Solution to Q. 1

(a). • tunnel diameter = 4 m
radius r_{co} = 2m

• Initial total stress in ground at tunnel axis level, $\sigma_o = \gamma z$

γ = unit weight of clay = 20 kN/m^3
z = depth of tunnel axis below ground level

$\sigma_o = 20 \times 25 = 500\ kN/m^2$

• Average radial stress on tunnel lining
$\sigma_c = 150\ kN/m^2$ (measured)

• Change in cavity radial stress during unloading of cylindrical cavity (the tunnel)
$\delta\sigma_c = \sigma_o - \sigma_c = 500 - 150 = 350\ kN/m^2$

• total (inward) radial ground movement at tunnel boundary, ρ_c = 20mm (measured)

• From Data Book, assuming tunnel construction as axisymmetric cavity contraction (analogous to cavity expansion):

$$\delta\sigma_c = c_u\left[1 + \ln\frac{G}{c_u} + \ln\frac{\delta A}{A}\right]$$

$$\frac{\rho_c}{r_{co}} \simeq \frac{1}{2}\frac{\delta A}{A_o} \simeq \frac{1}{2}\frac{\delta A}{A} = \frac{1}{2}\frac{c_u}{G}\exp\left[\frac{\delta\sigma_c}{c_u} - 1\right]$$

$\rho_c = 20 \times 10^{-3}$ m, r_{co} = 2m
$c_u = 175\ kN/m^2$ $\quad \delta\sigma_c = 350\ kN/m^2$

$$\therefore \frac{20\times 10^{-3}}{2} = \frac{1}{2} \times \frac{175}{G}\exp\left[\frac{350}{175} - 1\right]$$

$G = 13.4 \times 10^3\ kN/m^2 = \cancel{13.4}$ MPa
23.8 [40%]

(b) tunnel diameter = 8 m

tunnel radius, $r_{co} = 4$ m

$\sigma_c = 200$ kN/m^2 (estimated by designer)

$\therefore \delta\sigma_c = 500 - 200 = 300$ kN/m^2

as before

$$\frac{\rho_c}{r_{co}} = \frac{1}{2}\frac{c_u}{G}\exp\left[\frac{\delta\sigma_c}{c_u} - 1\right]$$

$r_{co} = 4$ m $\qquad c_u = 175$ kN/m^2

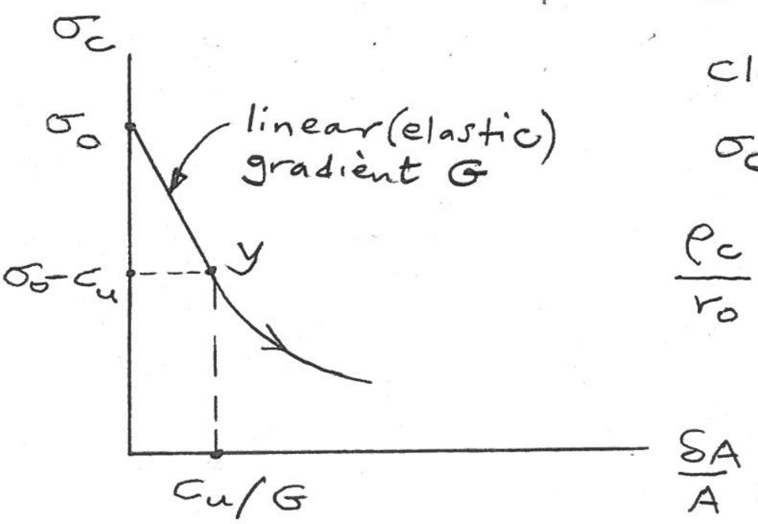
$\delta\sigma_c = 300$ kN/m^2 $\qquad G = \underset{23.8}{\cancel{13.4}} \times 10^3$ kN/m^2

$$\therefore \rho_c = 4 \times \frac{1}{2} \times \frac{175}{\underset{23.8}{\cancel{13.4}} \times 10^3}\exp\left[\frac{300}{175} - 1\right] = \overset{30.0}{\cancel{53.4}} \times 10^{-3}\ \text{m} = \underline{\overset{30.0}{\cancel{53.4}}\ \text{mm}}$$

[20%]

(c) zero pore pressure reduction if clay remains <u>elastic</u> $\Rightarrow$ <u>total</u> mean normal stress remains constant and, for undrained conditions (constant volume) <u>effective</u> mean normal stress remains constant

i.e. $\Delta p = \Delta p' = 0 \Rightarrow \Delta u = 0$



Clay yields at y, $\sigma_c = \sigma_o - c_u$

$\sigma_c = 500 - 175 = \underline{325\ \text{kN/m}^2}$

$$\frac{\rho_c}{r_o} \simeq \frac{1}{2}\frac{\delta A}{A} = \frac{1}{2}\frac{c_u}{G}$$

$$\therefore \rho_c = 4 \times \frac{1}{2} \times \frac{175}{\underset{23.8}{\cancel{13.4}} \times 10^3}\ \text{m}$$

$= \cancel{26.1 \times 10^{-3}}$

$= \cancel{26.1\ \text{mm}}$

14.7 mm

[40%]

2. London Clay $\phi_{crit} = 23°$

$K_p = 2.28$

$K_a = 0.44$

$K_{0,nc} = 1 - \sin\phi_{crit} = 0.61$

A $\sigma_V' = \underline{\underline{400\text{ kPa}}}$ $\sigma_H' = \underline{\underline{244\text{ kPa}}}$

$p' = 296\text{ kPa}$

$q = 156\text{ kPa}$

B $n = 8$ $n_{max} = 8$

$$K_0 = K_{0nc}\left[1 + \frac{(n-1)(n_{max}^{\alpha} - 1)}{n_{max} - 1}\right]$$

$\alpha = 1.2 \sin\phi_{crit} = 0.469$

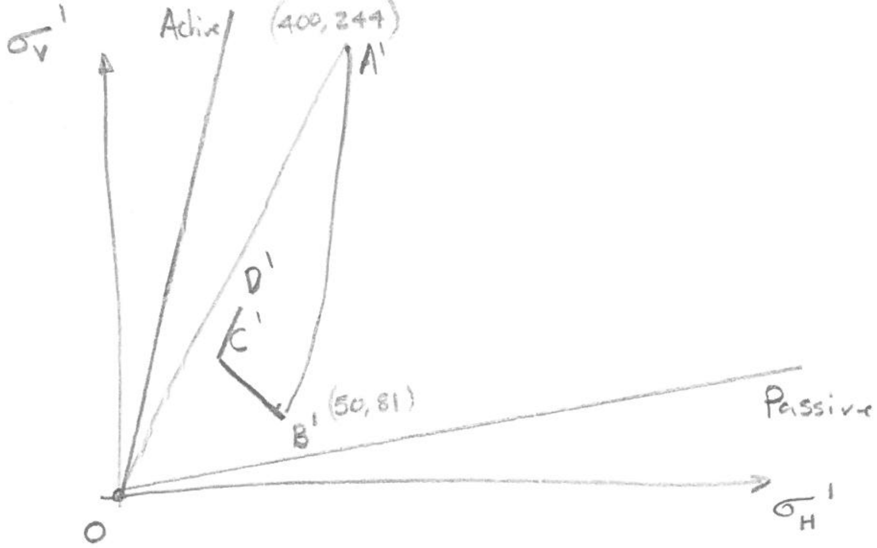
$K_0 = 1.62$

$\sigma_V' = \underline{\underline{50\text{ kPa}}}$ $\sigma_H' = \underline{\underline{81\text{ kPa}}}$

$p' = 70.7\text{ kPa}$ $q = -31\text{ kPa}$

$M_{comp} = 0.89$

$M_{extn} = 0.69$



c) Now plane-strain not triaxial

$C_u = \cancel{25}\ 45$ kPa

~~$\gamma \sim 16\ kN\,m^{-3}$~~

~~$\sigma_v \sim 64$ kPa~~

B $\sigma_v' = 50$ kPa $\qquad \sigma_H' = 81$ kPa

$U_{hydrostatic} = -20$ kPa

$\sigma_v = 30 \qquad \sigma_H = 61$ kPa

E $\sigma_H = 0 \qquad \sigma_v = 30$

undrained $\sigma_v' + \sigma_H' = \text{const}$ (assuming intermediate stress doesn't change)

$\sigma_v' + \sigma_H' = 131$

$(30 - u) + (0 - u) = 131$

$\underline{\underline{u = -50\tfrac{1}{2}\ \text{kPa}}}$

$\sigma_v' \approx 80\tfrac{1}{2}$ kPa $\qquad \sigma_H' \sim 50\tfrac{1}{2}$ kPa

Very high suction but stable provided no air entry

Drainage gives $u \longrightarrow 0$

At failure $\dfrac{\sigma_v'}{\sigma_H'} = 2.28$

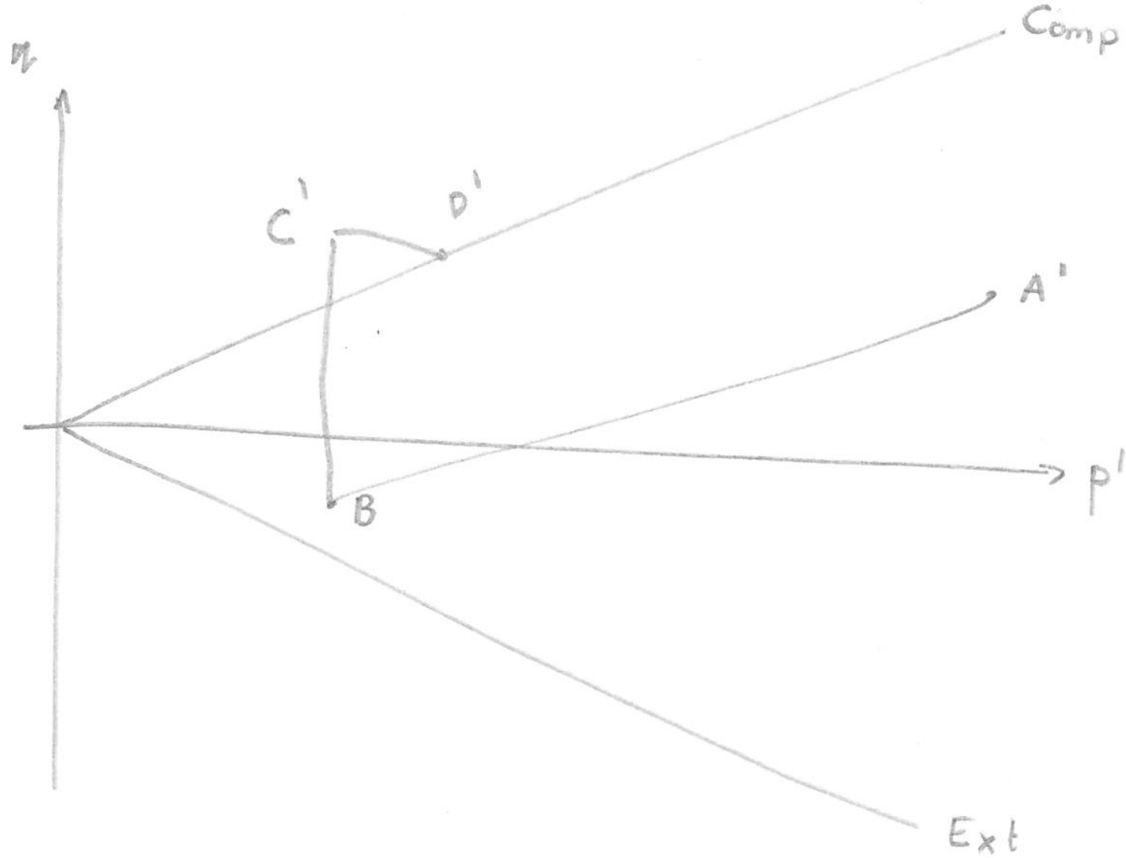
$$\frac{30 - u}{-u} = 2.28$$

$1.28\,u = -30$

$\underline{\underline{u = -23.4\ \text{kPa}}}$

hydrostatic is $\underline{\underline{-20\ \text{kPa}}}$

When ~~wall~~ pore-pressure drains, trench will collapse support needed.



C $\quad p' = 70.7$ kPa $\qquad q = 75$ kPa

$\sigma_H' = 45.7$ kPa $\qquad \sigma_v' = 120.7$ kPa

D $\quad q = 90$ kPa $\qquad \dfrac{q}{p'} = 0.89$

$p' = 101.1$ kPa

$\sigma_H' = 71$ kPa $\qquad \sigma_v' = 161$ kPa

3. a) Drained — constant pore pressure

Undrained - constant volume

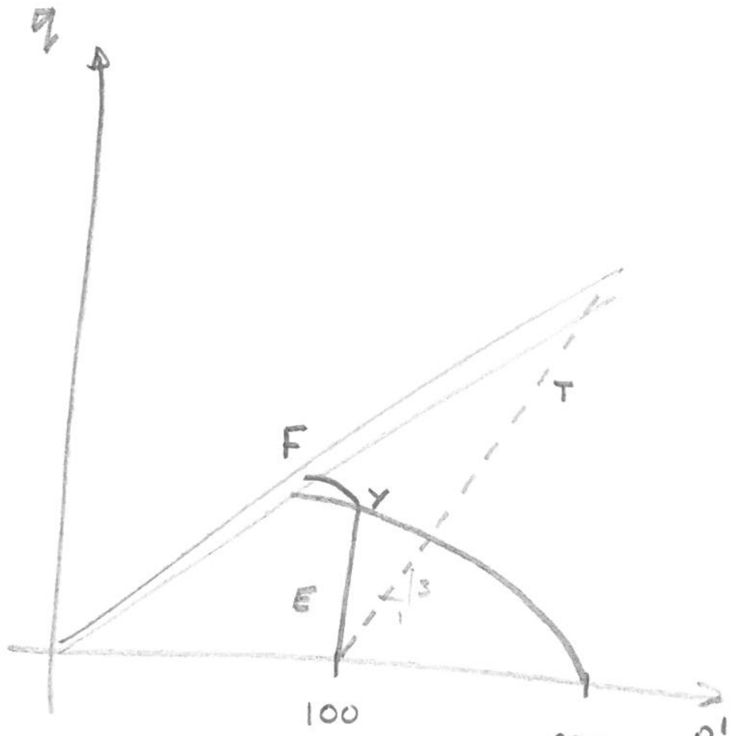
In field drained - slow relative to consolidation

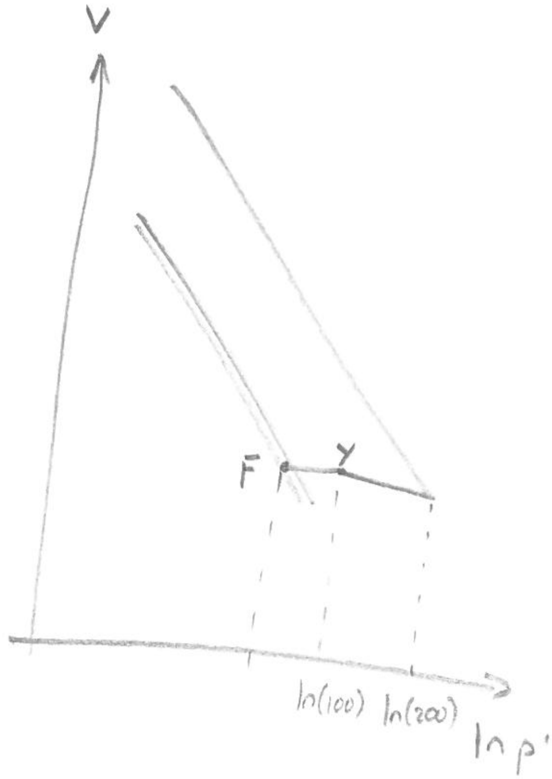
undrained fast "

In lab drained - slow with water allowed to flow in/out of sample

undrained - valves closed to prevent fluid flow.

b)





At Y $p' = 100$ kPa

$$\frac{q}{p'_{100}} = \cancel{M}^{1.02} \ln \frac{200}{100}$$

$q = \underline{70.7 \text{ kPa}}$

$p = 100 + \frac{q}{3}$
$= 123.6$

$v = \Gamma + \lambda - \kappa - \lambda \ln(200) + \kappa \ln(2)$

$= 2.634$

$u_{ex} = 23.6$ kPa

At F $v = 2.634$ on CSL

$$\Gamma - \lambda \ln p' = 2.634$$

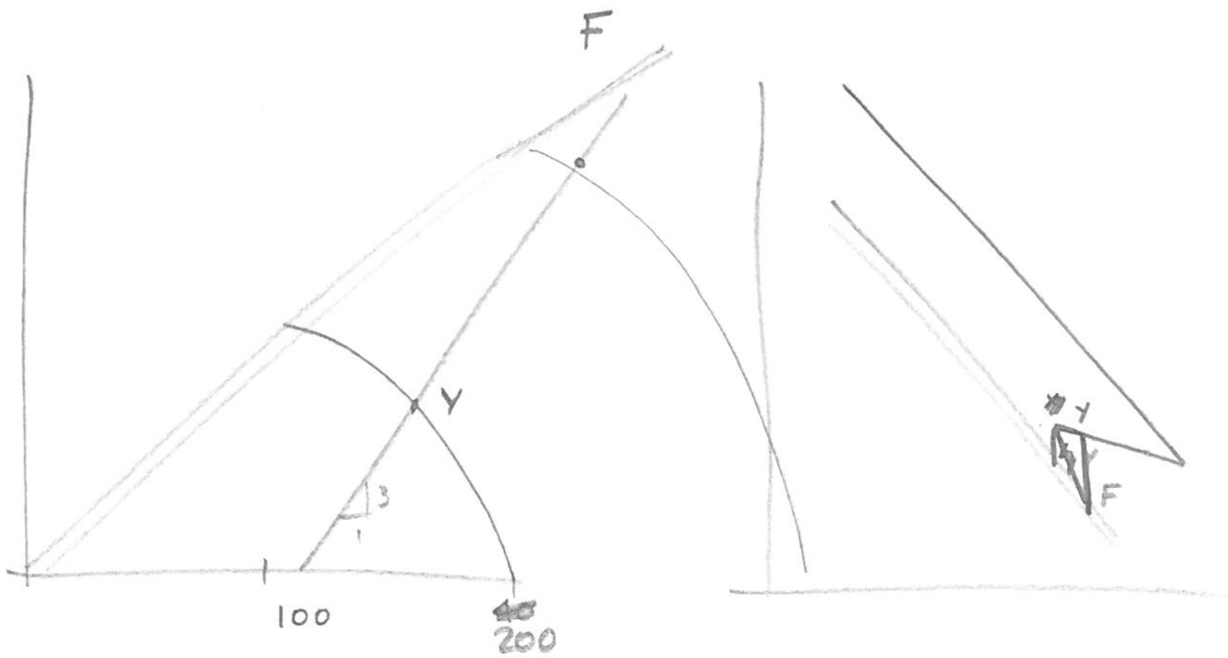
$$p' = \underline{78.1 \text{ kPa}}$$

$$q = 79.64 \text{ kPa} = Mp'$$

$$p = 100 + \frac{q}{3} = 126.55 \text{ kPa}$$

$$u_{ex} = \underline{48.5 \text{ kPa}}$$

c)



At Y $\quad q = q \quad p' = 100 + \frac{q}{3}$

$$\frac{\overset{3p'-300}{\cancel{q}}}{\cancel{M}p'} = M \ln \frac{p'_c}{p'} = \overset{1.02}{\cancel{M}} \ln \frac{200}{p'}$$

Iterate to get $q =$

$$p' = 121 \text{ kPa}$$

$$q = \underline{63 \text{ kPa}}$$

On K $\quad v = 2.634 - 0.05 \ln\left(\frac{121}{100}\right)$

$$= 2.6383 \qquad \varepsilon_v = \underline{0.36\%}$$

A) ~~ψ~~ $q = Mp'$

$$q = 1.02\left(100 + \frac{q}{3}\right)$$

$$\underline{q = 154.5 \text{ kPa}}$$

$$p' = \cancel{157.6}\ 151.4 \text{ kPa}$$

$$v = \Gamma - \lambda \ln 151.4$$

$$= 2.462$$

$$\varepsilon_v = \frac{0.173}{2.63\cancel{4}} = \underline{\underline{6.57\%}}$$

d) $$q = \frac{154.5}{1.2} = \underline{\underline{128.75 \text{ kPa}}}$$

$$p' = \underline{\underline{142.9 \text{ kPa}}}$$

$$\cancel{v = \Gamma - \lambda \ln 142.9 = 2.477}$$

~~On CSL @ v = 2.477 p' =~~

Project yieald surface

$$\frac{128.75}{142.9} = 1.02 \ln\left(\frac{p'_c}{142.9}\right)$$

$$p'_c = 3\overset{57.8 \text{ kPa}}{\cancel{45.67}}$$

$$v = \underset{+0.26-0.05}{3.767} - 0.26 \ln\left(\overset{357.8}{\cancel{345.67}}\right) + 0.05 \ln\left(\frac{\overset{357.8}{\cancel{345.67}}}{142.9}\right)$$

$$= 2.2\cancel{84} + 0.21 = 2.494$$

On CSL @ $v = 2.~~284~~$ 494 $p' =$ ~~291.67~~ 133.8 kPa

$q =$ ~~297.5 kPa~~ 136.45 kPa

$$FoS = \frac{136.45}{128.75} = \underline{\underline{1.06}}$$

$$\sigma_{axial} = \underline{\underline{236.45\ kPa}}$$

4 a) For a given slope define a slip surface as a circular arc Take moments about Centre of circle balancing self-weight against sliding resistance c_u on slip plane. This gives a lower bound on c_u required or upper bound on FoS. Hunt for optimal slip surface by changing circle radius and centre.

b) More complicated than a) owing to changing strength with stress.

Define slip circle and divide sliding mass into vertical slices.

For each slice make assumptions about interaction forces on vertical planes. In Bishop's method assume no net vertical force

Resolve to calculate stresses on slip plane, normal and shear

Find optimal mechanism for lowest FoS.

c) Slope angle = 30°

Lowest ϕ_{pk} at base of slope — 5m depth.

At 5m depth $\sigma_v' = \gamma z \cos^2 \beta$

$= 57.4$ kPa

$$\gamma_{dry} = \frac{\gamma_w G_s}{1+e} = 15.3 \text{ kN m}^{-3}$$

$$\gamma_{sat} = \frac{\gamma_w (G_s + e)}{1+e} = 19.4 \text{ kN m}^{-3}$$

$$I_D = \frac{0.25}{0.35} = 71.4\%$$

$$I_C = \ln \frac{5000}{57.4} = 4.47$$

$$I_R = I_D I_C - 1 = 2.19$$

$$\phi_{max} = 25 + 5 \times 2.19 = \underline{\underline{36^\circ}}$$

$$\phi_{mob} = 30^\circ$$

Slope is stable

$$FoS = \frac{\tan 36}{\tan 30} = \underline{\underline{1.26}}$$

d)

$u = 20 \cos^2\beta = 15$ kPa

$\sigma_n = \gamma z \cos^2\beta$

$= (15.3 \times 3 + 19.4 \times 2) \times \frac{3}{4} = 63.5$ kPa

$\sigma_n' = 48.5$ kPa

$I_C = \ln \frac{5000}{48.5} = 4.636$

$I_R = 2.31$

$\phi_{max} \simeq 36.5^\circ$

$\tau = \gamma z \cos\beta \sin\beta = 36.7$ kPa

$\frac{\tau}{\sigma'} = 0.757$

$\phi_{mob} = 37.1$

marginally unstable..

Assessor's Comments

Q1 Tunnelling:

Quite a popular question and generally well tackled by those who solved it. The main substantial error was by candidates who chose to represent the process as purely elastic and hence missed substantial parts of the complexity of the problem.

Q2 Stress Paths Quite a popular question but not very well dealt with on average. Parts a-c contained mostly minor errors, but there were few good attempts at any substantive analyses of stability in part d.

Q3 Triaxial testing

A popular question with many excellent attempts. The average mark was dropped by many incomplete solutions, probably due to lack of time. Where the question was completed, the marks were generally high.

Q4 Slope stability:

The least popular question, but well handled by most. Quite a few candidates lost marks on part a) as they considered infinite rather than finite slopes.