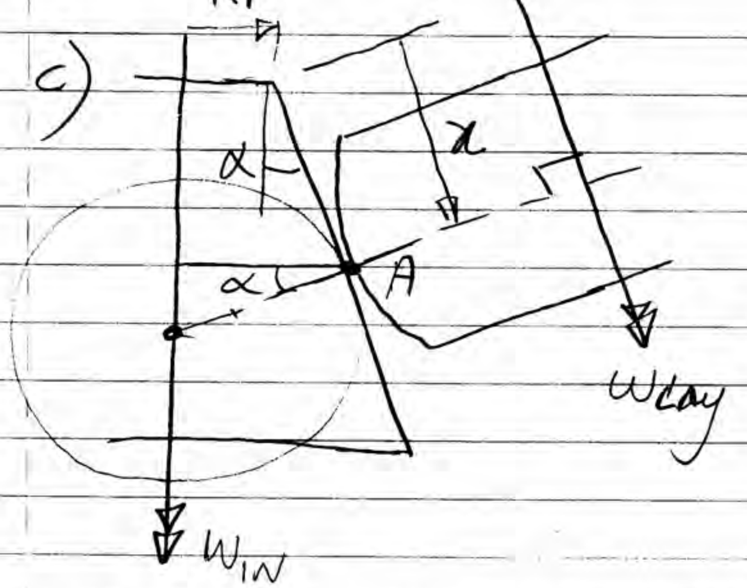
$$R_D = R_2 - x \sin\alpha$$

$$W_{lay} = -W_{IN} \cdot \frac{r_d}{r_a}$$

$$W_{OUT} = -W_{lay} \frac{r_a}{R_D}$$

$$\therefore \frac{W_{OUT}}{W_{IN}} = \frac{r_d}{R_D} = \frac{R_1 + x\sin\alpha}{R_2 - x\sin\alpha}$$

c)



ω_{cay} has no component normal to plane of contact

$\therefore$ $\omega_{spin} = \omega_{IN} \cdot \sin\alpha$

d) i) in plane of shafts
cone is flat $R = \infty$
wheel is r_b.

$\therefore$ $R_{effective} = r_b$

ii) in plane of wheel

wheel is r_a

cone is equivalent to sphere of radius:

$$\frac{R_1 + x\sin\alpha}{\cos\alpha}$$

$$\therefore \frac{1}{R_{effective}} = \frac{1}{r_a} + \frac{\cos\alpha}{R_1 + x\sin\alpha}$$

$$R_{effective} = \frac{r_a(R_1 + x\sin\alpha)}{R_1 + x\sin\alpha + r_a\cos\alpha}$$

e) Since input torque is half the limiting value,

$$I_F = \frac{F}{\mu N} = 0.5$$

Contact is circular so $\frac{a}{b} = 1.0$

From datasheet chart deduce $\frac{e}{\sqrt{ab}} = \frac{e}{a} = 0.44$

and $I_M = \frac{M_s}{\mu N \sqrt{ab}} = \frac{M_s}{\mu N a} = 0.47$

Now $$\frac{P_{loss}}{P_{IN}} = \frac{V_{slip} F + \omega_{spin} M_s}{V_{rolling} F}$$

$$= \frac{V_{slip} F + \omega_{spin} I_M \mu N a}{V_{rolling} F}$$

but $\mu N = \frac{F}{I_F}$

$$\therefore \frac{P_{loss}}{P_{IN}} = \frac{V_{slip} \cancel{F} + \omega_{spin} I_M \frac{\cancel{F}}{I_F} a}{V_{rolling} \cancel{F}}$$

but $V_{slip} = e . \omega_{spin}$

$$\frac{P_{loss}}{P_{IN}} = \frac{e\, \omega_{spin} + \omega_{spin} \frac{I_M}{I_F} a}{V_{rolling}}$$

$$= \frac{\omega_{spin}}{V_{rolling}} \left(e + \frac{I_M}{I_F} a \right)$$

$$= 10 \left(0.44 + \frac{0.47}{0.5} \right) 0.25.10^{-3} = 3.45.10^{-3}$$

$$\therefore \eta = 1 - 3.45.10^{-3} = \underline{99.66\%}$$

2 (a) From datasheet: $S_1 = (1+R)C_1 - RA_1$ (1)

$S_2 = (1+R)C_2 - RA_2$ (2)

Input: $\omega_i = S_1 = S_2$, $\alpha\omega_i = A_1$

Output: $\omega_0 = C_2$

Linkages: $C_1 =$

From (1) $\omega_i = (1+R)C_1 - R\alpha\omega_i$

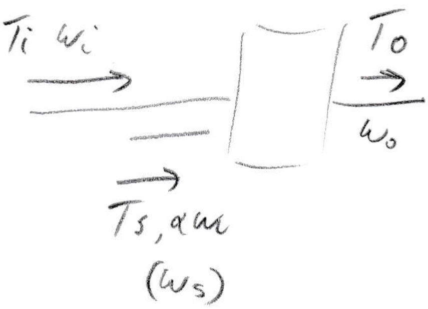
$\Rightarrow C_1 = \omega_i(1+R\alpha)/(1+R) = A_2$

Substituting into (2): $\omega_i = (1+R)\omega_0 - R\omega_i\dfrac{(1+R\alpha)}{(1+R)}$

$$\Rightarrow \omega_0(1+R) = \frac{\omega_i}{(1+R)}(1+R+R+\alpha R^2)$$

$$\omega_0 = \omega_i\frac{(1+2R+\alpha R^2)}{(1+R)^2}$$

Torques on shafts



(b)

Virtual work:

$T_i\omega_i' + T_s\omega_s' + T_0\omega_0' = 0$

Put $\omega_s' = 0$

$$\Rightarrow T_0 = -T_i\left(\frac{\omega_i}{\omega_0}\right)_{\omega_s'=0\ [\alpha=0]} = -T_i\frac{(1+R)^2}{(1+2R)} = -T_i \times \frac{25}{9} \quad \text{for } R=4$$

For $\alpha = 0.5, R=4 \Rightarrow \omega_0 = \omega_i\left(\dfrac{1+8+8}{25}\right)$

$T_i\omega_i + T_s\alpha\omega_i + T_0\omega_0 = 0 \Rightarrow T_i\cancel{\omega_i} + T_s\dfrac{\cancel{\omega_i}}{2} - T_i\dfrac{\cancel{25}}{9}\cancel{\omega_i}\times\dfrac{17}{\cancel{25}} = 0$

$T_s = \dfrac{16}{9}T_i$

$\left[\text{or } \omega_0 = 0 \Rightarrow \alpha.16 + 1 + 8 = 0,\ \alpha = -\dfrac{9}{16}\right.$

$\left.\dfrac{T_s}{T_i} = -\left(\dfrac{\omega_i}{\omega_s}\right)_{\omega_0=0} = \dfrac{-1}{\alpha} = \dfrac{16}{9}\right]$

b(ii) For 2nd epicyclic

$$T_S \omega_S + T_A \omega_A + T_C \omega_C = 0$$

$$\frac{T_S}{T_C} = -\left(\frac{\omega_C}{\omega_S}\right)_{\omega_A = 0} = -\left(\frac{1}{1+R}\right) = -\frac{1}{5}$$

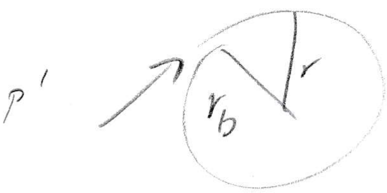
$$T_S = \left(-\frac{1}{5}\right) \cdot \left(-\frac{25}{9}\right) T_i = \frac{5}{9} T_i$$

Power through $S_2 = \frac{5}{9} T_i \omega_i$

(c)

$$p_0 = \left(\frac{P' E^*}{\pi R}\right)^{\frac{1}{2}} \quad \text{assuming line contact}$$

Critical case in both cases is Sun-planet contact



$$\underset{\text{number of planets}}{N} . P' \times r_b \times \underset{\text{facewidth}}{w} = T$$

$r \sim m$
$w \sim m$
$R \sim m$
E^*, N are the same for both epicyclics

$$1 = \left(\frac{p_{01}}{p_{02}}\right)^2 = \left(\frac{P'E^*}{\pi R}\right)_1 \Big/ \left(\frac{P'E^*}{\pi R}\right)_2 = \frac{P'_1}{P'_2} \frac{R_2}{R_1} = \frac{T_1}{T_2} \frac{r_2}{r_1} \frac{w_2}{w_1} \frac{R_2}{R_1}$$

But $\frac{T_1}{T_2}$ (torques on S_1 and S_2) $= \frac{4/9\, T_i}{5/9\, T_i}$ using the result of (b)(ii)

$$\Rightarrow 1 = \frac{4}{5}\left(\frac{m_2}{m_1}\right)^3 \qquad \frac{m_1}{m_2} = \left(\frac{4}{5}\right)^{\frac{1}{3}}$$

3 a)

	radial load	axial load	misalignment	axial displacement
deep groove ball	+	+	−	−
cylindrical roller	++	−−	−	+++
taper roller	++	+++*	−	−−
spherical roller	+++	+	+++	−−

* one direction only.

b) bearing NU 1010

$$L = a_1 a_{23} \left(\frac{C}{P}\right)^p$$

$a_1 = 0.62$ (95%) $\quad p = \frac{10}{3}$ (roller)

$P = 20\,kN$ (question) $\quad C = 30.8\,kN$ (data sheet)

to find a_{23} find $d_m = \frac{d+D}{2} = \frac{50mm + 80mm}{2} = 65mm$

from diagram 1 (data sheet) $\nu_1 = 30\ mm^2/s$

so $\kappa = \frac{\nu}{\nu_1} = \frac{8}{30} = 0.27$

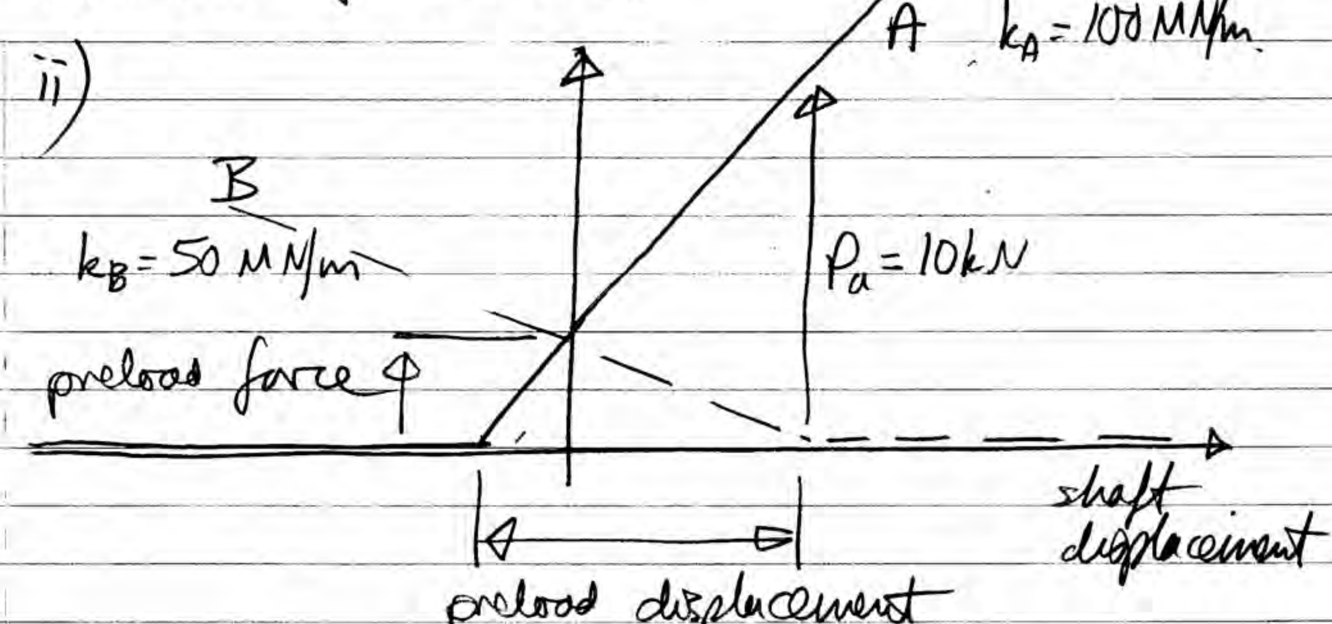
from diagram 3, $a_{23} = 0.16$

thus $L = 0.62 . 0.16 \left(\frac{30.8}{20}\right)^{10/3} = 0.418 . 10^6$ revs

$= 418$ thousand revs.

c) i) Zero preload and external axial force will cause clearance in one of the bearings, leading to poor shaft alignment accuracy and low axial stiffness. Radial force on a bearing with clearance may lead to premature failure, due to the force being carried by a small number of rollers.

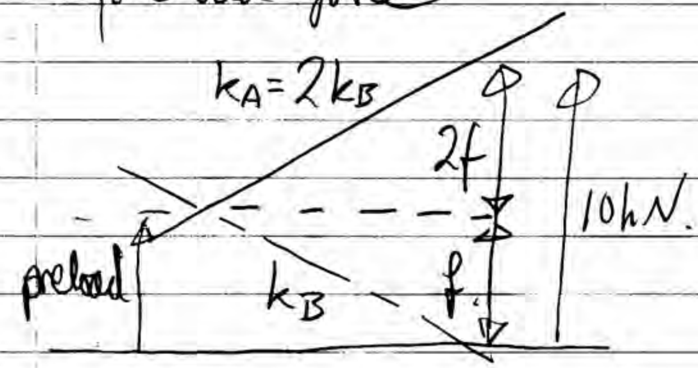
ii)



from the diagram :

$$\text{preload displacement} = \frac{P_a}{k_A} = \frac{10.10^3}{100.10^6} = \underline{\underline{0{\cdot}1\text{ mm}}}$$

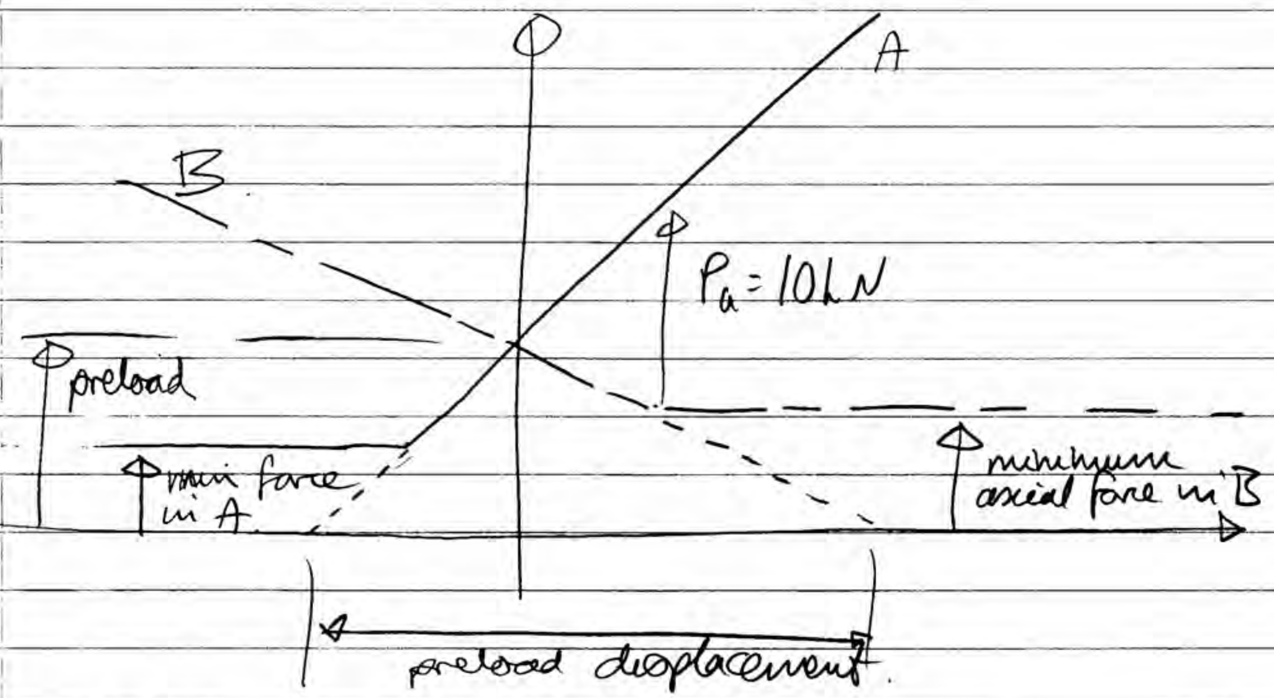
preload force :



preload force

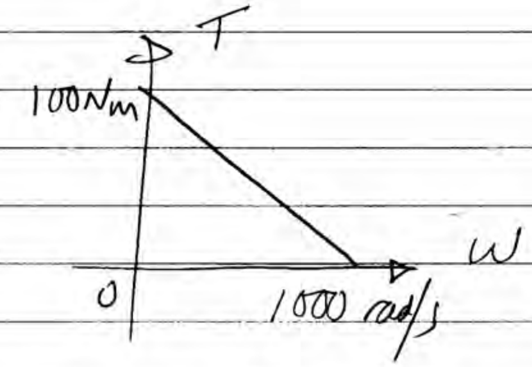
$$f = \frac{10\text{kN}}{3} = \underline{\underline{3{\cdot}33\text{ kN}}}$$

iii) A radial force means that the axial force on each bearing has a non-zero minimum value.



The minimum preload force required is increased. For the forces shown in the diagram, the preload force is increased by the amount of the minimum axial force in B.

4 a)



$T = m\omega + c = -0.1\omega + 100$

$P = T\omega = -0.1\omega^2 + 100\omega$ — (1)

$\frac{dP}{d\omega} = -0.2\omega + 100$

max P when $0.2\omega = 100$

$\omega = 500$ rad/s

$\therefore P = -0.1 \cdot 500^2 + 100 \cdot 500$

$= 25$ kW

From (1):

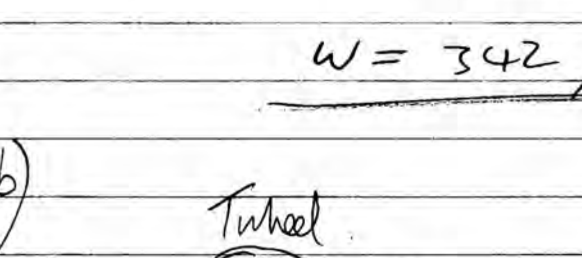
$0.1\omega^2 - 100\omega + 22,500 = 0.$

$\omega^2 - 1000\omega + 225,000 = 0$

$\omega = \frac{1000 \pm \sqrt{10^6 - 4 \cdot 225,000}}{2}$

$\omega = 342, 658$ rad/s

b)



$n = \frac{\omega}{\Omega}$

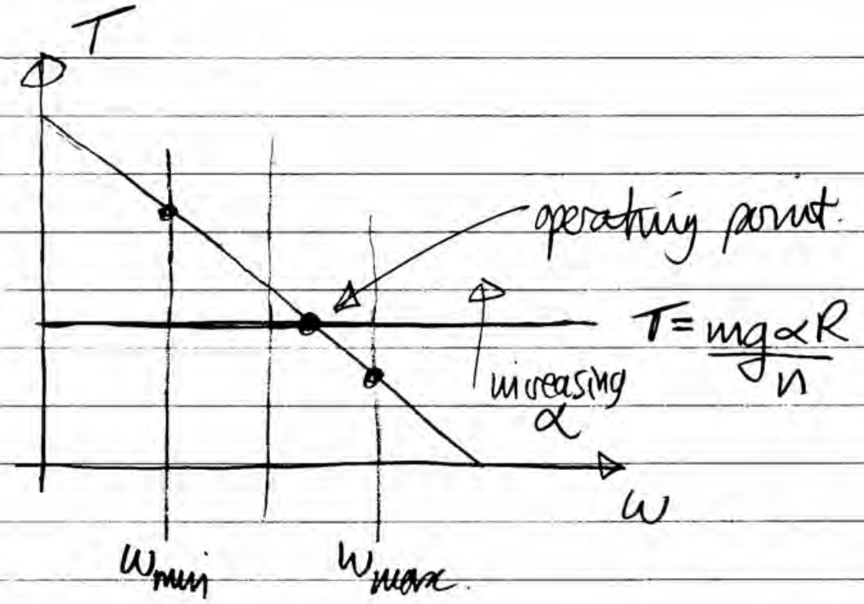
power conserved;

$T_{motor}\,\omega = T_{wheel}\,\Omega$

$T_{motor} = T_{wheel} \cdot \frac{\Omega}{\omega} = \frac{T_{wheel}}{n}$

also $T_{wheel} = mg\alpha R$

$\therefore T_{motor} = \frac{mg\alpha R}{n}$



c) Consider range of α that can be accommodated between ω_{min} and ω_{max} for fixed n:

let $\beta = \dfrac{\alpha_{\omega_{min}}}{\alpha_{\omega_{max}}} = \dfrac{\omega_{max}}{\omega_{min}} = \dfrac{658}{342} = 1.925$

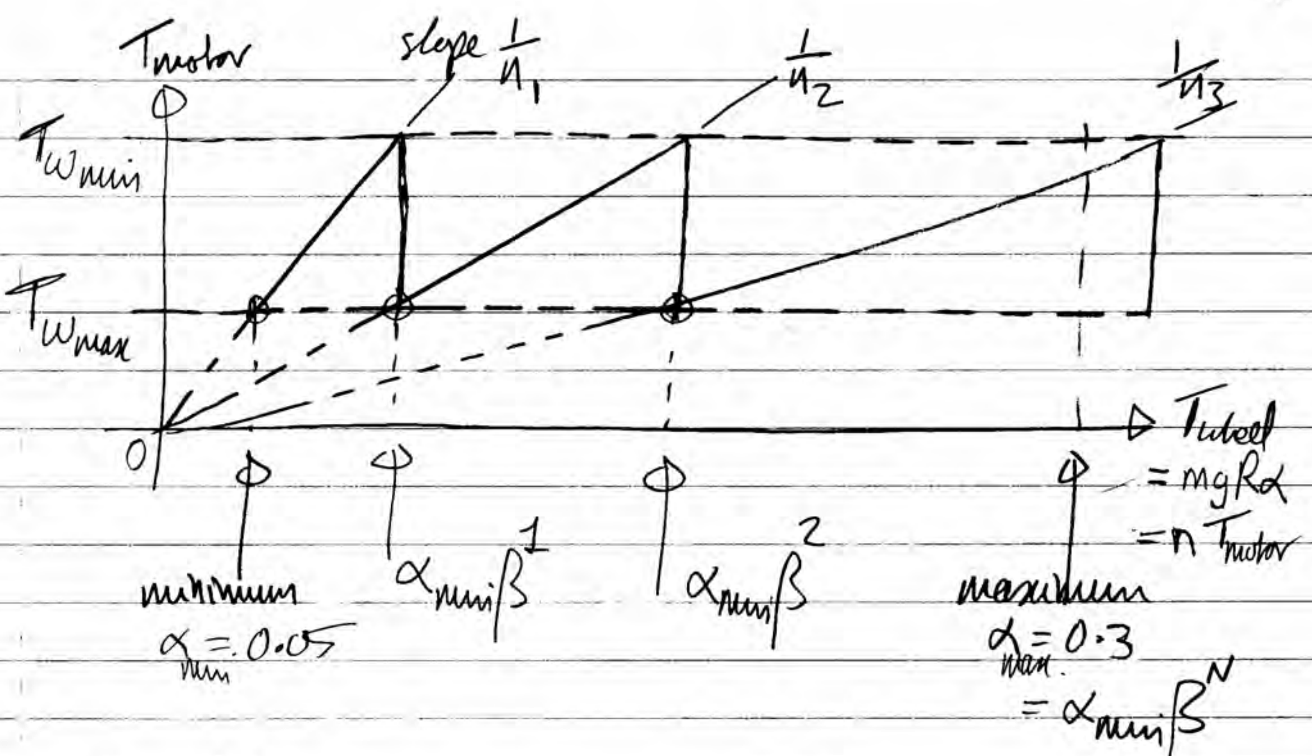
Hence number of ratios N required to span α_{min} to α_{max} is given by

$$\alpha_{max} = \alpha_{min}\,\beta^N$$

$$\beta^N = \frac{\alpha_{max}}{\alpha_{min}}$$

$$N = \frac{\log\left(\frac{0.3}{0.05}\right)}{\log 1.925} = 2.74$$

round up to nearest integer = $\underline{\underline{3 \text{ ratios}}}$



to find n_1

$$T_{wheel} = mgR\alpha_{min} = \underbrace{(100 - 0.1\omega_{MAX})}_{T_{\omega MAX}} n_1$$

$$n_1 = \frac{1000 \cdot 9.81 \cdot 0.3 \cdot 0.05}{100 - 0.1 \cdot 658}$$

$$n_1 = 4.303$$

$$n_2 = n_1\beta = 8.28$$

$$n_3 = n_1\beta^2 = 15.95$$