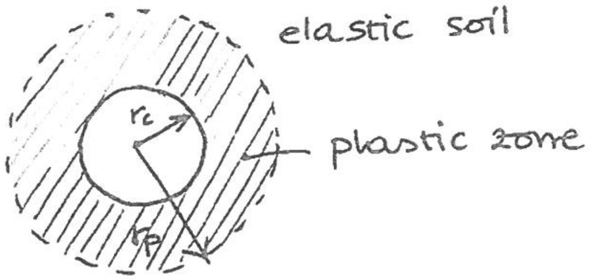
3D2

CRIB - 3D2 - 2019

1. (a)



r_c, radius of cavity (tunnel)

r_p, radius of plastic zone

For <u>elastic soil</u> and contracting cavity:

$$\sigma_r = \sigma_0 - G \frac{\delta A}{\pi r^2}$$

$$\sigma_\theta = \sigma_0 + G \frac{\delta A}{\pi r^2}$$

with δA reduction of cross-sectional area of cavity

hence:

$$\sigma_\theta - \sigma_r = 2G \frac{\delta A}{\pi r^2}$$

In <u>plastic zone</u>:

$$\sigma_\theta - \sigma_r = 2 s_u$$

At boundary between elastic soil and plastic zone ($r = r_p$):

$$\cancel{2} s_u = \cancel{2} G \frac{\delta A}{\pi r_p^2} \qquad (1)$$

current cross-sectional area of cavity:

$$A = \pi r_c^2 \Rightarrow \pi = A / r_c^2$$

substituting in eq. (1)

$$s_u = G \frac{\delta A}{A r_p^2 / r_c^2} \Rightarrow \frac{r_p^2}{r_c^2} = \frac{G}{s_u} \cdot \frac{\delta A}{A}$$

$$\text{or} \quad \frac{r_p}{r_c} = \left(\frac{G}{s_u} \frac{\delta A}{A} \right)^{0.5} \qquad [30\%]$$

1.(b) From data book:

$$\delta\sigma_c = s_u\left[1 + \ln\frac{G}{s_u} + \ln\frac{\delta A}{A}\right]$$

$$\frac{\delta A}{A} \cong \frac{2\,\rho_c}{r_c} = \frac{2 \times 0{,}08}{4} = 0{,}04$$

$$\delta\sigma_c = 100 \times \left[1 + \ln\frac{30\times10^3}{100} + \ln 0{,}04\right] \;(\mathrm{kPa})$$

$$\delta\sigma_c \cong 350 \text{ kPa}$$

$$\sigma_0 = \frac{\sigma_v + 2\sigma_h}{3} \qquad \text{assuming } k_0 \cong 1$$

$$\sigma_0 = \sigma_v = \gamma z_0 = 20 \times 30 = 600 \text{ kPa}$$

$$\sigma_{radial} = \sigma_0 - \delta\sigma_c \cong 600 - 350 = 250 \text{ kPa}$$

[25%]

1.(c) for cavity contraction under undrained conditions (constant volume), mass continuity requires that:

$$2\pi r \rho = \text{const}$$ ρ radial displacement at radius r

hence:

$$\rho = \frac{r_c\,\rho_c}{r}$$ r_c ($= 4$ m) and ρ_c ($= 0{,}08$ m) are radius and displacement at tunnel cavity

at toe of piles

$$r = r_c + 6 = 4 + 6 = 10 \text{ m}$$

$$\rho = \frac{4 \times 0{,}08}{10} = 0{,}032 \text{ m} = 32 \text{ mm}$$

[20%]

1. (d)

$$\frac{r_p}{r_c} = \left(\frac{G}{s_u} \frac{\delta A}{A} \right)^{0.5} \quad \text{from question } 1(a)$$

$$r_p = r_c \left(\frac{G}{s_u} \frac{\delta A}{A} \right)^{0.5} = 4 \left(\frac{30 \times 10^3}{100} \times 0.04 \right)^{0.5} = 13.86 \text{ m}$$

at toe of piles

$r = 10$ m

centre pile extend into plastic zone by:

$r_p - r = 13.86 - 10 = 3.86$ m [25%]

2. (a)

The main challenges are brittleness and progressive failure. For clays, progressive softening due to cycles of drying and wetting may also be an issue.

Soils yielding on the dry side of critical do so with dilation and softening. Therefore, these soils are brittle, with their supercritical shear strength falling towards critical state with deformation.

For dense sands, the peak friction angle, φ_{max}, depends on relative density and stress. If peak strength is mobilised at any point of a slip surface, soil fails locally. Because dense sand has brittle stress-strain characteristic the stress at the point of failure is reduced, and the stress is transferred at adjacent points, which in turn may fail. Deformations will localise in a shear band. This is progressive failure, which cannot happen if $\varphi_{mob} < \varphi_{cs}$.

To avoid catastrophic failure, one should design slopes in dilatant materials so that they would only mobilise their ultimate critical state angle of friction.

For clays, excavation is undrained, it unloads and shears the soil, and the pore water pressures become depressed ($\Delta u < 0$) so that $\sigma'_{short\,term} > \sigma'_{final}$ and temporary undrained shear strength exceeds ultimate drained strength

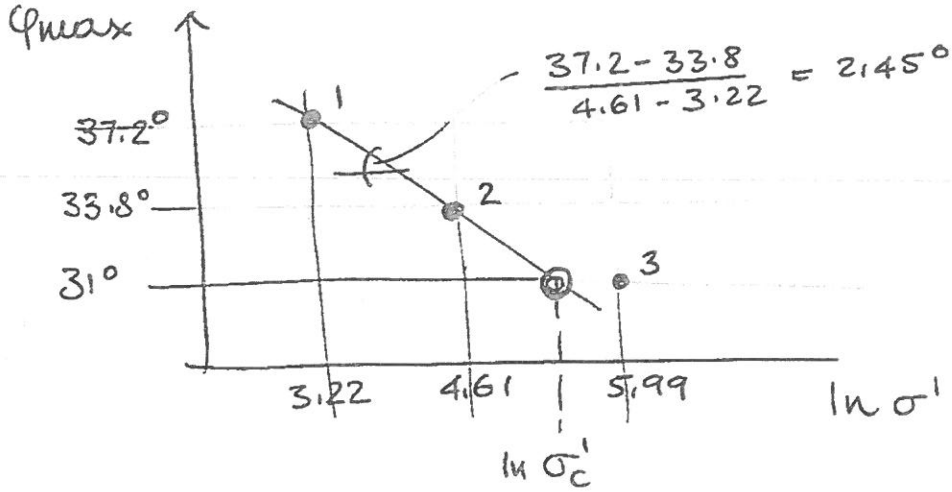
[20%]

2.(b)

sample #	1	2	3
σ' [kPa]	25	100	400
τ_{max} [kPa]	19	67	240
φ'_{max} [°]	37.2	33.8	31
$\ln \sigma'$	3.22	4.61	5.99

[20%]

i) from sample 3: $\varphi_{cs} = 31°$, consistent with the desciption of silty sand.



$$\frac{37.2-33.8}{4.61-3.22} = 2.45°$$

$$\frac{\ln \sigma'_c - 3.22}{31-37.2} = \frac{4.61-3.22}{33.8-37.2}$$

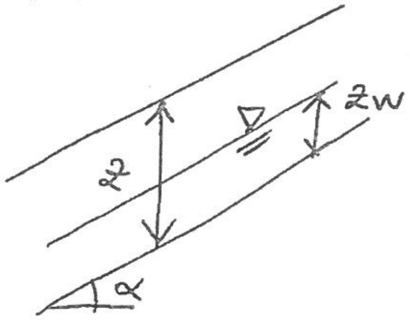
$$\ln \sigma'_c = \frac{4.61-3.22}{33.8-37.2}(31-37.2) + 3.22 = 5.47$$

$$\sigma'_c = e^{5.47} \simeq 237 \text{ kPa}$$

$$\varphi'_{max} = 37.2° - 2.45° \ln \frac{\sigma'}{237}$$

[20%]

2. (ii)



$\gamma = 20\,kN/m^3$ $\alpha = 25^\circ$ $z = 6\,m$

$\sigma = \gamma z \cos^2\alpha$ $u = \gamma_w z_w \cos^2\alpha$

$\tau_{mob} = \gamma z \sin\alpha \cos\alpha$ $\sigma' = \sigma - u$

$\varphi_{mob} = tg^{-1} \dfrac{\tau_{mob}}{\sigma'}$ $\varphi_{cs} = 31^\circ$

$\varphi_{max} = 37.2^\circ - 2.45 \ln \dfrac{\sigma'}{237}$ from 2 (b)

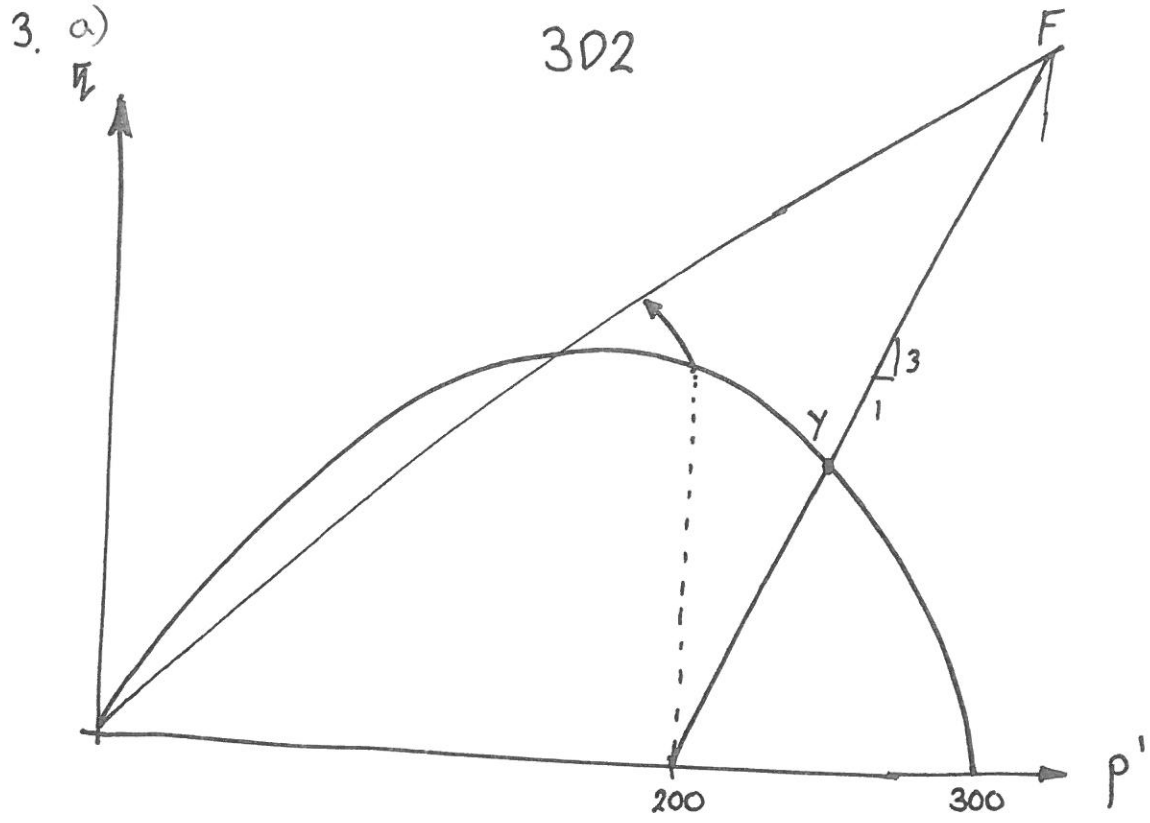
iii)

	Dry season	Wet seas.	water@g.l.
z_w (m)	1	4	6
σ (kPa)	98.6	98.6	98.6
u (kPa)	8.2	32.9	49.3
σ' (kPa)	90.4	65.7	49.3
τ_{mob} (kPa)	46.0	46.0	46.0
φ_{mob} (°)	27.0	35.0	43
φ_{max} (°)	39.6	40.3	41

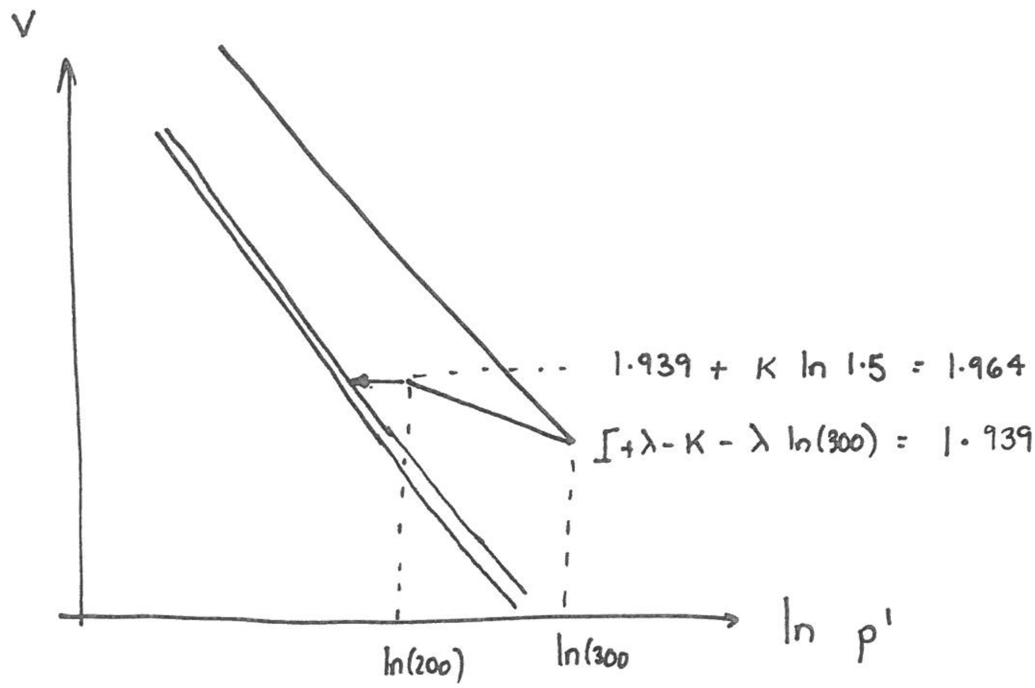
Both in the dry season and in the wet season, the mobilised angle of friction is less than the peak angle of friction $\varphi_{mob} < \varphi_{max}$, so no first time failure if the wet season is duplicated. [30%]

2 (iv) dilatant creep might be expected as in the wet season $\varphi_{mob} > \varphi_{cs}$. Also, the worst wet season could be with $z_w = 6\,m$. In this case, $\varphi_{mob} > \varphi_{max}$ (see above) and the slope will experience first time catastrophic failure [10%]

3. a)



~~KB~~ London Clay $\lambda = 0.161$ $K = 0.062$ $\Gamma = 2.759$ $M = 0.89$



At F $\quad q = Mp'$

$$p' = 200 + \frac{q}{3} = 200 + Mp'/3$$

$$p' = \frac{200}{\left(1 - \frac{M}{3}\right)} = 284\text{ kPa} \qquad q = 253\text{ kPa}$$

$C_u = 126.5\text{ kPa}$

$v = \Gamma - \lambda \ln(284) = 1.849$

$\varepsilon_v = 5.8\%$.

At Y : $\frac{q}{3} = p' - 200$

$$\frac{q}{p'} = M \ln \frac{p'_{oc}}{p'} = 0.89 \ln \frac{300}{p'}$$

$p' = \underline{\underline{220\,kPa}}$ $q = \underline{\underline{60\,kPa}}$ τ at yield $= \underline{\underline{30\,kPa}}$

$v = 1.939 - \kappa \ln(1.1) = 1.933$

$\varepsilon_v = \frac{0.006}{1.939} = 0.3\%$

b)

Undrained

At failure $q = Mp'$

$\Gamma - \lambda \ln p' = 1.964$

$p' = 139.5\,kPa$

$q = 124\,kPa$

$\tau = \underline{\underline{62\,kPa}}$

$u = 200 + \frac{124}{3} - 139.5$

$= 101.8\,kPa$

At Y $q = Mp' \ln \frac{300}{200} = 72\,kPa$

$u = 200 + \frac{72}{3} - 200 = \underline{24\,kPa}$

c) Rate to achieve drained loading.

max vol strain rate relative to shear strain at yield.

$\frac{d\varepsilon_v}{d\varepsilon_p}$ is ~~pp~~ normal to yield surface.

$$\frac{q}{\cancel{p'}} = Mp' \ln \frac{300}{p'}$$

$$\frac{\partial}{\partial p'}\left(\ln \frac{300}{p'}\right) = -\cancel{\frac{1}{3}}\cancel{\frac{300}{p'}}$$

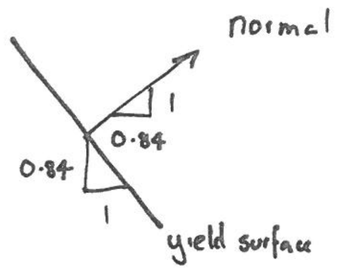
$$\frac{\partial}{\partial p'}\left(\ln(300) - \ln p'\right)$$

$$= -\frac{1}{p'}$$

$$\frac{\partial q}{\partial p'} = M \ln \frac{300}{p'} - \frac{M\cancel{p'}}{\cancel{p'}}$$

$$p' = 284 \quad \text{so} \quad \frac{\partial q}{\partial p'} = -0.84$$

$$\frac{\varepsilon_{vp}}{\varepsilon_{sp}} = \frac{0.84}{1} = \frac{(\varepsilon_a + 2\varepsilon_r)}{2/3\,(\varepsilon_a - \varepsilon_r)}$$



$$(\varepsilon_a + 2\varepsilon_r) = 0.84 \times 2/3\,(\varepsilon_a - \varepsilon_r)$$

$$0.44\,\varepsilon_a = -2.56\,\varepsilon_r$$

$$\varepsilon_a = -5.82\,\varepsilon_r$$

$$\varepsilon_v = \left(1 - \frac{2}{5.82}\right)\varepsilon_a = \underline{\underline{0.65\,\varepsilon_a}}$$

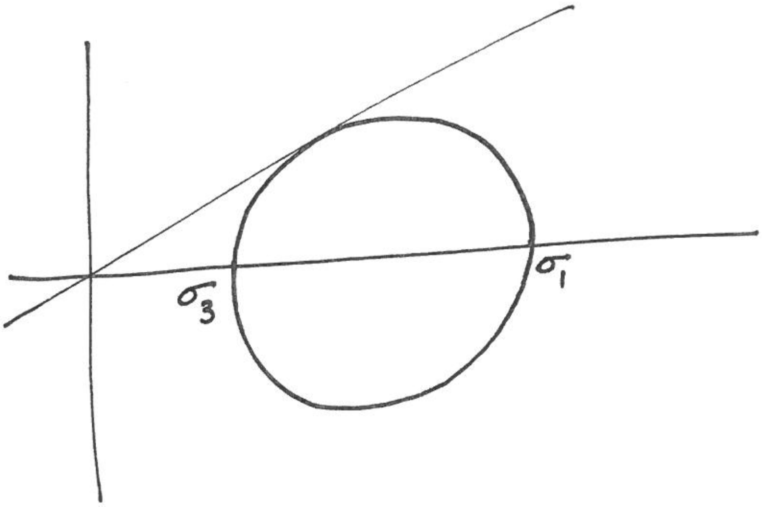
$\Delta u = 1$ kPa on 50 mm $\quad i = \frac{0.1}{0.05} = 2$

$v = 2k = 2 \times 10^{-8}$ m/s

$\varepsilon_v = 2 \times 10^{-7}/0.1 = 2 \times 10^{-7}$ $\quad \varepsilon_a = \underline{\underline{3 \times 10^{-7}}}$ m/s

4 a) Critical state is a set of σ', τ, v at which soil will shear with constant conditions, not contracting or dilating. Increasing σ' will give increased tendency to contract thus making the c.s. denser.

b) The Mohr's circle looks the same:



However as M is defined by q/p' and

$$p' = \frac{2\sigma_{cell} + \sigma_{axial}}{3},$$

it matters which is bigger as p' is not at the centre of the Mohr's circle.

c) 1) Plastic work done in volumetric + shear strain is all dissipated in friction.

2) Normality applies, maximising plastic work done.

d)

Work equation

$$\sigma' \, d\varepsilon_{vp} + \tau \, d\gamma_p = \mu_{crit} \, \sigma' \, d\gamma_p$$

Normality

Plastic strain normal to yield surface:

$$\frac{d\tau}{d\sigma'} = - \frac{d\varepsilon_{vp}}{d\gamma_p}$$

$$-\sigma' \frac{d\tau}{d\sigma'} d\gamma_p + \tau \, d\gamma_p = \mu_{crit} \, \sigma' \, d\gamma_p$$

$d\gamma_p = 0$ or

$$\tau - \sigma' \frac{d\tau}{d\sigma'} = \mu_{crit} \, \sigma'$$

Substitute $\mu = \frac{\tau}{\sigma'}$ $\quad \sigma' \, d\mu + \mu \, d\sigma' = d\tau$

$$\mu \sigma' - \sigma' \left(\mu + \sigma' \frac{d\mu}{d\sigma'} \right) = \mu_{crit} \, \sigma'$$

$$\sigma' \frac{d\mu}{d\sigma'} = - \mu_{crit}$$

$$\Big[\ln \sigma' \Big]^{\sigma'_c} = \left[\frac{\mu}{\mu_{crit}} \right]^{\mu}$$

Q.E.D.