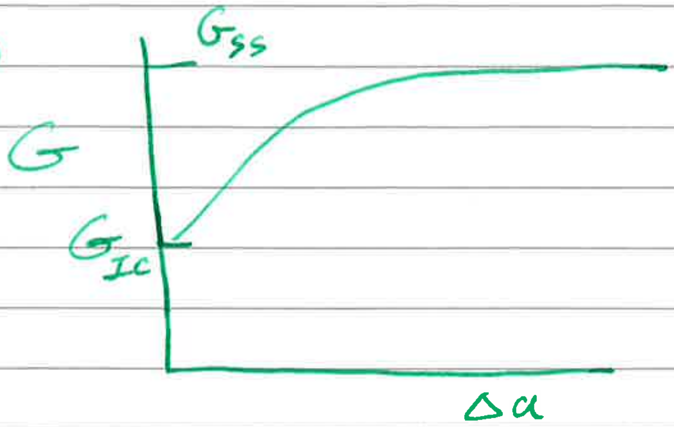
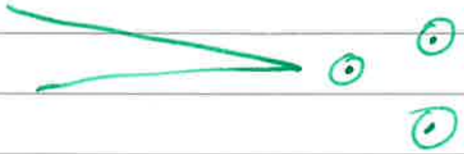
3C9 Crib 2014/15

Q1. (a)





Fracture mechanism is microvoid coalescence, such that $G^D_{IC} \sim \sigma_Y \ell \varepsilon_f$

where σ_Y = yield strength
ℓ = precipitate / inclusion spacing
$\varepsilon_f \approx 1$ is material ductility

Cleavage toughness $G^C_{IC} \approx 2\gamma_s \sim E\underbrace{b}_{\text{atomic spacing}}$

So $$\frac{G^D_{IC}}{G^C_{IC}} = \frac{\sigma_Y \ell \varepsilon_f}{Eb} = \underbrace{\varepsilon_Y}_{10^{-2}} . \underbrace{\frac{\ell}{b}}_{10^{\cancel{3}4}} . \underbrace{\varepsilon_f}_{1} \approx 10^2$$

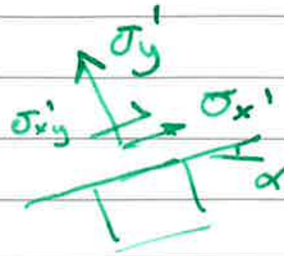
Additionally, plasticity surrounding the crack tip process zone leads to additional plastic dissipation such that $G_{ss} / G_{Ic} \approx 4$.

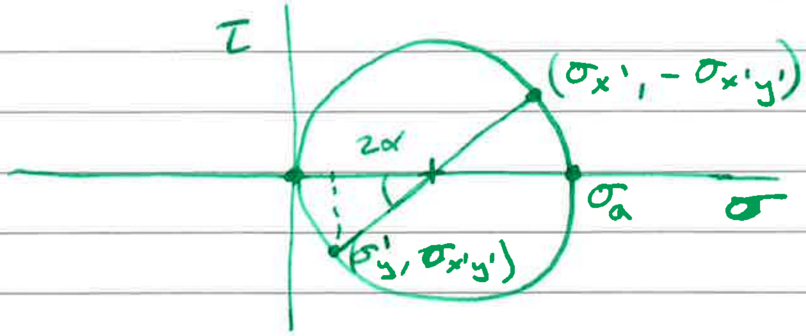
[Comment: few candidates mentioned microvoids.]

3C9 : 2014/15 crib

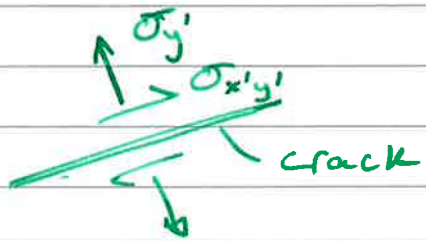
contd.

Q1.(b) Axial stress $\sigma_a = \dfrac{F}{2\pi R t}$





$$\sigma_{y'} = \frac{\sigma_a}{2}\left(1 - \cos 2\alpha\right) \qquad \sigma_{x'y'} = -\frac{\sigma_a}{2} \sin 2\alpha$$



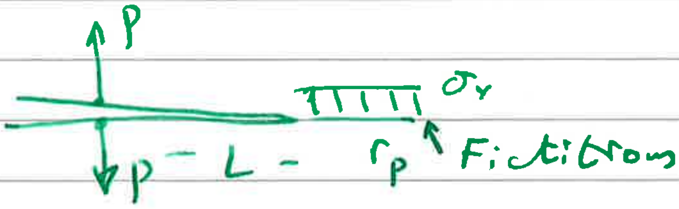
So, $K_I = \sigma_{y'} \sqrt{\pi a}$ $\qquad K_{II} = \sigma_{x'y'} \sqrt{\pi a}$

(c) $K_I = \dfrac{2P}{\sqrt{2\pi L}}$ $\qquad$ (i) $r_p = \dfrac{1}{\pi} \dfrac{K_I^2}{\sigma_y^2}$

$$\Rightarrow r_p = \frac{1}{\pi} \frac{4P^2}{2\pi L^2 \sigma_y^2} = \frac{2}{\pi^2} \frac{P^2}{L^2 \sigma_y^2}$$

(plane stress, Irwin model)

(ii)



K at fictitious crack tip is

$K = K_A + K_B$ $\qquad$ K_A – due to P
$= 0$ $\qquad$ K_B – due to σ_Y

Q1. (c) contd.

$$K_A = \frac{2P}{\sqrt{2\pi}\sqrt{L+r_p}} = \sqrt{\frac{2}{\pi}}\frac{P}{\sqrt{L+r_p}}$$

$$K_B = -\int_0^{r_p}\sqrt{\frac{2}{\pi}}\frac{\sigma_y\,dx}{\sqrt{x}} = \frac{2\sqrt{2}}{\sqrt{\pi}}\sigma_y\sqrt{r_p}$$

$$K_A = -K_B \Rightarrow$$

$$\frac{P}{\sqrt{L+r_p}} = 2\sigma_y\sqrt{r_p}$$

$$\frac{P^2}{L+r_p} = 4\sigma_y^2 r_p$$

$$4\sigma_y^2(r_p^2 + r_pL) = P^2$$

$$\Rightarrow r_p^2 + r_pL = \frac{P^2}{4\sigma_y^2}$$

$$\Rightarrow \left(r_p + \tfrac{1}{2}L\right)^2 = \frac{P^2}{4\sigma_y^2} + \frac{1}{4}L^2$$

$$\Rightarrow r_p = -\tfrac{1}{2}L + \sqrt{\left(\frac{P}{2\sigma_y}\right)^2 + \left(\frac{L}{2}\right)^2}$$

If $\dfrac{P}{2\sigma_y} \ll \dfrac{L}{2}$ ie $P \ll L\sigma_y$

then $r_p = -\tfrac{1}{2}L + \tfrac{1}{2}L\left[1 + \tfrac{1}{2}\left(\dfrac{P}{L\sigma_y}\right)^2 + \dots\right]$

$$\Rightarrow r_p = \frac{1}{4}\frac{P^2}{\sigma_y^2 L}$$

Q1. (c) contd.

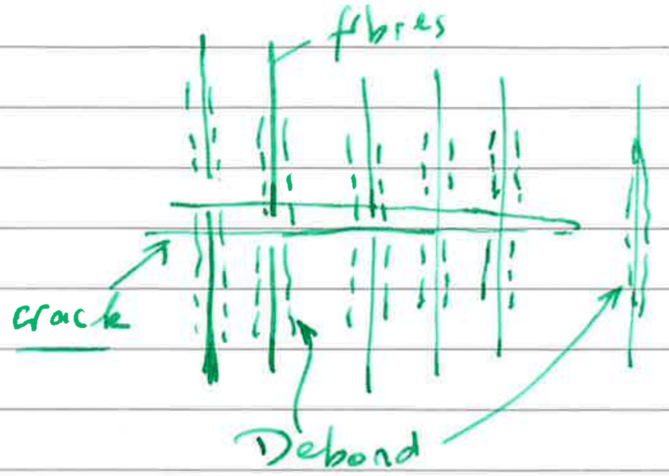
check : $r_p = \frac{\pi}{8} \frac{K_I^2}{\sigma_Y^2}$ by Dugdale

$$K_I \approx \sqrt{\frac{2}{\pi}} \frac{P}{\sqrt{L}} \Rightarrow r_p \approx \frac{\pi}{8} \frac{2}{\pi} \frac{P^2}{\sigma_Y^2 L} = \frac{1}{4} \frac{P^2}{\sigma_Y^2 L}.$$

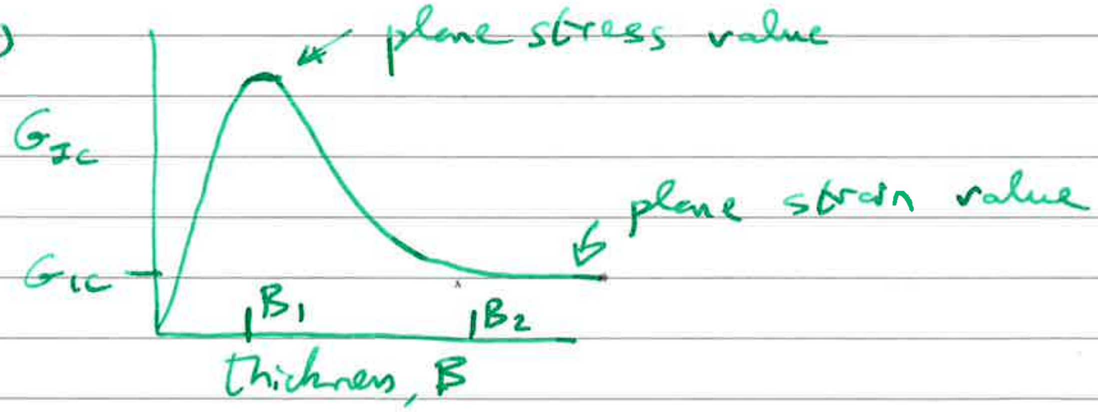
[Comment: most understood how to use Mohr's circle, most did not realise that K_{II} could be negative. Most could perform the Dugdale analysis].

3C9 : 2014/15 crib

Q2 (a) Glass fibres give bridging & pull-out behind the crack tip, and possibly crack kinking ahead of the crack tip to blunt the crack.



(b)



So arrange for sheets of thickness B_1 rather than sheets of thickness B_2.

[Comment : few candidates mentioned fibre pull-out.]

Q1. Examiner's Comment:
A popular and straightforward question, well-answered by most candidates. Surprisingly few understood that the toughness of engineering alloys if governed by spacing of inclusions rather than the presence of plasticity. Most could do the last part of the question, although some used the Dugdale result from lectures rather than solving the different problem here.

Q2. Examiner's Comment:
A popular question. Most understood the basics of toughening mechanisms. Few were able to do the final part of the question although it was relatively straightforward.

C6.

(c)

Q2. (i) $K_R = K_0\left(1 - \frac{1}{2}\exp(-\Delta a/\lambda)\right)$

At $\Delta a = 0$, $K_R = \frac{1}{2}K_0 \Rightarrow K_{Ic} = \frac{1}{2}K_0$

$= \infty$, $K_R = K_0$

(ii) $K = \sigma\sqrt{\pi(a_0 + \Delta a)}$

$\Rightarrow K_R' = \frac{K_0}{2\lambda}\exp(-\Delta a/\lambda)$ $\qquad K' = \frac{\pi}{2}\frac{\sigma}{\sqrt{\pi(a_0+\Delta a)}}$

$K' = K_R' \Rightarrow \qquad \sigma = \frac{\cancel{2}}{\pi}\sqrt{\pi(a_0+\Delta a)}\,\frac{K_0}{\cancel{2}\lambda}\exp\left(-\frac{\Delta a}{\lambda}\right)$

$K = K_R \Rightarrow \frac{(a_0+\Delta a)}{\lambda}K_0\exp\left(-\frac{\Delta a}{\lambda}\right) = K_0 - \frac{K_0}{2}\exp\left(-\frac{\Delta a}{\lambda}\right)$

$\Rightarrow \left(\left(\frac{a_0+\Delta a}{\lambda}\right) + \frac{1}{2}\right) = \exp\left(\frac{\Delta a}{\lambda}\right)$

Write $x = \frac{\Delta a}{\lambda} \Rightarrow \left(\frac{1}{2} + \frac{a_0}{\lambda} + x\right) = \exp x$

Now : $A + x_n = \exp x_{n+1}$

$A = \frac{1}{2} + \frac{a_0}{\lambda}$ $\qquad \frac{a_0}{\lambda} = 1 \Rightarrow A = \frac{3}{2}$

Guess $x_1 = 1 \Rightarrow x_2 = \ln\left(\frac{3}{2} + x_1\right) = 0.9163$

$\Rightarrow x_3 = \ln\left(\frac{3}{2} + x_2\right) = 0.8822$

$x_4 = \ln\left(\frac{3}{2} + x_3\right) = 0.8680$

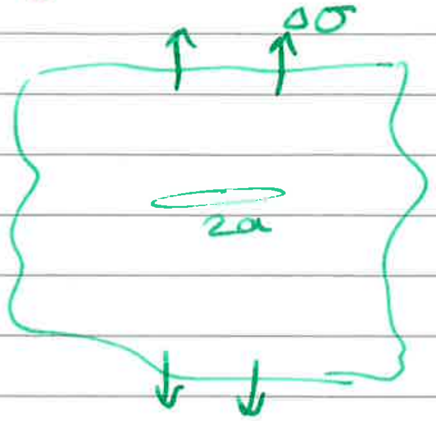
$0.8680 \to 0.8621 \to 0.8595 \to 0.8585$

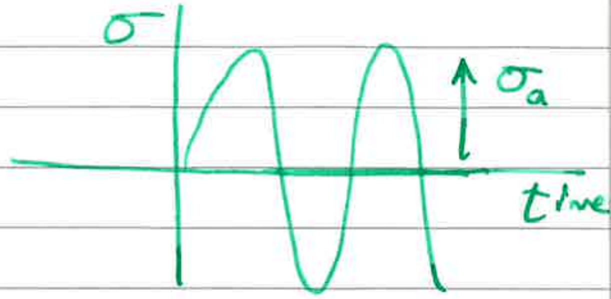
$\to 0.8580$ $\qquad$ Ans. $= 0.858 \Rightarrow \frac{\Delta a}{\lambda} = 0.858$

$\Rightarrow K_R = K_0\left(1 - \frac{1}{2}\exp(-0.858)\right) = 0.788\,K_0$

$\sigma = \frac{K_R}{\sqrt{\pi(a_0+\Delta a)}} = \frac{K_R}{\sqrt{\pi\lambda}}\left(\frac{a_0}{\lambda} + \frac{\Delta a}{\lambda}\right)^{-\frac{1}{2}} = \frac{0.626\,K_0}{\sqrt{\pi a_0}}$

3. (a) A sharp notch behaves like a crack, although it gives a somewhat greater fatigue strength.





$$\Delta\sigma = 2\sigma_a$$

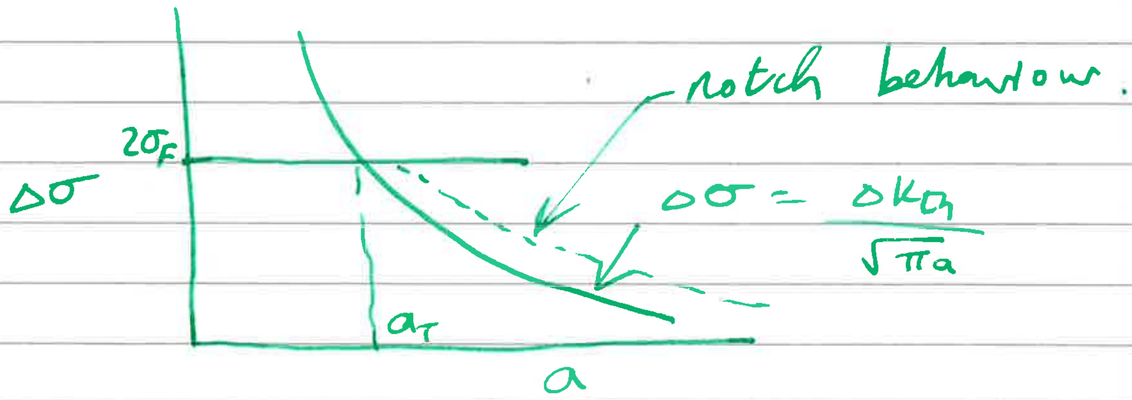
Consider a long crack:

$$\Delta\sigma \approx \frac{\Delta K_{th}}{\sqrt{\pi a}}$$

Consider a short crack:

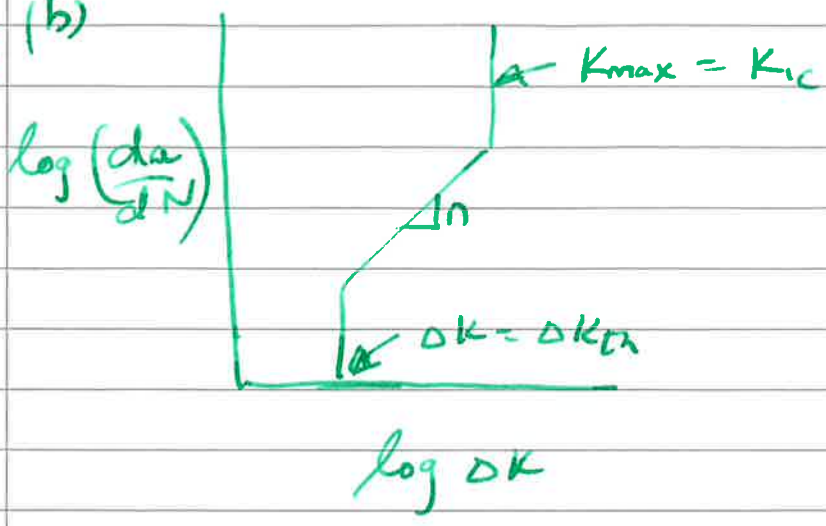
$$\Delta\sigma \approx 2\sigma_F$$

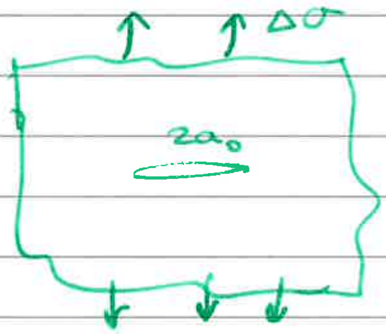
where σ_F is the fatigue limit.



$$a_T = \frac{1}{4\pi^2}\frac{\Delta K_{TH}^2}{\sigma_F^2}$$

3 (b)





At infinite life : $\Delta K = \Delta\sigma\sqrt{\pi a} = \Delta K_{th}$

$$\Rightarrow \Delta\sigma = \frac{\Delta K_{th}}{\sqrt{\pi a_0}} = 2\sigma_F$$

fatigue limit (amplitude)

For a growing crack :

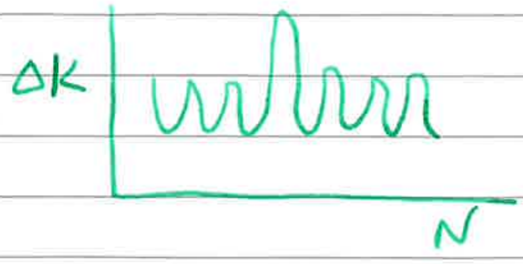
$$\frac{da}{dN} = C\Delta K^n = C\left(\Delta\sigma\sqrt{\pi a}\right)^n$$

$$\Rightarrow \int_{a_0}^{a_F} \frac{da}{a^{n/2}} = C\,\Delta\sigma^n\,\pi^{n/2}\,N_f$$

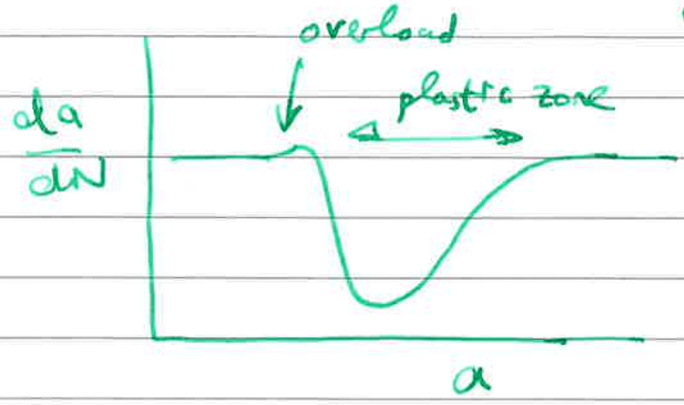
Now $a_F \gg a_0 \Rightarrow \frac{2}{2+n}\,a_0^{\frac{n+2}{2}} = C\Delta\sigma^n\pi^{n/2}N_f$

Hence slope of $\Delta\sigma$ vs. N_f plot is $-1/n$.

3 (c)



Overload in ΔK leads to a large plastic zone. When crack grows through this zone, crack closure retards crack growth.



3 (d) $a_{TF} = \frac{1}{4\pi^2} \frac{\Delta K_{TH}^2}{\sigma_F^2}$. for fatigue

$a_{TM} = \frac{1}{\pi^2} \frac{K_{IC}^2}{\sigma_Y^2}$. for monotonic loading

$\Rightarrow \quad \frac{a_{TF}}{a_{TM}} \simeq \frac{1}{4} \left(\frac{\Delta K_{th}}{K_{IC}}\right)^2 \left(\frac{\sigma_F}{\sigma_Y}\right)^2$

Now, $\frac{\Delta K_{TH}}{K_{IC}} \sim \frac{1}{10} \qquad \frac{\sigma_F}{\sigma_Y} \simeq 1$

$\Rightarrow \quad \frac{a_{TH}}{a_{TM}} << 1.$

3 (c) A corrosive environment leads to chemical attack at crack tip & shortens fatigue crack initiation.

Fatigue crack growth rate can be similarly accelerated, or retarded by crack closure effects (build-up of corrosion products on the crack faces).

A tensile mean stress holds crack open, allows atmospheric attack & reduces crack closure. Hence faster crack initiation and growth.

[General comment: many answers were glib and did not address the specific question.]

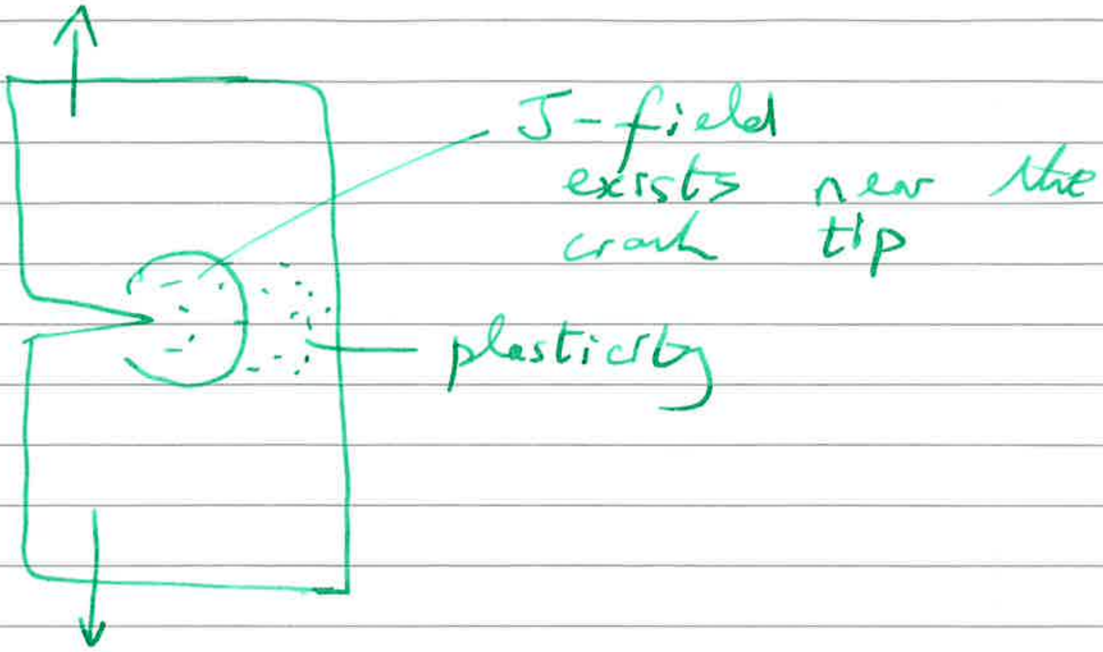
Q3. Examiner's Comment:

An unpopular question that probed understanding of various aspects of fatigue. A very wide range in quality of answer was provided. It appears that several candidates did not attend the lectures, as this material was emphasized during the course. Many answers were superficial and did not address the questions being asked.

Q4. Examiner's Comment:
A popular question. The physical basis of J and its relevance were well understood, but mentioned the HRR field. The non-linear beam calculation was understood in broad terms, although several candidates used the linear elastic beam bending formulae for a non-linear beam.

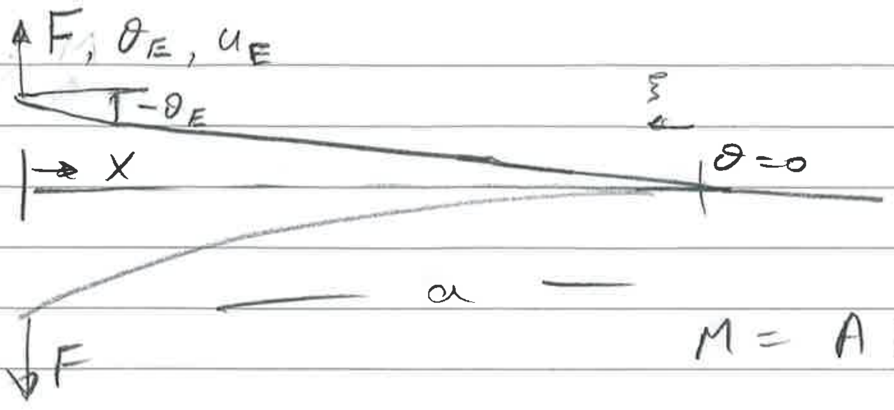
C 11

4. 1(a)



For such a specimen, no K-field exists.

Q4. (b)



$$M = A K^N \frac{K}{|K|}$$

$$M = F.x \Rightarrow K = \left(\frac{Fx}{A}\right)^{1/N} = \frac{d\theta}{dx}$$

$$\theta(x) = \left(\frac{F}{A}\right)^{1/N} \frac{N}{N+1} x^{\frac{1+N}{N}} - \theta_E$$

Now $\theta(a) = 0 \Rightarrow \theta_E = \left(\frac{F}{A}\right)^{1/N} \frac{N}{N+1} a^{\frac{1+N}{N}}$

$$\frac{du}{dx} = \theta$$

$$\theta = \frac{du}{dx} = \left(\frac{F}{A}\right)^{1/N} \frac{N}{N+1} \left[x^{\frac{N+1}{N}} - a^{\frac{N+1}{N}} \right]$$

$$u(x) = \left(\frac{F}{A}\right)^{1/N} \frac{N}{N+1} \left[\frac{N}{2N+1} x^{\frac{2N+1}{N}} - a^{\frac{N+1}{N}} x + C \right]$$

At $x = a$: $u(x) = 0 \Rightarrow C = a^{\frac{2N+1}{N}}\left(1 - \frac{N}{2N+1}\right)$

$x = 0$: $u = u_E$

$$C = \frac{N+1}{2N+1} a^{\frac{2N+1}{N}}$$

$$\Rightarrow u(x) = \left(\frac{F}{A}\right)^{\frac{1}{N}} \frac{N}{N+1} \left[\frac{N}{2N+1} x^{\frac{2N+1}{N}} - x a^{\frac{N+1}{N}} + \frac{N+1}{2N+1} a^{\frac{2N+1}{N}} \right]$$

$$\Rightarrow u_E = \left(\frac{F}{A}\right)^{\frac{1}{N}} \frac{N}{2N+1} a^{\frac{2N+1}{N}}$$

Q4.(b) contd.

$$J = -\left.\frac{\partial \Psi}{\partial a}\right|_F$$

$$\Psi = 2\int_0^F F(u_E')\,du_E' - 2F.u_E$$

Now $F(u_E) = A.\left[\frac{2N+1}{N} a^{-\left(\frac{2N+1}{N}\right)} u_E\right]^N$

$$\int F\,du = (\ )\,u_E^{N+1}\frac{1}{N+1}$$

So, $2\int_0^F F(u_E')\,du_E' = 2A\left[\frac{2N+1}{N} a^{-\left(\frac{2N+1}{N}\right)}\right]^N \frac{1}{N+1} u_E^{N+1}$

$$2F.u_E = 2A\left[\frac{2N+1}{N} a^{-\left(\frac{2N+1}{N}\right)}\right]^N u_E^{N+1}$$

$$\Rightarrow \Psi = -2A\left[\left(\frac{2N+1}{N}\right) a^{-\left(\frac{2N+1}{N}\right)}\right]^N \frac{N}{N+1} u_E^{N+1}$$

$$u_E^{N+1} = \left(\frac{F}{A}\right)^{\frac{N+1}{N}}\left[\frac{N}{2N+1} a^{\frac{2N+1}{N}}\right]^{N+1}$$

$$\Rightarrow \Psi = -2A\frac{N}{N+1} F^{\frac{N+1}{N}} A^{-\frac{N-1}{N}} \left(\frac{2N+1}{N}\right)^{\cancel{N-N-1}}$$

$$\times\ \cancel{a^{-(2N+1)}} . \cancel{a^{(2N+1)}}\ a^{\frac{2N+1}{N}}$$

$$\Rightarrow \Psi = -2\frac{N^2}{(N+1)(2N+1)} F^{\frac{N+1}{N}} A^{-\frac{1}{N}} a^{\frac{2N+1}{N}}$$

$$J = -\frac{\partial \Psi}{\partial a} = 2\frac{N}{N+1} F^{\frac{N+1}{N}} A^{-\frac{1}{N}} a^{\frac{N+1}{N}}$$

At $J = J_{Ic}$, $F = F_c$

[Comment: many candidates treated the beam as linear elastic.]