Engineering Part II A 2018 - 19

Paper 3C9 : Fracture Mechanics of Materials & Structures

CRIB

(a)

1. Energy release rate $\mathcal{G}$ at a crack tip is ~~the~~ energy released per unit area of crack advance. It is the crack driving force for growth.

Write Ψ = Potential energy of body

$$\Psi = \underbrace{W}_{\text{stored internal strain energy}} - \underbrace{P^{o}}_{\text{prescribed applied load}} \underbrace{u}_{\text{displacement}}$$

Then $$\mathcal{G} = -\frac{1}{B}\frac{\partial \Psi}{\partial a}$$

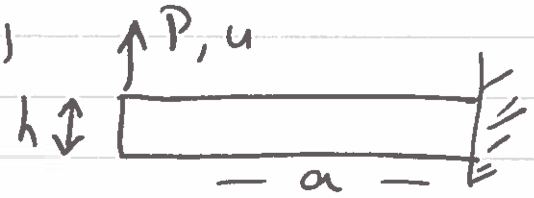
B: thickness; a: crack length a

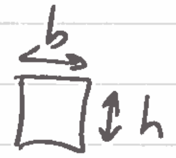
If displacements, rather than loads, are prescribed then $\Psi = W$.

Energy ejected to crack tip $= \mathcal{G}.\delta a$

Crack growth occurs if $\mathcal{G} > \mathcal{G}_{Ic}$ ← toughness

1. b) (i)





For beam : $u = \dfrac{Pa^3}{3EI}$ $\quad I = \frac{1}{12} bh^3$

where b = thickness of beam (into page).

Compliance of beam arm is

$$C = \frac{u}{P} = \frac{a^3}{3EI}$$

(ii)

Now insert a wedge between the 2 arms. The end load P on the 2 arms is the same. Then,

$$\delta = u_1 + u_2 = \frac{Pa^3}{3EI_1} + \frac{Pa^3}{3EI_2}$$

where $I_1 = \frac{1}{12} bh_1^3$ and $I_2 = \frac{1}{12} bh_2^3$

Thus $\quad \delta = \dfrac{4Pa^3}{Eb} \left(\dfrac{1}{h_1^3} + \dfrac{1}{h_2^3} \right)$

ie. $\quad \delta = \dfrac{4Pa^3}{Eb} \cdot \dfrac{(h_1^3 + h_2^3)}{(h_1 h_2)^3}$

Energy release rate $\quad \mathcal{G} = \dfrac{1}{2b} P^2 \dfrac{\partial C}{\partial a}$

Here, $\quad C = \dfrac{\delta}{P} = \dfrac{4a^3}{Eb} \dfrac{(h_1^3 + h_2^3)}{(h_1 h_2)^3}$

1.(b) (ii) contd.

Thus, $$\mathcal{G} = \frac{1}{2} P^2 \frac{12 a^2}{E b^2} \frac{(h_1^3 + h_2^3)}{(h_1 h_2)^3}$$

$$\Rightarrow \quad \mathcal{G} = 6 \frac{P^2 a^2}{E b^2} \frac{(h_1^3 + h_2^3)}{(h_1 h_2)^3}$$

Now eliminate P by noting that

$$P = \frac{\delta E b}{4 a^3} \frac{(h_1 h_2)^3}{(h_1^3 + h_2^3)}$$

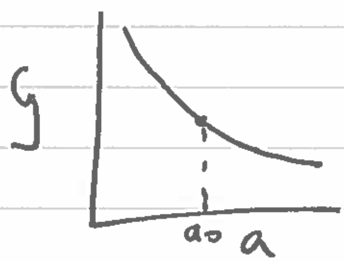
Thus, $$\mathcal{G} = \frac{6 a^2}{E} \frac{\delta^2 E^2}{16 a^6} \frac{(h_1 h_2)^3}{(h_1^3 + h_2^3)}$$

So, $$\mathcal{G} = \frac{3}{8} \frac{\delta^2 E}{a^4} \cdot \frac{(h_1 h_2)^3}{(h_1^3 + h_2^3)}$$

(iii) Assume that $\mathcal{G}_{IC}$ = constant,

i.e. the R - curve is flat.

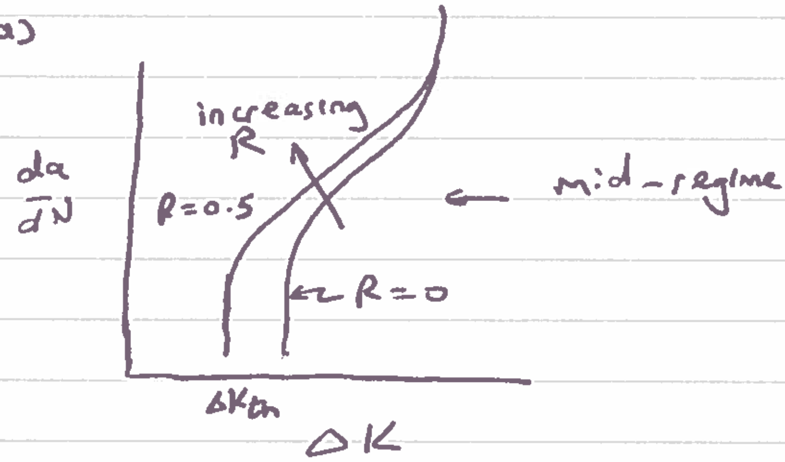
Note that, for δ held constant, $\mathcal{G}$ drops sharply with increasing a, since $\mathcal{G} \propto \frac{\delta^2}{a^4}$.



Hence, crack growth is STABLE.

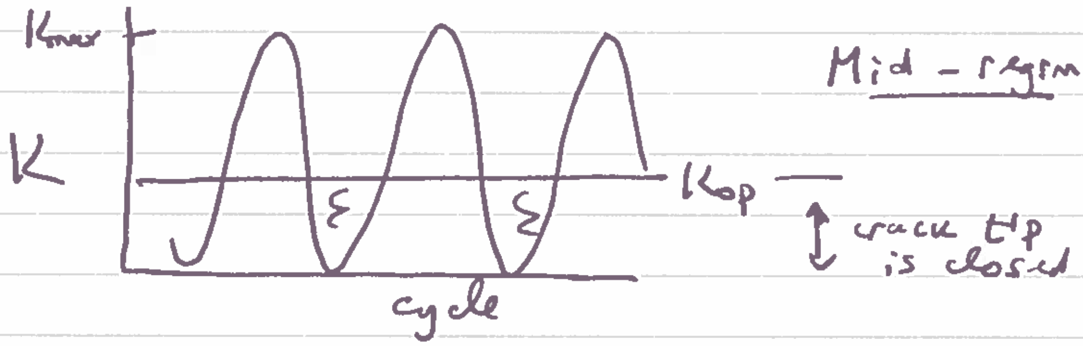
Crib for 3C9, 2018-19

2. (a)

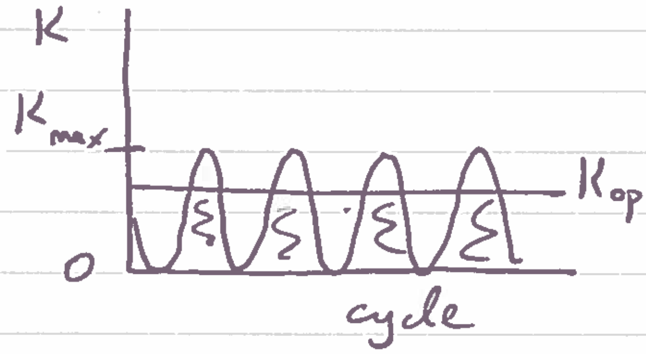


In mid-regime of Paris plot, plasticity-induced crack closure results in a crack opening value of K, termed K_{op} of the order of $K_{op} = 0.3\,K_{max}$ where K_{max} = max. stress intensity of cycle.

Mid-regime



Near threshold



Near threshold, other closure mechanisms are also active such as oxide-induced crack closure and roughness-induced crack closure. This raises K_{op}.

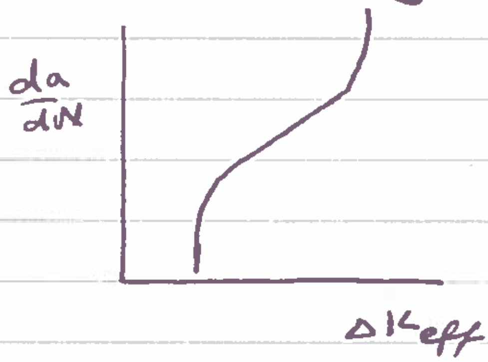
Now, the value of K_{op} is not sensitive to K_{min}.

2. (a) contd.

The effective stress intensity range

$$\Delta K_{eff} = K_{max} - K_{op} \quad \text{for} \quad K_{op} > K_{min}$$
$$= K_{max} - K_{min} \quad \text{for} \quad K_{op} < K_{min}$$

The relation $\frac{da}{dN} = f(\Delta K_{eff})$ is unique for a given material.

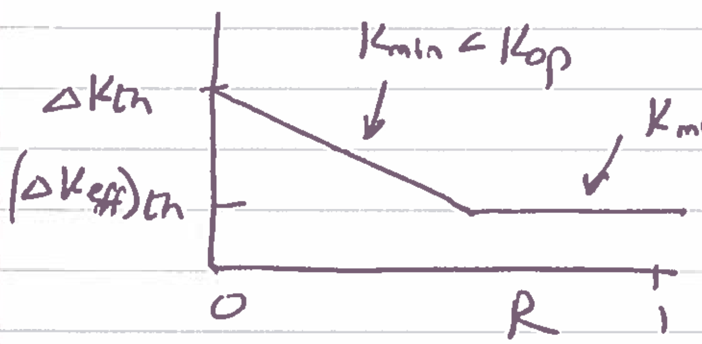


Near threshold, $\Delta K \approx \Delta K_{th} = K_{max} - K_{min}$

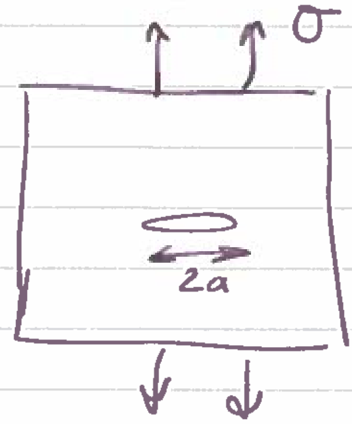
Assume $K_{op} \approx$ constant

Then $(\Delta K_{eff})_{th} = K_{max} - K_{op}$
$$= \frac{\Delta K_{th}}{1-R} - K_{op}$$

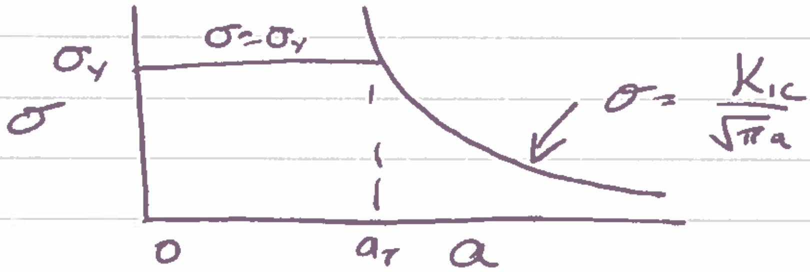
Thus $\Delta K_{th} = \left(K_{op} + (\Delta K_{eff})_{th}\right)(1-R)$



2. (b) Consider a cracked panel

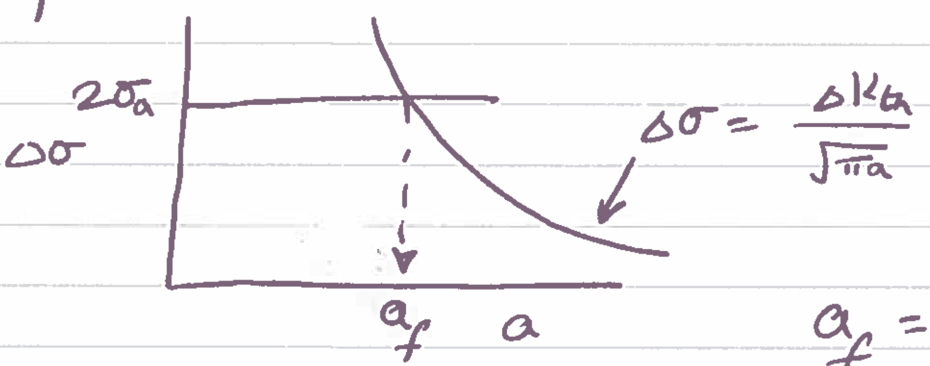


Under <u>monotonic</u> loading:



At $a = a_T$: $\sigma = \sigma_Y = \dfrac{K_{IC}}{\sqrt{\pi a_T}} \Rightarrow a_T = \dfrac{1}{\pi}\dfrac{K_{IC}^2}{\sigma_Y^2}$

Now consider fatigue crack initiation under cyclic loading. Take the case of infinite life and write σ_f as the fatigue limit (amplitude) in the absence of a crack, and ΔK_{th} when a crack is present.



$$a_f = \frac{1}{4\pi}\frac{\Delta K_{th}^2}{\sigma_a^2}$$

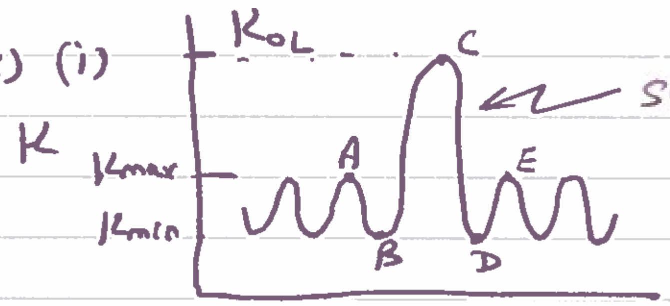
2\. (b) cont'd.

Now $\Delta K_{th} \sim \frac{1}{10} K_{IC}$ for metallic alloys

but $\sigma_a \sim \sigma_Y$

$$\Rightarrow \quad a_f \sim \frac{1}{400} \cdot a_T$$

2\. (c) (i)



single peak overload

$$R = \frac{K_{min}}{K_{max}} = 0.5$$

plastic zone size varies as follows :

A : $r_p = \frac{1}{\pi} K_{max}^2 / \sigma_Y^2$, tensile

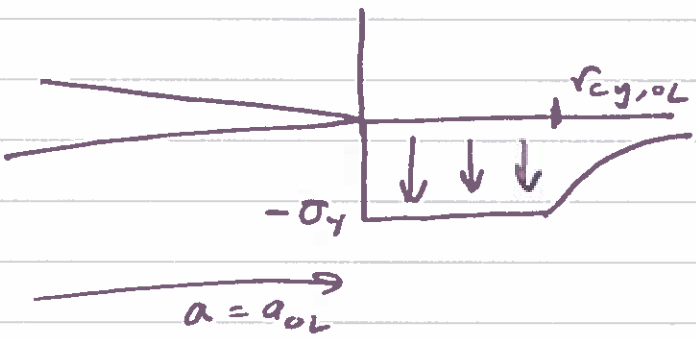
B : $r_{cy} = \frac{1}{\pi} \frac{(K_{max} - K_{min})^2}{4\sigma_Y^2}$, compressive

C : $r_{OL} = \frac{1}{\pi} \frac{K_{OL}^2}{\sigma_Y^2}$, tensile

D : $r_{cy,OL} = \frac{1}{\pi} \frac{(K_{OL} - K_{min})^2}{4\sigma_Y^2}$, compressive

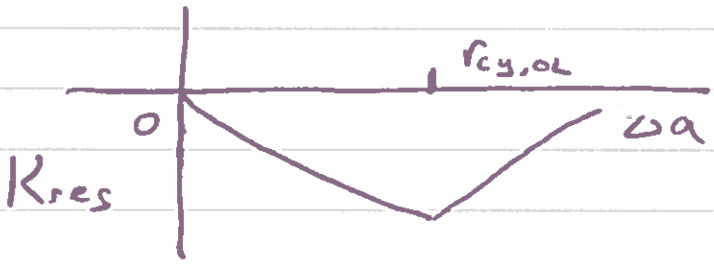
E : $r_{max} = \frac{1}{\pi} \frac{(K_{max} - K_{min})^2}{4\sigma_Y^2}$, tensile

At D :

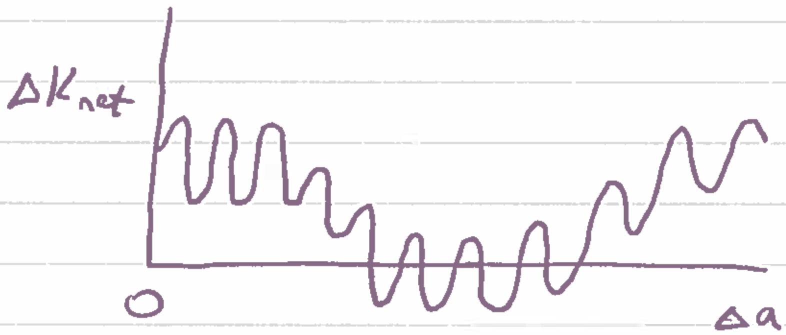


2. (c)(i) contd.

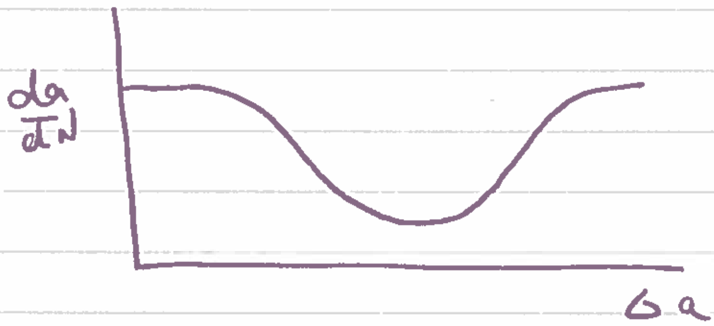
After the single peak overload the fatigue crack grows into a compressive zone and this generates a -ve. K, termed K_{res}.



Add this residual K to the applied ΔK to get :

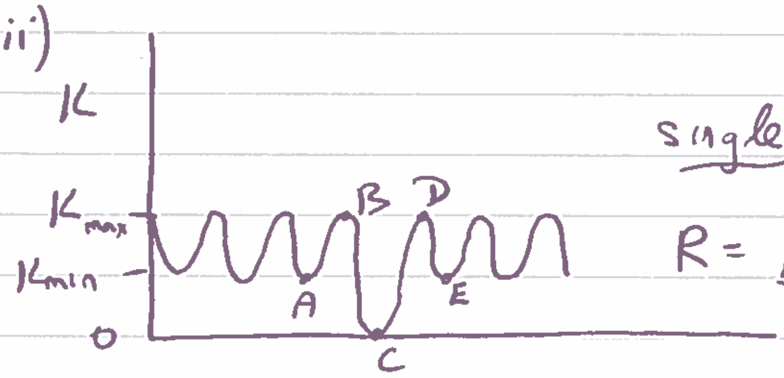


Hence



Transient retardation!

2. (c) (ii)



single peak underload

$$R = \frac{K_{min}}{K_{max}} = 0.5$$

PLASTIC ZONE SIZE VARIES AS FOLLOWS :

A : $r_{cy} = \frac{1}{\pi} \frac{(K_{max} - K_{min})^2}{4\sigma_y^2}$

B : $r_p = \frac{1}{\pi} \quad K_{max}^2 / \sigma_y^2$

C : $r_u = \frac{1}{\pi} \quad K_{max}^2 / (4\sigma_y^2)$

D : $r_p = \frac{1}{\pi} \quad K_{max}^2 / \sigma_y^2$

E : $r_{cy} = \frac{1}{\pi} (K_{max} - K_{min})^2 / (4\sigma_y^2)$

Conclusion : the underload does not change the residual stress state ahead of the crack tip. Thus, no crack growth transient is expected.

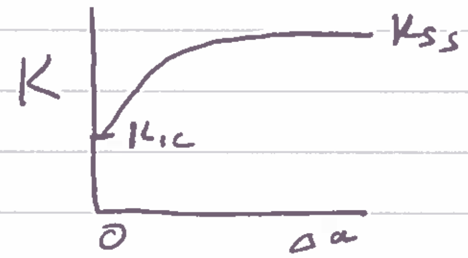
Minor effect : the underload may compress residual material on the crack flanks and lead to a transient drop in Kop. This would lead to slight transient crack growth acceleration.

CRIB for 3C9 (2018-19)

Q3. (a) The small scale yielding (ssy) case of fracture assumes that the plastic zone size is sufficiently small for an outer K-field to exist. The usual A.S.T.M. requirement is

$$a,\ (W-a) > 2.5\ \frac{K_{IC}^2}{\sigma_y^2}$$

Fracture initiation is at $K = K_{IC}$. Crack growth, Δa, can be predicted on the basis of an assumed $K(\Delta a)$ R-curve.



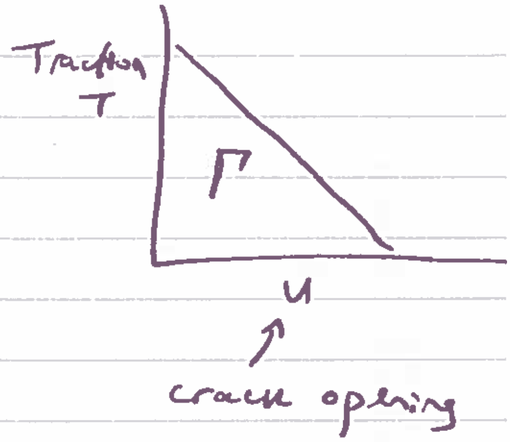
In the large scale yielding case, there is no outer K field. But there may be a J-field near the crack tip. In order for a J-field to exist we must have

$$a > 25\,\delta \quad \text{where } \delta = \text{crack tip opening}$$

$$\delta = \frac{J}{\sigma_Y} \quad \text{so} \quad \underline{\underline{a > 25\frac{J}{\sigma_Y}}}$$

3(b) The R-curve is due to plastic hysteresis in a metallic alloy. Non-proportional loading of material elements occur as the crack advances and this leads to additional plastic work.

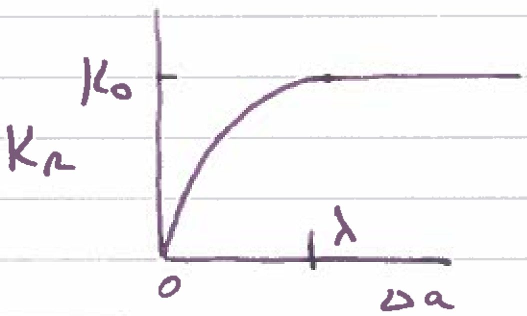
A long fibre composite exhibits fibre bridging behind the crack tip, with fibre pull-out.



Toughness due to fibre bridging $= \Gamma$.

3. (c) $K_R = K_0 \sin\left(\frac{\pi \Delta a}{2\lambda}\right)$ for $0 \le \Delta a \le \lambda$

$= K_0$ for $\Delta a > \lambda$



$K = \sigma^{\infty} \sqrt{\pi(a_0 + \Delta a)}$

3. (c) contd

Hence, $K = \sigma^\infty \sqrt{\pi\lambda} \left(\frac{a_0}{\lambda} + \frac{\Delta a}{\lambda}\right)^{1/2}$

$$= K_0 \sin\left(\frac{\pi}{2}\frac{\Delta a}{\lambda}\right) \qquad (1)$$

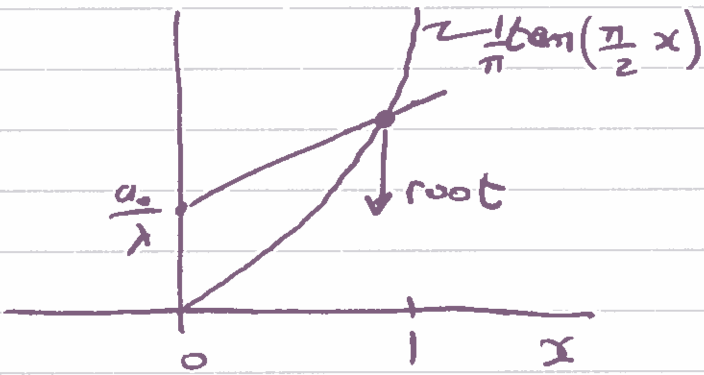
At max^m. load, $\frac{\partial K}{\partial a} = \frac{\partial K_R}{\partial a}$

$$\Rightarrow \frac{\sigma^\infty \sqrt{\pi\lambda}}{2\left(\frac{a_0}{\lambda} + \frac{\Delta a}{\lambda}\right)^{1/2}} \frac{1}{\lambda} = K_0 \frac{\pi}{2\lambda} \cos\left(\frac{\pi}{2}\frac{\Delta a}{\lambda}\right) \qquad (2)$$

Now divide (1) by (2) $\Rightarrow$

$$\frac{a_0}{\lambda} + \frac{\Delta a}{\lambda} = \frac{1}{\pi} \tan\left(\frac{\pi}{2}\frac{\Delta a}{\lambda}\right)$$

Write $x = \frac{\Delta a}{\lambda}$. Then $\frac{a_0}{\lambda} + x = \frac{1}{\pi} \tan \frac{\pi}{2} x$



(i) solve by iteration, for $\frac{a_0}{\lambda} = 0.2$

So $0.2 + x = \frac{1}{\pi} \tan \frac{\pi}{2} x$

$\Rightarrow x = ?$ Guess $x = 0.5$

$x_0 = 0.5 \Rightarrow x_1 = 0.728 \Rightarrow x_2 = 0.790$

$\Rightarrow x_3 = 0.802 \Rightarrow x_4 = 0.804$

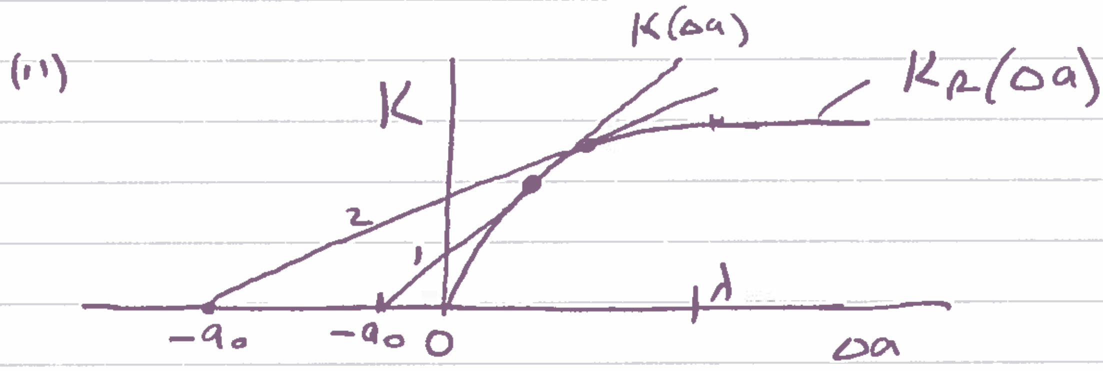
Converged solution is $x_5 = 0.805$.

$$\sigma^\infty \sqrt{\pi\lambda}\,(0.2 + x_5)^{1/2} = K_0 \sin\left(\frac{\pi}{2} x_5\right)$$

3(c)(i) contd.

$$\Rightarrow \quad \sigma^{\infty} = \frac{K_0}{\sqrt{\pi\lambda}} \times 0.951 = 0.537 \frac{K_0}{\sqrt{\lambda}}$$

(ii)



Compare p/c-cracks (1) and (2).

Instability is reached when the K−Δa loading curve kisses the $K_R-\Delta a$ resistance curve.

The instability occurs after a larger amount of crack extension for case (2) compared to case (1).

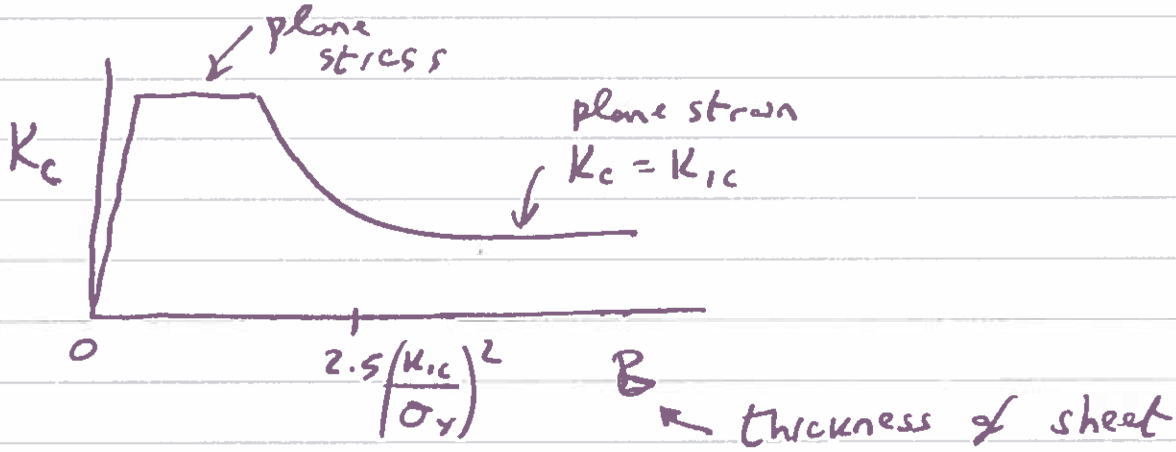
CRIB for 3C9 (2018-19)

4. (a) Goodman's rule is used in combination with Basquin's law.

$$\underset{\text{for } R>-1}{\Delta\sigma} = \underset{\text{for } R=-1}{\Delta\sigma_0}\left(1 - \frac{\overset{\text{mean stress}}{\sigma_m}}{\sigma_{uts}}\right)$$

where $\Delta\sigma_0 N_f^{\alpha} = C_1$.

(b)



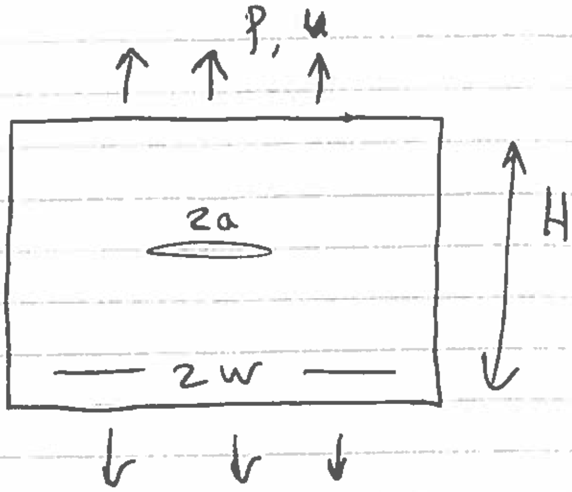
The level of hydrostatic stress is sensitive to the thickness of sheet.

If: $B > 2.5\left(\frac{K_{Ic}}{\sigma_Y}\right)^2$

Then plane strain conditions apply and void growth at the crack tip is promoted by the high hydrostatic stress there (on the order of $3\sigma_Y$).

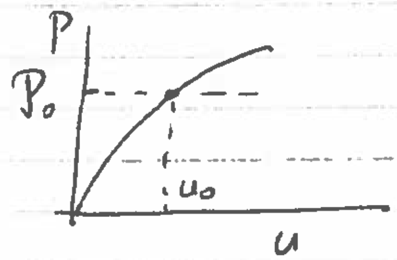
Thin sheet $\Rightarrow$ plane stress $\Rightarrow$ reduced void growth $\Rightarrow$ higher toughness K_c.

4. (c)



$$P = \sigma_Y \, B \, (w - a) \left(\frac{u}{H}\right)^{1/3}$$

(i)



Internal energy $\quad W = \int_0^{u_o} P \, du$

$$\Rightarrow \quad W = \int_0^{u_o} \sigma_Y \, B \, (w - a) \, \frac{u^{1/3}}{H^{1/3}} \, du$$

$$= \frac{3}{4} \, \frac{u_o^{4/3}}{H^{1/3}} \, \sigma_Y \, B \, (w - a)$$

Potential Energy $\quad \Psi = W - P_o u_o$

$$\Rightarrow \quad \Psi = -\frac{1}{4} \, \frac{u_o^{4/3}}{H^{1/3}} \, \sigma_Y \, B \, (w - a) \quad \left(\equiv -\tfrac{1}{2} W\right)$$

$$= -\frac{1}{4} \left[\frac{P_o}{\sigma_Y B (w - a)}\right]^4 H \, \sigma_Y \, B \, (w - a)$$

4. (c) contd.

(ii)
$$\Psi = -\frac{1}{4}\,\frac{P_0^4\,H}{\sigma_y^3\,B^3(w-a)^3}$$

$$2J = -\frac{\partial\Psi}{B\,\partial a} = +\frac{1}{4}\,\frac{P_0^4\,H}{\sigma_y^3\,B^4}\,\frac{\partial}{\partial a}(w-a)^{-3}$$

$$\Rightarrow \quad J = \frac{3}{8}\,\frac{P_0^4\,H}{\sigma_y^3\,B^4}\,(w-a)^{-4}$$