Crib for Engineering Tripos Part IIA

Module 3C9 : Fracture Mechanics of Materials and Structures

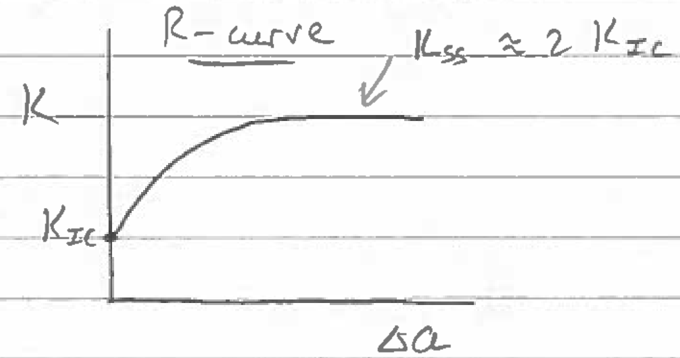
Exam on 5 May 2017.

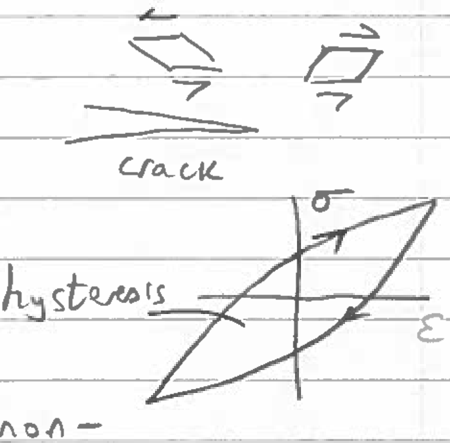
Q1. (a) When small scale yielding applies, the crack tip plastic zone is embedded within an outer K-field. In the K-field, the stress σ and strain ε scale as $K r^{-1/2}$ with radius r from the crack tip. Thus K gives the intensity of crack tip loading.

Small scale yielding applies when the crack length a, and ligament size $(W-a)$ satisfy

$$a,\ (W-a) > 2.5 \frac{K^2}{\sigma_Y^2}$$

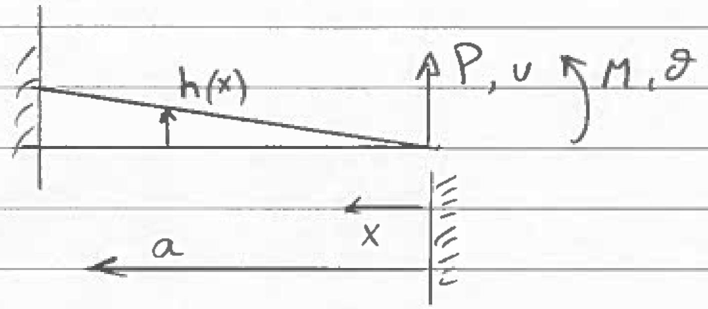
where W = specimen width and σ_Y = yield strength of the solid.





Plastic dissipation due to non-proportional loading at the crack tip occurs with crack advance, and K increases with Δa.

1. (b) (i)



Write $y(x)$ as the upward transverse deflection of the beam. $y(x)$ is related to the bending moment at a typical section by

$$EI\,y'' = m(x) = P.x + M$$

where $I = \frac{1}{12} B h^3 = \frac{1}{12} B \alpha^3 x^3$

So $\frac{\alpha^3}{12} BE y'' = P x^{-2} + M x^{-3}$

$\Rightarrow \quad \frac{\alpha^3}{12} BE y' = -P x^{-1} - \frac{1}{2} M x^{-2} + c_1$

$y' = 0$ at $x = a$ and $y' = -\theta$ at $x = 0$

$\Rightarrow \quad \frac{\alpha^3}{12} BE y' = P(a^{-1} - x^{-1}) + \frac{1}{2} M (a^{-2} - x^{-2})$

So $\theta \to \infty$ as $x \to 0$

$$\frac{\alpha^3}{12} BE y = P\left(\frac{x}{a} - \ln x\right) + \frac{1}{2} M\left(\frac{x}{a^2} + \frac{1}{x}\right) + C$$

Now $y = 0$ at $x = a \Rightarrow$

$$\frac{\alpha^3}{12} BE y = P\left(\frac{x}{a} - \ln\frac{x}{a} - 1\right) + \frac{1}{2} M\left(\frac{x}{a^2} + \frac{1}{x} - \frac{2}{a}\right)$$

Again, $y \to \infty$ as $x \to 0$.

$u = y(0) = \infty$.

$-\frac{1}{2}\Psi = \frac{1}{2} P u + \frac{1}{2} M \theta \quad \Rightarrow -\Psi = Pu + M\theta$

$G = -\frac{1}{B} \frac{\partial \Psi}{\partial a}$

1. (b) (i) contd.

To proceed, take $x = \varepsilon \to 0$

Then, $$\frac{\alpha^3}{12} BE\theta = \frac{P}{\varepsilon} + \frac{1}{2}\frac{M}{\varepsilon^2} - \frac{P}{a} - \frac{1}{2}\frac{M}{a^2}$$

and $$\frac{\alpha^3}{12} BEu = -P\ln\varepsilon + \frac{1}{2}\frac{M}{\varepsilon} + P\left(\frac{\varepsilon}{a} + \ln a - 1\right) + \frac{1}{2}M\left(\frac{\varepsilon}{a^2} - \frac{2}{a}\right)$$

$$-\Psi = Pu + M\theta \quad \text{and} \quad G = -\frac{1}{B}\frac{\partial\Psi}{\partial a}$$

$$\Rightarrow -\Psi = \frac{12}{\alpha^3 BE}\left[-P^2\ln\varepsilon + \frac{1}{2}\frac{PM}{\varepsilon} + P^2\left(\frac{\varepsilon}{a} + \ln a - 1\right) + \frac{1}{2}PM\left(\frac{\varepsilon}{a^2} - \frac{2}{a}\right) + \frac{MP}{\varepsilon} + \frac{1}{2}\frac{M^2}{\varepsilon^2} - \frac{PM}{a} - \frac{1}{2}\frac{M^2}{a^2}\right]$$

$$\Rightarrow G = \frac{12}{\alpha^3 B^2 E}\left[\frac{-\varepsilon P^2}{a^2} + \frac{P^2}{a} + \frac{1}{2}PM\left(\frac{-2\varepsilon}{a^3} + \frac{2}{a^2}\right) + \frac{PM}{a^2} + \frac{M^2}{a^3}\right]$$

Now let $\varepsilon \to 0$ and

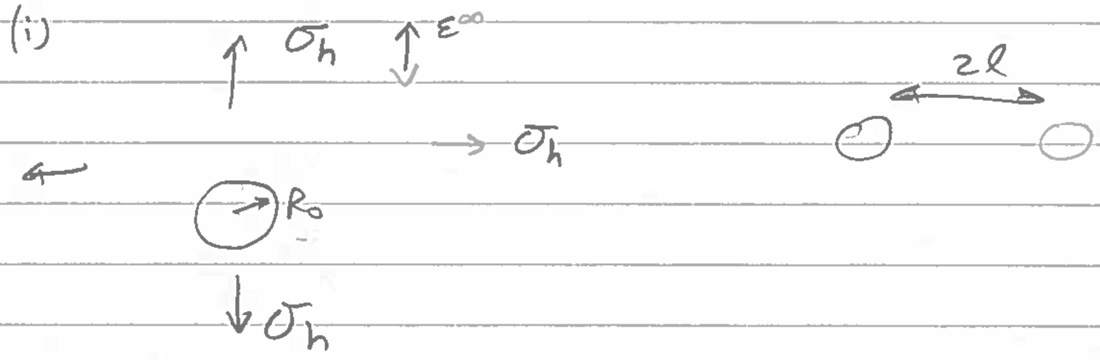
$$G = \frac{12}{\alpha^3 B^2 E}\left[\frac{P^2}{a} + \frac{2PM}{a^2} + \frac{M^2}{a^3}\right]$$

(ii) $\Psi \to \Psi/2$ and $G \to G/2$.

The lower arm is not loaded, and the crack tip is now in mixed mode loading.

<u>Note</u> Part (b)(i) is challenging and candidates were given high marks for following the standard method as given in lecture notes, without considering a finite value of ε.

2. (a) (i)



The void grows with increasing ε^∞ such that
$R / R_0 = \varepsilon^\infty \exp(3\sigma_h / 2\sigma_Y)$

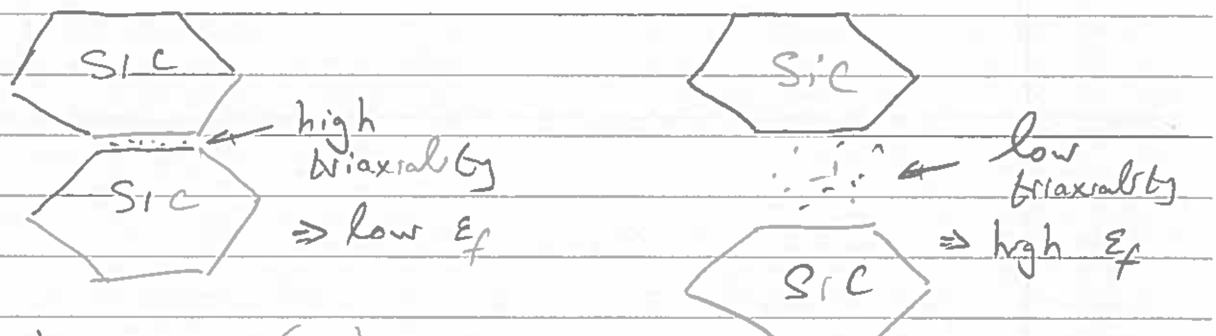
Assume a void spacing of 2ℓ. Then, the void volume fraction f is $f = (R_0/\ell)^2$. Assume that the voids coalesce with $R = \ell$.

Hence, $$\frac{\ell}{R_0} = \varepsilon_f \exp(3\sigma_h / 2\sigma_Y)$$

$$\Rightarrow \quad \varepsilon_f = \frac{1}{\sqrt{f}} \exp(-3\sigma_h / 2\sigma_Y)$$

(i) Assume that failure is by microvoid coalescence. In plane strain, the hydrostatic stress σ_h is of order $\sigma_h \approx 3\sigma_Y$. This promotes void growth and reduces ε_f. No such elevation in hydrostatic stress occurs in a thin sheet.

(ii)



high ceramic (SiC) content

low ceramic content

2. (b) (i) $\varepsilon = \frac{u}{a}$ $\qquad P = \pi R^2 \sigma$ where $R = D/2$

$$= \frac{\pi}{4} D^2 \sigma$$

$$\sigma = A\varepsilon^N \Rightarrow P = \frac{\pi}{4} D^2 A \left(\frac{u}{a}\right)^N$$

(ii) $$W = \int_0^u P\, du = \frac{\pi}{4}\frac{D^2 A}{a^N}\int_0^u u^N du$$

$$\Rightarrow W = \frac{\pi}{4}\frac{D^2 A}{a^N}\frac{u^{N+1}}{N+1}$$

(iii) $$\pi D J = -\left.\frac{\partial W}{\partial a}\right|_u = \frac{\pi}{4}\frac{D^2 A\, u^{N+1}}{(N+1)} N a^{-(N+1)}$$

$$\Rightarrow J = \frac{1}{4}\frac{N}{N+1} DA \left(\frac{u}{a}\right)^{N+1}$$

Now $$\left(\frac{u}{a}\right)^N = \frac{4}{\pi}\frac{P}{D^2 A}$$

$$\Rightarrow J = \frac{1}{4}\frac{N}{N+1} DA \left(\frac{4}{\pi}\frac{P}{D^2 A}\right)^{\frac{N+1}{N}}$$

$$\Rightarrow \frac{4}{\pi}\frac{P_c}{D^2 A} = \left[4\left(\frac{N+1}{N}\right)\left(\frac{J_{Ic}}{DA}\right)\right]^{\frac{N}{N+1}}$$

$$\Rightarrow P_c = \frac{\pi}{4}\, 4^{\frac{N}{N+1}} \left(\frac{N+1}{N}\right)^{\frac{N}{N+1}} \frac{D^2 A}{(DA)^{N/(N+1)}} J_{Ic}^{\frac{N}{N+1}}$$

2b (iii) contd.

$$\Rightarrow \quad P_c = \pi \; 4^{-\frac{1}{N+1}} \left(\frac{N+1}{N}\right)^{\frac{N}{N+1}} D^{\frac{N+2}{N+1}} A^{\frac{1}{N+1}} J_{Ic}^{\frac{N}{N+1}}$$

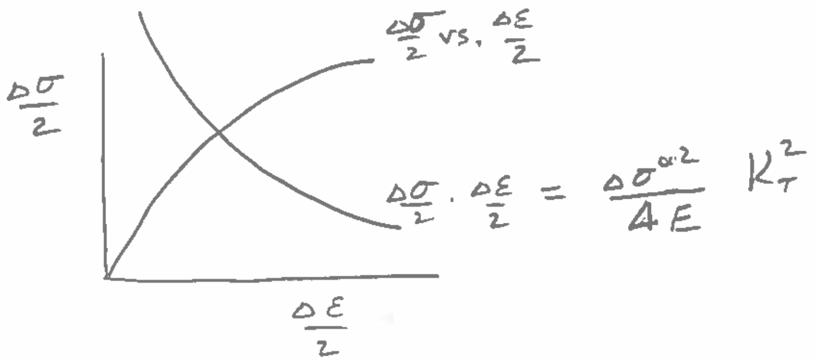
3. (a) A cold shock induces transient tension in the surface layers and compression in the core (to give no net force on any section). Tension is induced parallel to the surface and cracks can channel in this tensile zone but will not penetrate into the compression zone. A hot shock has the reverse effect.

(b) Oxide, roughness and plasticity induced crack closure are all active near threshold and consequently the stress intensity range must increase to give crack growth at small R.

(c) Residual stress of tensile yield magnitude exists, and this any pre-existing crack is held open. Once the crack grows beyond this zone of residual tension, the crack will close and not grow further.

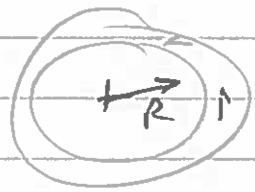
(d) Wear removes short fatigue cracks and so they need to re-nucleate. Need to ensure that the wear rate is optimal to get maximum life.

4. (a) Neuber's rule states that the product of stress and strain at a notch root is the same for a plastic state as for an elastic state. It is an approximation, and detailed finite element calculations are more accurate. No account is taken of mean stress effects are made but mean stress relaxation will occur in a cyclic plastic zone at the notch root, so mean stress effects may not be very important for the prediction of crack initiation.



Then use Coffin – Manson law to get N_f from $\Delta\varepsilon^{pl}$.

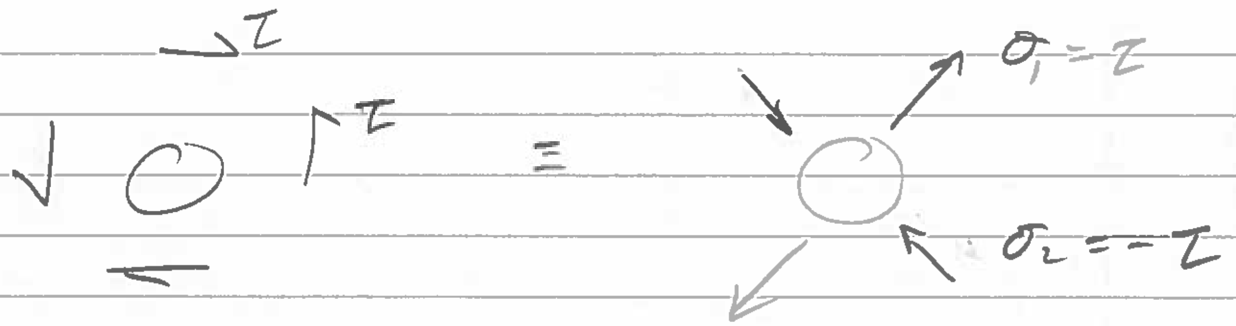
4 (b) (i)

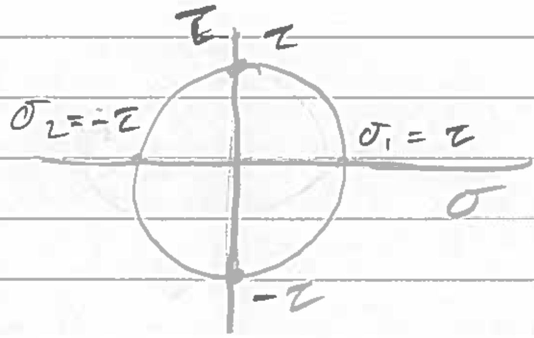


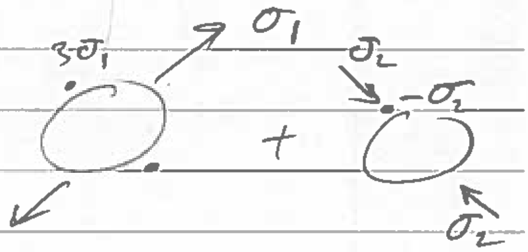
$$Q = \tau \cdot 2\pi R^2 t$$

Thin wall, so uniform τ.

(ii)







So local stress $= 3\sigma_1 - \sigma_2$
$= 4\tau$

(iii)

$$\sigma_{max} = \frac{4\,Q_{max}}{2\pi R^2 t} \qquad \sigma_{min} = 0$$

$\Delta\sigma = \sigma_{max}$

$\sigma_m = \frac{1}{2}\sigma_{max}$ $\qquad \Delta\sigma = \Delta\sigma_0\left(1 - \dfrac{\sigma_m}{\sigma_{ts}}\right)$

$$\Rightarrow \Delta\sigma_0 = \Delta\sigma\left(1 - \frac{\sigma_m}{\sigma_{ts}}\right)^{-1} = \sigma_{max}\left(1 - \frac{\sigma_{max}}{2\sigma_{ts}}\right)^{-1}$$

Then apply Basquin:

$$\Delta\sigma_0 N_f^{\alpha} = C_1 \Rightarrow N_f.$$