ENGINEERING TRIPOS PART IIA 2014/2015
MODULE 3D2: GEOTECHNICAL ENGINEERING II

1. (a) $v = 2.724$
   (b) $q = 154.5$ kPa, $v = 2.462$, yielding $p' = 107$ kPa and $q = 21.2$ kPa
   (c) (i) –, (ii) $q = 26.8$ kPa, $u = 100$ kPa, (iii) $q = 56.3$ kPa, $u = 144.8$ kPa, (iv) –

2. (a) –
   (b) –
   (c) –
   (d) –

3. (a) 16.7 mm
   (b) $\sigma_c = 450$ kPa, $u = 350$ kPa
   (c) –

4. (a) (i) $\sigma_v = 660$ kPa, $\sigma'_v = 330$ kPa, $\sigma_h = 505$ kPa, $\sigma'_h = 175$ kPa
   (ii) $\sigma_v = 160$ kPa, $\sigma'_v = 90$ kPa, $\sigma_h = 160$ kPa, $\sigma'_h = 90$ kPa, OCR = 3.7
   (b) 30 kPa
   (c) 26 kPa

1 $\sigma_a' = \sigma_r' = 130$ kPa then swell back to 100 kPa

(a) From the data book

$$v = N - \lambda \ln \sigma'$$

$$= \Gamma + \lambda - \kappa - \lambda \ln \sigma'$$

$$= 3.767 + 0.28 - 0.05 - 0.26 \ln 130$$

$$= \underline{2.711} \quad \text{after compressing to 130 kPa}$$

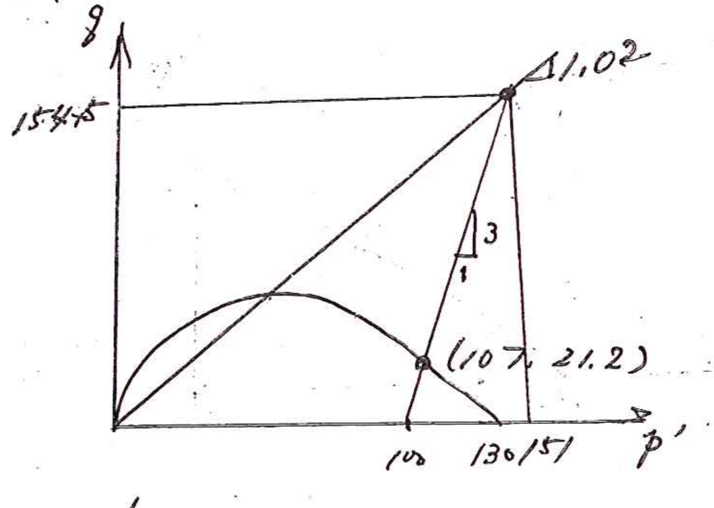
Swelling

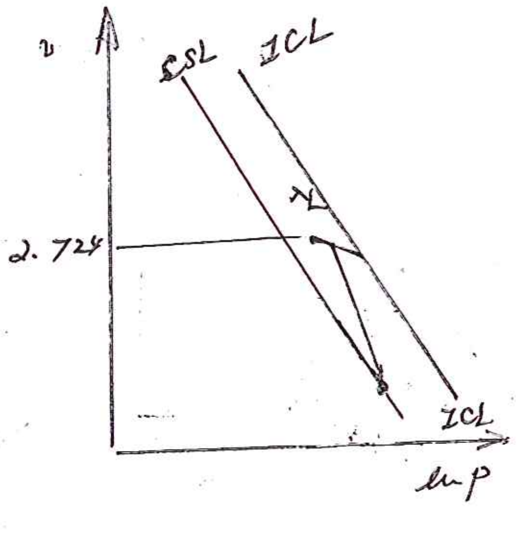
$$v = 2.711 + \kappa \ln (130/100)$$

$$= 2.711 + 0.05 \ln (130/100)$$

$$= \underline{2.724}$$

(b) (i)&(ii)





$$q/(100 + q/3) = 1.02$$

$$q = 1.02(100 + q/3) \qquad v = \Gamma - \lambda \ln p'$$

$$(1 - \tfrac{1.02}{3})q = 102 \qquad = 3.767 - 0.26 \ln 151.5$$

$$\underline{q = 154.5 \text{ kPa}} \qquad = \underline{2.462}$$

$$p' = 151.5 \text{ kPa}$$

The stress state at yield $= (p', q) = \underline{(107, 21.2) \text{ kPa}}$

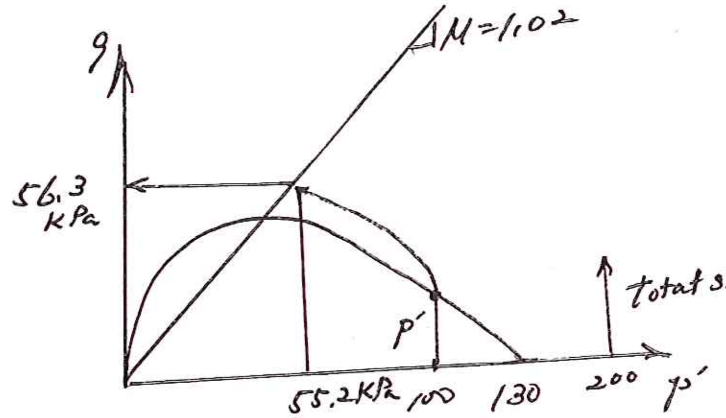
(c)

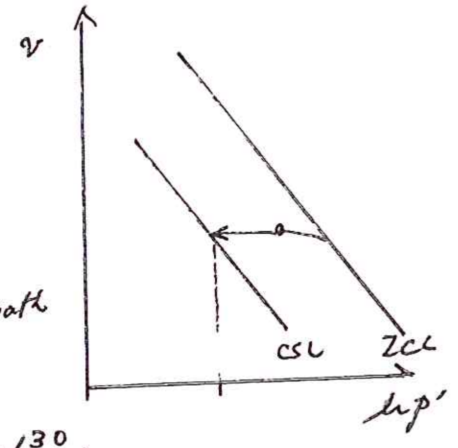
(i) $\Delta p = \dfrac{\Delta\sigma_a + 2\Delta\sigma_r}{3} = 0$

$\Delta\sigma_a = -2\Delta\sigma_r$

So if axial stress is increased $\Delta\sigma$, then the radial stress needs to decrease $\dfrac{\Delta\sigma}{2}$.

(ii)





$q/p' = M \ln\left(\dfrac{\sigma_c'}{p'}\right) = 1.02 \ln\left(\dfrac{130}{p'}\right)$

$q = 100 \times 1.02 \cdot \ln\left(\dfrac{130}{100}\right) = \underline{\underline{26.8 \text{ KPa}}}$

pore pressure $= \underline{\underline{100 \text{ KPa}}}$

(iii) At critical state.

$2.724 = \Gamma - \lambda \ln p' = 3.767 - 0.26 \ln p'$

$\ln p' = \dfrac{3.767 - 2.724}{0.26} = 4.01$

$p' = \underline{\underline{55.2 \text{ KPa}}}$

$q = 1.03 \cdot p' = \underline{\underline{56.3 \text{ KPa}}}$

pore pressure $= 200 - 55.2 = \underline{\underline{144.8 \text{ KPa}}}$

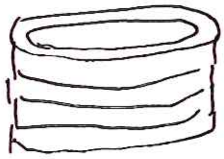
(iv) No it will be the same.

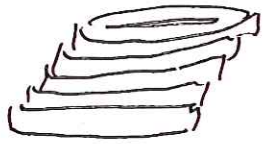
Q1. Examiner's Comment:

Most of the students attempted this question. The student did well in computing void ratios of an overconsolidated clay from confining pressures. Most students were able to evaluate the critical shear strengths in drained triaxial compression conditions. Some students were confused with the mechanical behaviour under a more complex stress path, which was a constant total mean pressure in undrained conditions in this case.

2.

(a) A simple shear apparatus has a series of rings stacked together as shown below. The rings will keep the radial strain to be zero (i.e. K_0 condition) the verical strain then becomes equal to the volumetric strain.





Shearing is done by moving the rings relative to each other and the gradient of the tilt is related to the shear strain.

(b) From the Data book

$$I_R = I_D I_C - 1$$

$$= \frac{e_{max} - e}{e_{max} - e_{min}} \cdot \ln\left(\frac{\sigma_c}{\sigma'}\right) - 1$$

The critical state is when $I_R = 0$

$$\frac{e_{max} - e}{e_{max} - e_{min}} \cdot \ln\left(\frac{20{,}000}{\sigma'}\right) = 1$$

$e_{max} = 0.9$, $e_{min} = 0.5$

$$\frac{0.9 - e}{0.4} = 1/\ln\left(\frac{20{,}000}{\sigma'}\right)$$

$$e = 0.9 - 0.4\left[1/\ln\left(\frac{20{,}000}{\sigma'}\right)\right]$$

(c)(i) Under $\sigma' = 500\,kPa$, e at critical state

is $e = 0.9 - 0.4\left[1/\ln \frac{20{,}000}{500}\right]$

$= 0.791$.

dense sample

$e = 0.7$

$$I_D = \frac{0.9 - 0.7}{0.9 - 0.5} = 0.5$$

$$I_C = \ln\left(\frac{20000}{500}\right) = 3.69$$

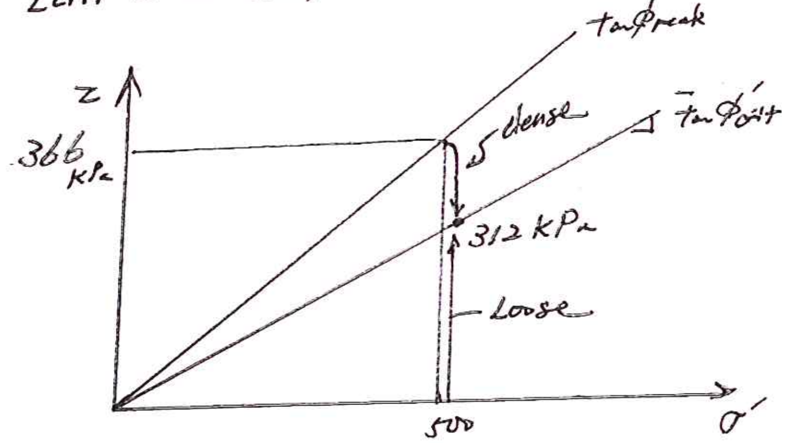
$$I_R = I_D \cdot I_C - 1 = 0.5 \cdot 3.69 - 1 = 0.845$$

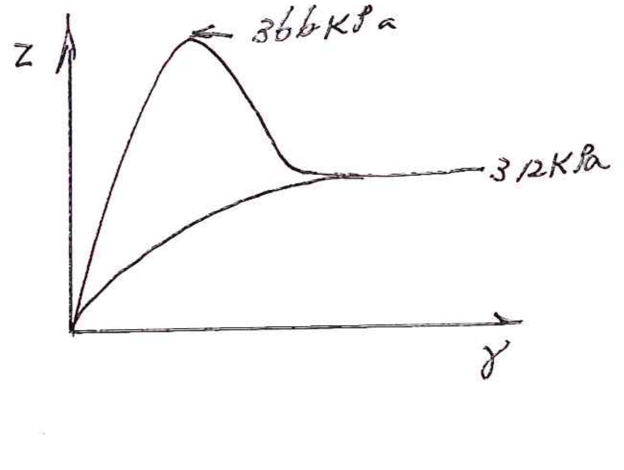
$$\phi_{max} - \phi_{crit} = 5 \cdot I_R = 5 \times 0.845 = 4.23$$

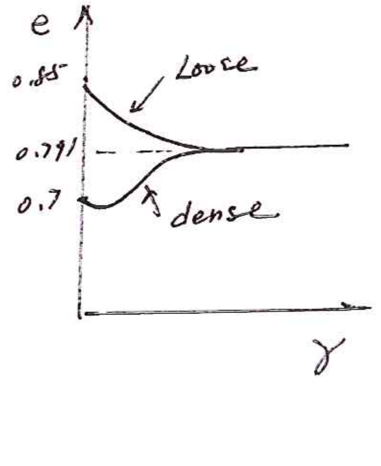
$$\phi_{max} = 4.23 + \phi_{crit} = 4.23 + 32 = 36.2^\circ$$

$$\tau_{max} = \sigma' \tan\phi'_{max} = 500 \cdot \tan 36.2^\circ = 366 \text{ KPa}$$

$$\tau_{crit} = \sigma' \tan\phi_{crit} = 500 \cdot \tan 32 = 312 \text{ KPa}$$







(ii)

dense sand

$$\ln\left(\frac{20{,}000}{\sigma'}\right) = \frac{e_{max} - e_{min}}{e_{max} - e} = \frac{0.9 - 0.5}{0.9 - 0.7} = 2$$

$\sigma' = 2.70$ MPa at critical state

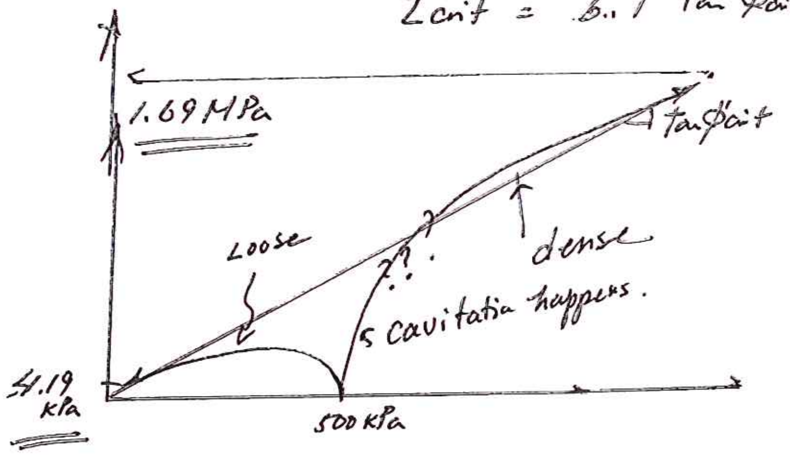
$$\tau_{crit} = 2.70 \tan\phi'_{crit} = \underline{1.69 \text{ MPa}}$$ //

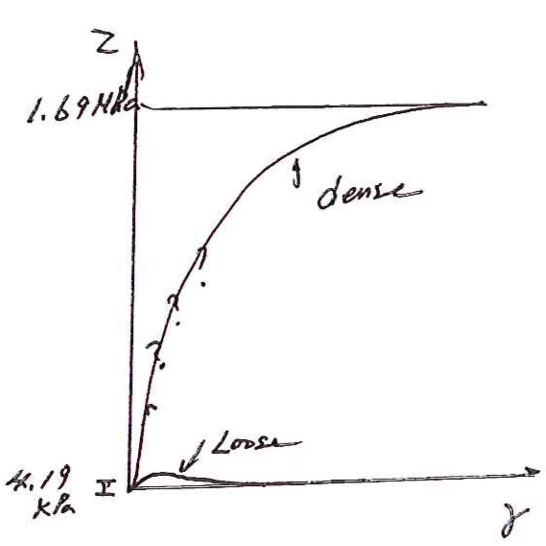
loose sand

$$\ln\left(\frac{20{,}000}{\sigma'}\right) = \frac{e_{max} - e_{min}}{e_{max} - e} = \frac{0.9 - 0.5}{0.9 - 0.85} = 8$$

$\sigma' = 6.7$ kPa at critical state

$$\tau_{crit} = 6.7 \tan\phi'_{crit} = \underline{4.19 \text{ kPa}}$$ //





excess pore pressure

dense

$= 0.5 - 2.70 = -2.20$ MPa

possibly cavitates //

Loose

$= 500 - 6.7 = \underline{493}$ kPa //

Q2. Examiner's Comment:

Most of the students seem to understand the critical state of sand and the mechanical behaviour of dense and loose sands. Some students were not able to derive the critical state in undrained conditions using Bolton's relative dilatancy index. It was disappointing that most students could not distinguish the difference between simple shear and direct shear.

Q.3

(a) Radial ground deformation at tunnel boundary $= \rho_c$

Assume small displacements, treat contracting cylindrical cavity as analagous to expanding cavity. Using Data Book:

shear strain $\gamma = \dfrac{2\rho}{r}$

At $r = r_o$, $\rho = \rho_c \Rightarrow \gamma = \dfrac{2\rho_c}{r_o}$ .... (1)

If soil remains elastic, $\tau = G\gamma$ ..... (2)

where τ = shear stress, γ = shear strain, G = elastic shear modulus

Maximum radial ground deformation $= \rho_{cm}$ before shear stress at tunnel boundary just reaches the maximum possible value $\tau = c_u$

$$\rho_{cm} = \frac{r_o \gamma_{max}}{2} \quad \text{from (1)}$$

$$\gamma_{max} = \frac{\tau}{G} = \frac{c_u}{G} \quad \text{from (2)}$$

$$\therefore \quad \rho_{cm} = \frac{r_o}{2}\frac{c_u}{G}$$

$r_o = 4\text{m}$, $G = 30\ \text{MPa}$, $c_u = 250\ \text{kPa}$

$$\rho_{cm} = \frac{4}{2} \times \frac{250}{30 \times 10^3}\ \text{m} = 16.7 \times 10^{-3}\ \text{m}$$
$$= \underline{\underline{16.7\ \text{mm}}}$$

[40%]

(b) From Data Book, contraction $\delta A = A_o - A$ caused by reduction of pressure $\delta\sigma_c = \sigma_o - \sigma_c$

(σ_o = original in-situ stress; σ_c = tunnel support pressure = total radial stress at tunnel boundary)

and $\delta\sigma_c = G\dfrac{\delta A}{A}$

$$\frac{\delta A}{A} = \frac{2\rho_c}{r_o}$$

$$\therefore \quad \delta\sigma_c = \frac{2G\rho_c}{r_o}$$

σ_o = original in-situ stress = γz_o

$\gamma = 20\ kN/m^3 \quad z_o = 35m \Rightarrow \sigma_o = 20 \times 35$
$= 700\ kPa$

$$\rho_c = \rho_{cm} = \frac{r_o}{2} \cdot \frac{c_u}{G}$$

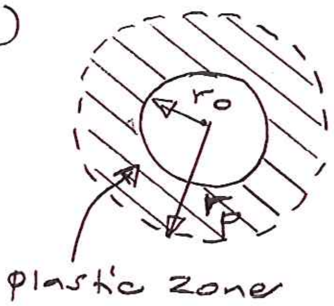
$$\therefore \quad \delta\sigma_c = \frac{2\cancel{G}}{\cancel{r_o}} \cdot \frac{\cancel{r_o}}{2} \cdot \frac{c_u}{\cancel{G}} = c_u$$

$$\therefore \quad \sigma_c = \sigma_o - \delta\sigma_c = 700 - 250 = \underline{\underline{450\ kPa}}$$

Soil elastic, no change in pore pressure

$\therefore \quad u = u_o = 10 \times 35 = \underline{350\ kPa}$ [30%]

(c)



r_o = radius of tunnel

r_p = radius of plastic zone

$$\left.\begin{aligned} \sigma_r &= \sigma_o - G\frac{\delta A}{\pi r^2} \\ \sigma_\theta &= \sigma_o + G\frac{\delta A}{\pi r^2} \end{aligned}\right\} \text{within the elastic zone at any radius } r$$

$$\therefore \quad \sigma_\theta - \sigma_r = 2G\frac{\delta A}{\pi r^2} \quad \text{in elastic zone}$$

In plastic zone $\sigma_\theta - \sigma_r = 2c_u$

$\therefore$ at boundary of elastic/plastic zones $(r = r_p)$

$$2G\frac{\delta A}{\pi r_p^2} = 2c_u$$

for small deformations $A \simeq \pi r_o^2$

$$\therefore \quad \frac{G}{c_u}\delta A = \pi r_p^2$$

$$\therefore \quad \frac{G}{c_u} \cdot \frac{\delta A}{A} = \frac{\pi r_p^2}{\pi r_o^2}$$

$$\frac{r_p}{r_o} = \left(\frac{G}{c_u} \cdot \frac{\delta A}{A}\right)^{1/2}$$

[30%]

Q3. Examiner's Comment:

A straightforward question using the cavity contraction equation to assess the deformation and stresses around a tunnel. Students did well on this question. Mistakes come from either applying a wrong equation or making numerical errors. Some students did not recognise that excess pore pressure is zero in elastic cavity contraction.

Q. 4

(a) For normally consolidated deposition

(i) $K_{onc} = 1 - \sin \phi_{crit}$ (Data Book)

$= 1 - \sin 28° = 1 - 0.47 = 0.53$

<u>original</u> maximum stresses for soil element X :

$\sigma_v = 33 \times 20 = 660$ kPa

$u = 33 \times 10 = 330$ kPa

$\therefore \sigma_v' = \sigma_v - u = 660 - 330 = 330$ kPa

$\sigma_h' = K_{onc} \cdot \sigma_v' = 0.53 \times 330 = 175$ kPa

$\sigma_h = \sigma_h' + u = 175 + 330 = 505$ kPa

$s = \frac{1}{2}(\sigma_v + \sigma_h) = \frac{1}{2}(660 + 505) = 582$ kPa

$s' = \frac{1}{2}(\sigma_v' + \sigma_h') = \frac{1}{2}(330 + 175) = 253$ kPa

$t = \frac{1}{2}(\sigma_v - \sigma_h) = \frac{1}{2}(660 - 505) = 78$ kPa

} effective Stress State A'

(ii) <u>present day</u> stresses before installation of wall :

$\sigma_v = 8 \times 20 = 160$ kPa

$u = 7 \times 10 = 70$ kPa

$\therefore \sigma_v' = 160 - 70 = 90$ kPa

$\therefore$ Overconsolidation ratio $= \frac{330}{90} = 3.7$

$\sigma_h' = K_o \sigma_v' = 1.0 \times 90 = 90$ kPa

$\sigma_h = \sigma_h' + u = 90 + 70 = 160$ kPa

$s = \frac{1}{2}(160 + 160) = 160$ kPa

$s' = \frac{1}{2}(90 + 90) = 90$ kPa

$t = \frac{1}{2}(160 - 160) = 0$

} effective stress state B'

(iii) <u>graph</u> : OA' effective stress path corresponding to 1D deposition (normal consolidation).
A'B' is stress path during erosion of 25m of soil.

[40%]

(b) $p_h = \sigma_h = 100$ kPa

σ_v remains constant, i.e. $\Delta\sigma_v = 0$

$s = \frac{1}{2}(\sigma_v + \sigma_h)$ $\therefore$ $\Delta s = \frac{1}{2}\Delta\sigma_h$

$t = \frac{1}{2}(\sigma_v - \sigma_h)$ $\therefore$ $\Delta t = -\frac{1}{2}\Delta\sigma_h$

hence $\frac{\Delta t}{\Delta s} = -1$

soil remains elastic (and isotropic)

$\therefore$ $s' = $ constant $= 90$ kPa

$\sigma_v = 160$ kPa

$s = \frac{1}{2}(160 + 100) = 130$ kPa

$\therefore$ $\Delta s = 130 - 160 = -30$ kPa

$\frac{\Delta t}{\Delta s} = -1$ $\therefore$ $\Delta t = 30$ kPa

Stress state C, C' (see graph):

$t = 0 + 30 = 30$ kPa

$s = 130$ kPa

$s' = 90$ kPa

$\Delta u = \Delta s = -30$ kPa

i.e. pore pressure reduces by 30 kPa (from 70 kPa to 40 kPa)

[30%]

(c) As pore pressure increases with time (drainage occurring) ESP C' → D'

$\sigma_v = 160$ kPa $=$ constant

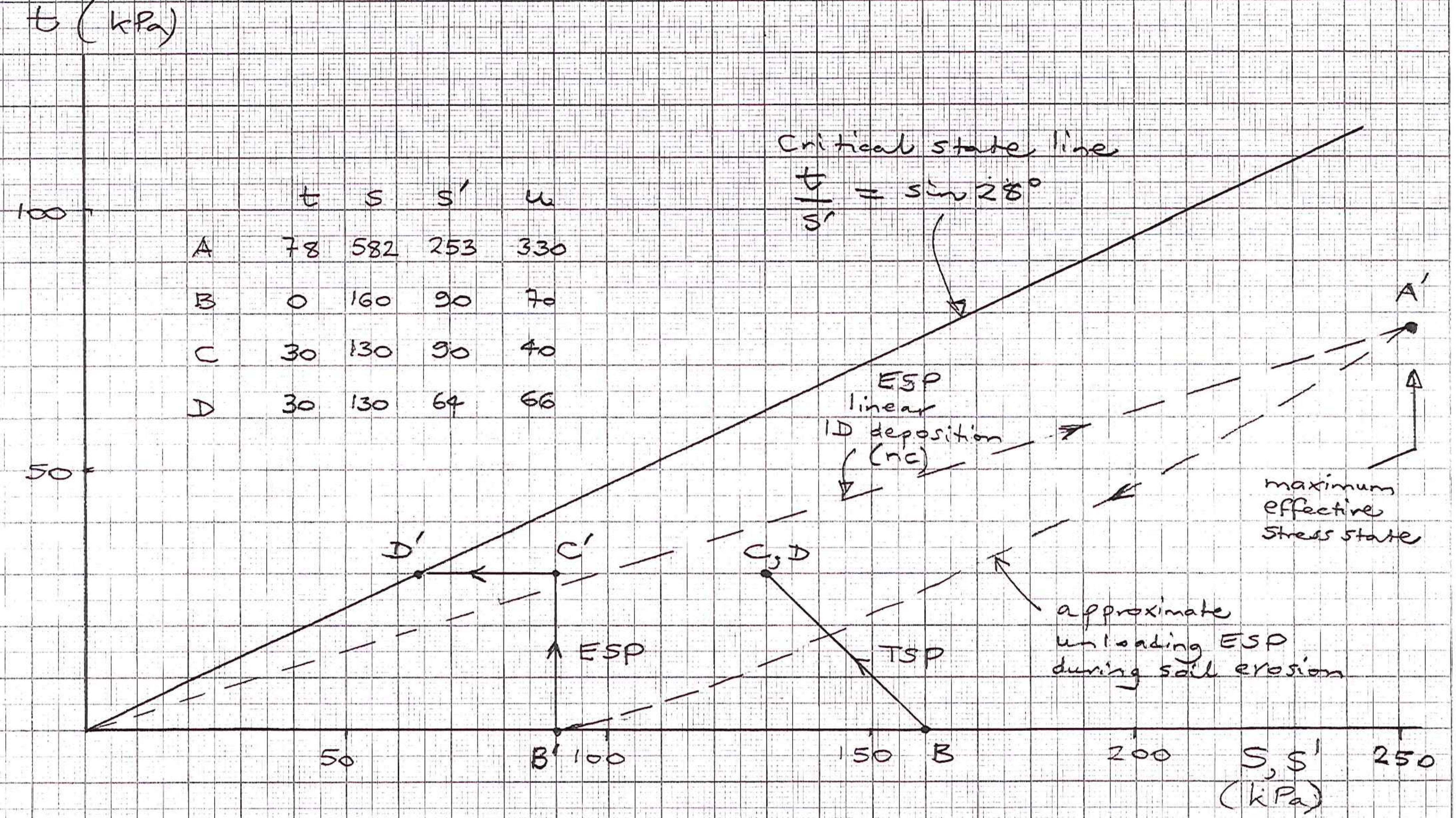
$\sigma_h = 100$ kPa $= p_h =$ constant

$\therefore$ t and s both remain constant

At critical state line $s' = \frac{t}{\sin 28°} = \frac{30}{0.47} = 64$ kPa

$\therefore$ pore pressure increases by $90 - 64 = 26$ kPa (from 40 kPa to 66 kPa)

[30%]



	t	s	s'	u
A	78	582	253	330
B	0	160	90	70
C	30	130	90	40
D	30	130	64	66

Q4. Examiner's Comment:

Although this was a relatively long question, students did well in answering the questions. They understand how to compute insitu stresses and overconsolidation ratio. Most students understood the undrained stress path of retaining wall excavation. The answers to the subsequent consolidation behaviour were mostly correct.