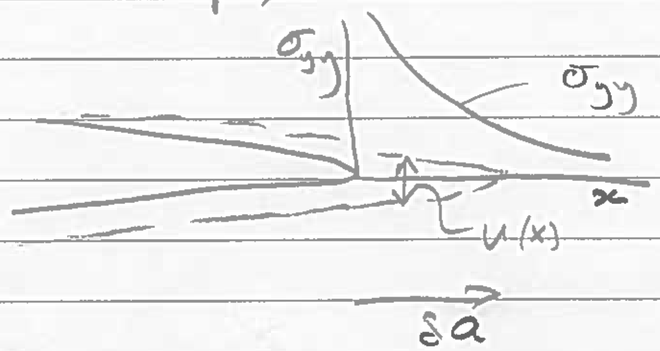
3C9, 2016 crib.

1. (a) Irwin relation : $K^2 = EG$

G = energy release per unit area of crack advance.

Suppose a K-field exists at the crack tip; advance the crack by δa.



Proof

crack opening is $u(x) \sim \frac{K}{E}\sqrt{\delta a - x}$ for $x < \delta a$

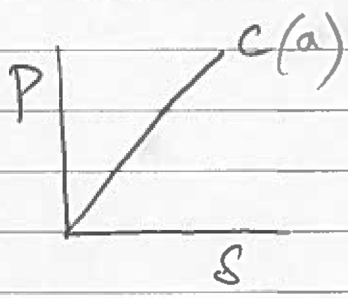
Energy release $= G\,\delta a = \frac{1}{2}\int_0^{\delta a} dx \; u(x)\,\sigma_{yy}(x)$

$$\simeq \frac{K^2}{E}\,\delta a$$

G is an energy criterion for fracture; K is a stress-based criterion.

Fracture at $G = G_c \Leftrightarrow K = K_c$

(b) (i)



Potential energy is

$\Psi = W - P.u$

stored elastic energy

$G B \delta a = -\delta\Psi = -\frac{\partial \Psi}{\partial a}\Big|_P$ $\quad u = CP$

(b) contd.

Now, $W = \frac{1}{2} P u \Rightarrow \Psi = -\frac{1}{2} P u$

$$\Rightarrow \quad \Psi = -\tfrac{1}{2} C P^2$$

$$\Rightarrow \quad G = -\frac{\partial \Psi}{B \partial a} = \frac{P^2}{2B} \frac{\partial C}{\partial a}.$$

(ii)

$$u = u_1 + u_2$$

$$u_1 = \frac{P a^3}{3 E I_1} \qquad u_2 = \frac{P a^3}{3 E I_2}$$

$$I_1 = \tfrac{1}{12} B h_1^3 \qquad I_2 = \tfrac{1}{12} B h_2^3$$

$$\Rightarrow \quad u = \frac{4 P a^3}{E B} \left(\frac{1}{h_1^3} + \frac{1}{h_2^3} \right) \quad \Rightarrow \quad C = \frac{u}{P} = \frac{4 a^3}{E h^3 B}$$

where $\frac{1}{h^3} = \frac{1}{h_1^3} + \frac{1}{h_2^3} = \frac{h_1^3 + h_2^3}{h_1^3 h_2^3} \Rightarrow h = \frac{h_1 h_2}{(h_1^3 + h_2^3)^{1/3}}$

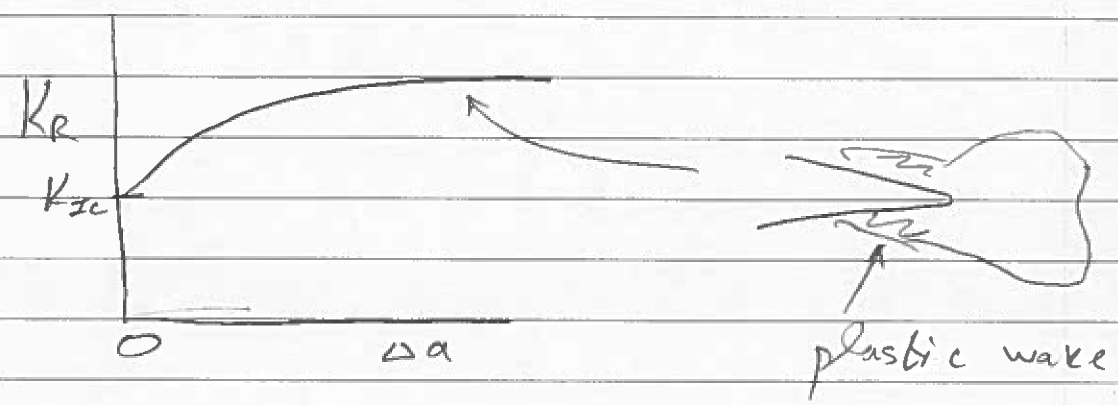
$$G = \frac{1}{2} \frac{P^2}{B} \frac{\partial C}{\partial a} = \frac{1}{2} \frac{P^2}{B} \frac{12 a^2}{E B h^3}$$

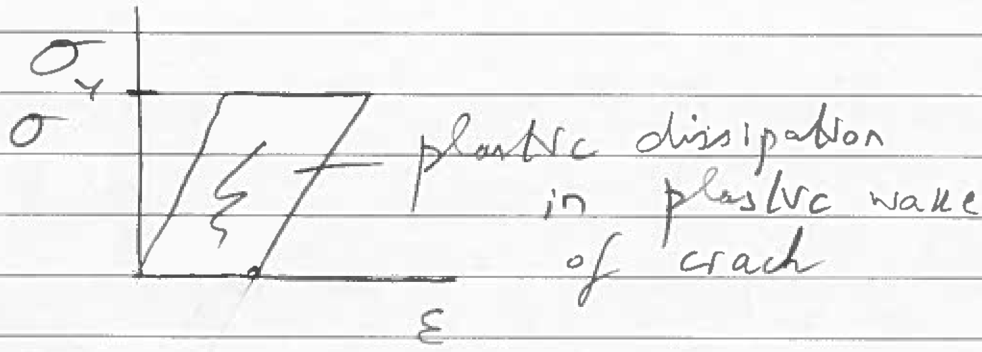
$$\Rightarrow \quad G = \frac{6 P^2 a^2}{E B^2 h^3} \quad \Rightarrow \quad P_c^2 = \frac{G_c E B^2 h^3}{6 a^2}$$

$$\Rightarrow \quad P_c = \frac{\sqrt{E G_c}}{\sqrt{6}} \frac{B h \sqrt{h}}{a}$$

(iii) Mixed mode I & II.

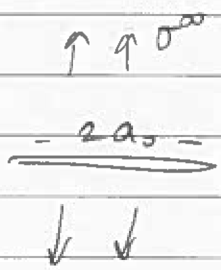
2. (a)





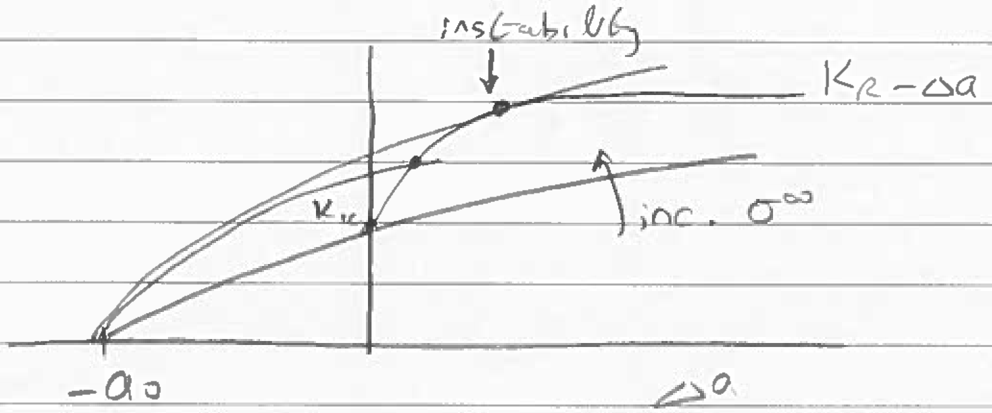
Consider a crack of initial length $2a_0$.

$$a = a_0 + \Delta a$$



$$K \approx \sigma^\infty \sqrt{\pi(a_0 + \Delta a)}$$

$$K_R = K_R(\Delta a)$$



At onset of instability : $K = K_R$

and $\dfrac{\partial K}{\partial a} = \dfrac{\partial K_R}{\partial \Delta a}$

(b) (i)

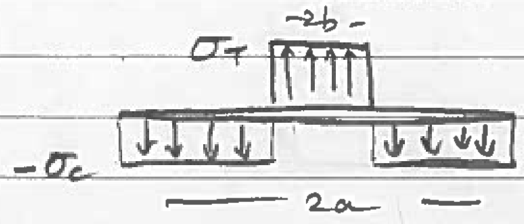
2. $K = \sigma^{\infty}\sqrt{\pi a}$ where $\sigma^{\infty} = 100\,\text{MPa}$, $a = 10\,\text{mm}$

$\Rightarrow K = \underline{17.7\ \text{MPa}\sqrt{\text{m}}}$

(ii) For $0 \le a \le 30\,\text{mm}$ $\Rightarrow K = \sigma^{\infty}\sqrt{\pi a}$

$\sigma^{\infty} = 100\,\text{MPa}$ At $\underline{a = 30\,\text{mm}}$ $\Rightarrow K = \underline{30.7\ \text{MPa}\sqrt{\text{m}}}$

For $30 \le a \le 90\,\text{mm}$ write $b = 30\,\text{mm}$

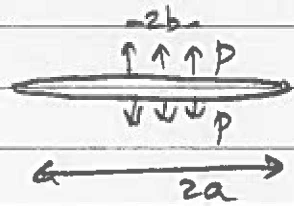


$\Rightarrow p = (\sigma_T + \sigma_c)$ over $|x| < b$

$+$

$p = -\sigma_c$ over $|x| < a$

Make use of



$K = 2p\sqrt{\dfrac{a}{\pi}}\sin^{-1}\left(\dfrac{b}{a}\right)$ from data sheet.

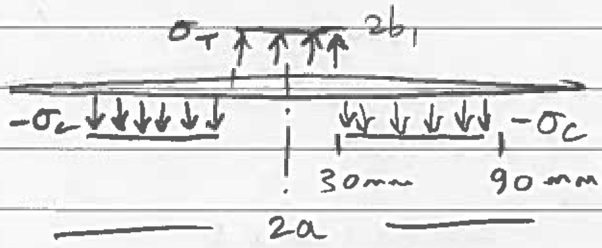
$$\Rightarrow K = 2(\sigma_T + \sigma_c)\sqrt{\frac{a}{\pi}}\sin^{-1}\left(\frac{b}{a}\right) - \sigma_c\sqrt{\pi a}$$

where $\sigma_T = 100\,\text{MPa}$ $\sigma_c = 50\,\text{MPa}$ $\underline{a = 90\,\text{mm}}$ $b = 30\,\text{mm}$

$\Rightarrow K = 17.3\ \text{MPa}\sqrt{\text{m}} - 26.6\ \text{MPa}\sqrt{\text{m}} = \underline{-9.3\ \text{MPa}\sqrt{\text{m}}}$

For $90\,\text{mm} \le a \le 100\,\text{mm}$: write $b_1 = 30\,\text{mm}$ $b_2 = 90\,\text{mm}$

$\sigma_T = 100\,\text{MPa}$, $\sigma_c = 50\,\text{MPa}$



$\Rightarrow p = -\sigma_c$ over $|x| < b_2$

$+$

$p = (\sigma_T + \sigma_c)$ over $|x| < b_1$

So $K = 2(\sigma_T + \sigma_c)\sqrt{\dfrac{a}{\pi}}\sin^{-1}\left(\dfrac{b_1}{a}\right) - 2\sigma_c\sqrt{\dfrac{a}{\pi}}\sin^{-1}\left(\dfrac{b_2}{a}\right)$

$a = 100\,\text{mm}$ $\Rightarrow K = 16.3 - 20.0 = \underline{-3.7\ \text{MPa}\sqrt{\text{m}}}$

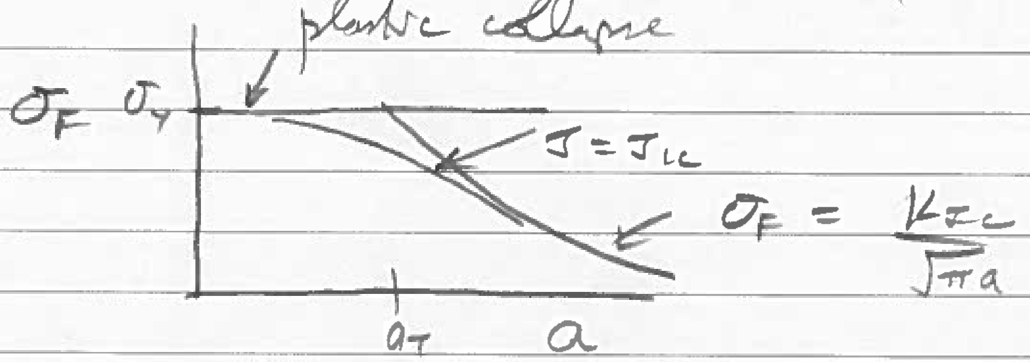
2. (b) (iii)

Can use linear superposition provided stresses are below yield.

Note; if $K_{TOTAL} < 0$ put $K_{TOTAL} = 0$.

3. (a) cracks are short in weldments, below the transition flaw size a_T

Use LEFM for $a > 2.5\left(\frac{K_{IC}}{\sigma_Y}\right)^2$



Fracture at $J = J_{IC}$

$J \sim \sigma^{\infty} \varepsilon^{\infty} a$

so can have bulk plasticity prior to failure.

(b) Metals yield at $\sigma_Y <<$ ideal cohesive strength $E/20$, by dislocation flow.

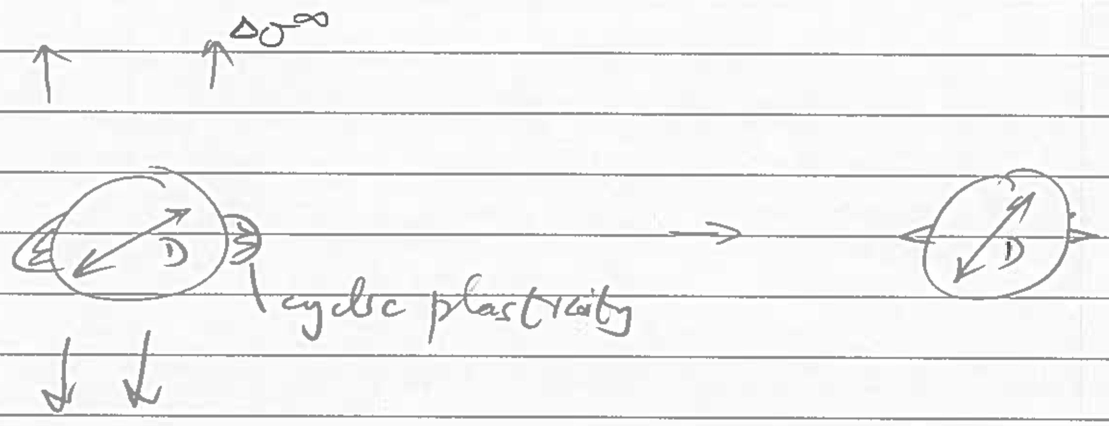
For glass $G_c \simeq 2\gamma_s$ ← surface energy

For metals, yielding occurs at the crack tip & cracking is by ductile microvoid coalescence.

$\sigma \varepsilon \sim \frac{J}{r}$ ahead of crack tip with $\varepsilon \sim 1$

$\Rightarrow J_c \sim \sigma_Y l$ ← ~~inclusion spacing~~

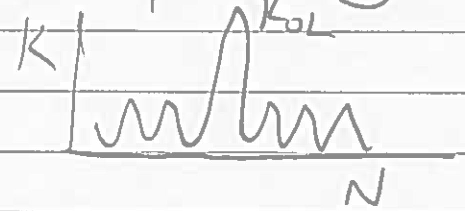
3 (c) cracks initiate by low cycle fatigue in a cyclic plastic zone at a notch root.

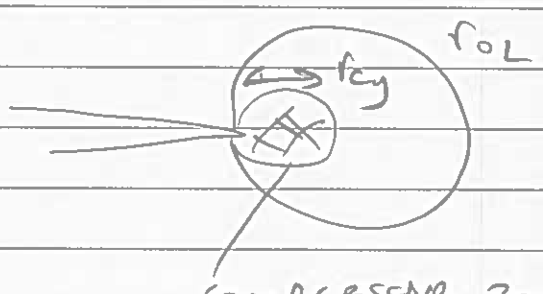


The cracks will continue to grow if $\Delta\sigma^\infty \sqrt{\pi D} > \Delta K_{TH}$

So if $\Delta\sigma^\infty < \frac{\Delta K_{TH}}{\sqrt{\pi D}}$ the cracks will arrest

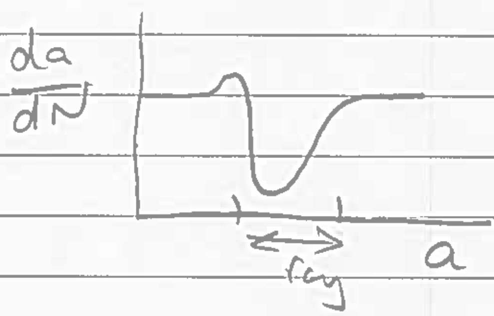
(d)
overloads retard fatigue crack growth via plasticity induced crack closure.





$r_{OL} \sim \frac{1}{\pi} \frac{K_{OL}^2}{\sigma_Y^2}$

compressive zone
$\Rightarrow$ cracks close



4. (a) $\Delta \varepsilon^{pl} N_f^{\beta} = C_2$

$$\Rightarrow \log_{10} \Delta\varepsilon^{pl} + \beta \log_{10} N_f = \log_{10} C_2$$

$$\log_{10} 0.01 + \beta \log_{10} 545 = \log_{10} C_2$$

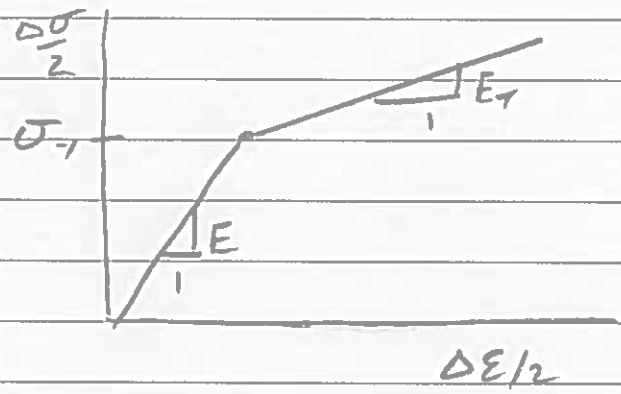
and

$$\log_{10} 0.005 + \beta \log_{10} 2000 = \log_{10} C_2$$

subtract : $\beta \log_{10} \dfrac{2000}{545} = 0.301$

$$\Rightarrow \quad \beta = 0.533 \quad \text{and} \quad C_2 = 0.287$$

(b)



$$k_\sigma = \frac{\sigma}{\sigma^\infty} \qquad k_\varepsilon = \frac{\varepsilon}{\varepsilon^\infty}$$

$$k_\sigma k_\varepsilon = k_T^2 \Rightarrow \frac{\Delta\sigma}{2\sigma^\infty} \frac{\Delta\varepsilon}{2\varepsilon^\infty} = 9$$

$$\Rightarrow \frac{\Delta\sigma}{2} \cdot \frac{\Delta\varepsilon}{2} = \frac{9 \sigma^{\infty 2}}{E}$$

Value of σ^∞ for yield ?

$$\frac{\Delta\sigma}{2} = \sigma_Y \qquad \frac{\Delta\varepsilon}{2} = \frac{\sigma_Y}{E} \Rightarrow \sigma_Y^\infty = \frac{\sigma_Y}{3} = \frac{280 \text{ MPa}}{3}$$

4. (b) contd.

So, for $\sigma^\infty = \frac{\Delta\sigma}{2} = 250$ MPa

$\Rightarrow$ yield at the notch root

$$\frac{\Delta\sigma}{2} = \sigma_Y + E_T\left(\frac{\Delta\varepsilon}{2} - \frac{\sigma_Y}{E}\right)$$

and $$\frac{\Delta\sigma}{2}\frac{\Delta\varepsilon}{2} = \frac{9\,\sigma^{\infty 2}}{E}$$

Write $\sigma_Y' = \sigma_Y - \frac{E_T}{E}\sigma_Y = \left(\frac{E - E_T}{E}\right).\sigma_Y$

Then, $$\left(\sigma_Y' + E_T\frac{\Delta\varepsilon}{2}\right)\frac{\Delta\varepsilon}{2} = \frac{9\,\sigma^{\infty 2}}{E}$$

$$\Rightarrow \left(\frac{\Delta\varepsilon}{2}\right)^2 + \frac{\sigma_Y'}{E_T}\frac{\Delta\varepsilon}{2} = \frac{9\,\sigma^{\infty 2}}{E E_T}$$

$$\Rightarrow \left(\frac{\Delta\varepsilon}{2} + \frac{\sigma_Y'}{2E_T}\right)^2 = \frac{9\,\sigma^{\infty 2}}{E E_T} + \left(\frac{\sigma_Y'}{2E_T}\right)^2$$

Now put in values : $\sigma_Y' = 280 - \frac{7}{70}280 = 252$ MPa

$$\left(\frac{\Delta\varepsilon}{2} + \frac{252}{2 \times 7{,}000}\right)^2 = \frac{9\;250^2}{70{,}000 \times 7{,}000} + \left(\frac{252}{2 \times 7{,}000}\right)^2$$

$$\Rightarrow \frac{\Delta\varepsilon}{2} = 0.0204$$

$$\Rightarrow \frac{\Delta\sigma}{2} = 280 + 7{,}000\,(0.0204 - 0.004) \text{ MPa}$$
$$= 395 \text{ MPa}$$

$$\frac{\Delta\varepsilon^{PL}}{2} = \frac{\Delta\varepsilon}{2} - \frac{\Delta\sigma}{2E} = 0.0204 - 0.0056 = 0.01476$$

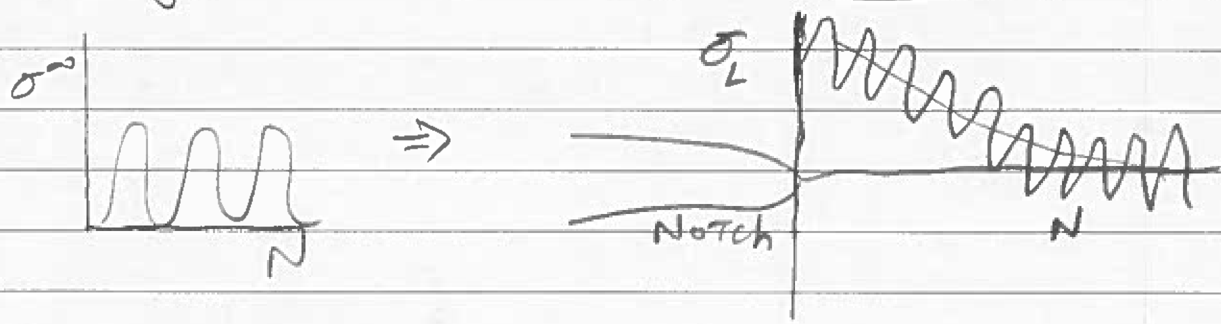
4. (b) contd.

$$\Delta\varepsilon^{PL} N_f^{\beta} = C_2$$

$$\Rightarrow \quad N_f^{0.533} = \frac{C_2}{\Delta\varepsilon^{PL}} = \frac{0.287}{2 \times 0.01476}$$

$$\Rightarrow \quad N_f = \underline{71 \text{ cycles}}.$$

(c) cyclic creep relaxes mean stress



Neuber's rule does not have this degree of sophistication.