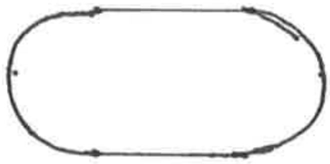
Qu 1

a)

(i) - thin-walled closed cross-section

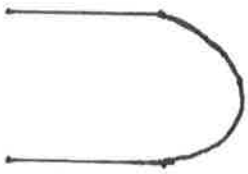


$$A = 2r \cdot 2r + \pi r^2 = 7{,}142\, r^2$$

$$\int \frac{ds}{t} = \frac{2 \cdot 2r + 2\pi r}{t} = 10{,}28\, \frac{r}{t}$$

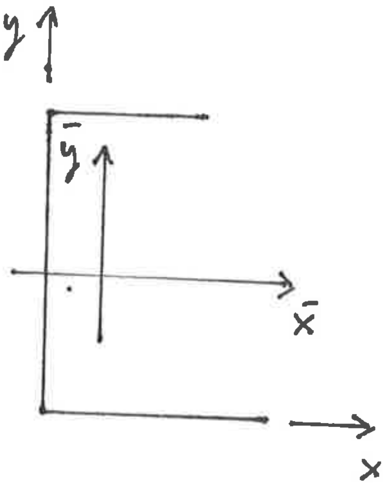
$$J = \frac{4 \cdot (7.142\, r^2)^2 t}{10{,}28} = \underline{19.84\, r^3 t}$$

- thin-walled open cross-section



$$J = \frac{1}{3} \int t^3\, ds = \frac{t^3}{3}\left(4r + \pi r\right) = 2{,}381\, r t^3$$

b)



$$A = 9\cancel{b}t$$

$$x_s = \frac{2b \cdot t \cdot 1b + 3b t \cdot 1.5 \cdot b}{9\, b t}$$

$$= 0{,}7222\, b$$

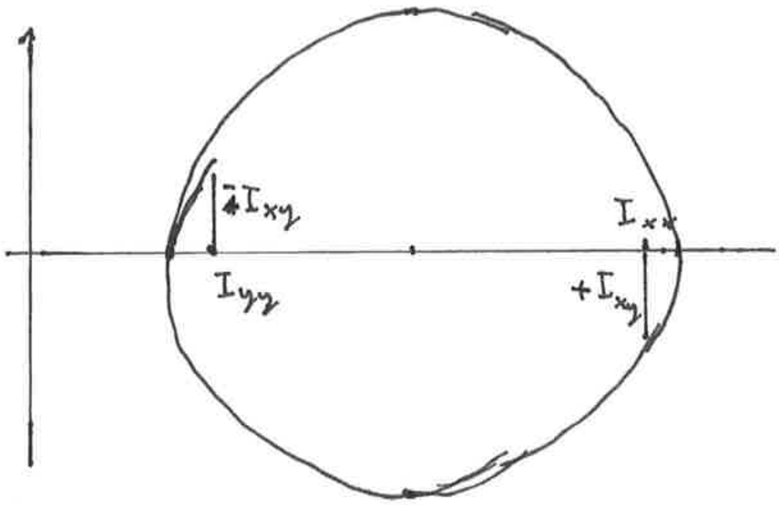
$$y_s = \frac{2bt \cdot 4b + 4bt \cdot 2b}{9bt}$$

$$= 1{,}778\, b$$

$$I_{xx} = 2bt \cdot (2.222b)^2 + 3bt(1.778b)^2$$
$$+ \frac{(4b)^3 t}{12} + 4bt \cdot (0.222b)^2$$
$$= \underline{24.89\, b^3 t}$$

$$I_{yy} = 4bt \cdot (0.7222b)^2 + \frac{(3b)^3 t}{12} + 3bt(0.778b)^2$$
$$+ \frac{(2b)^3 t}{12} + 2bt \cdot (0.278b)^2$$
$$= \underline{6.972\, b^3 t}$$

$$I_{xy} = 3bt \cdot 0.7778b(-1.778b) + 2bt(2.222b)(0.2778b)$$
$$+ 4bt \cdot (0.222b)(-0.7222b)$$
$$= \underline{-3.556\, b^3 t}$$



Radius

$$R^2 = \left[(24.89 - 15.93)^2 + 3.556^2\right] b^6 t^2$$
$$\Longrightarrow R = \underline{9.640\, b^3 t}$$

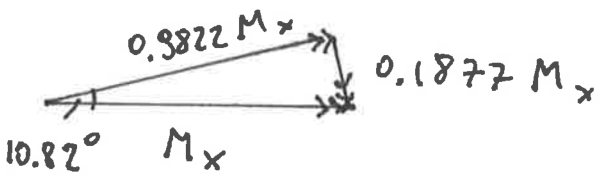
Centre

$$C = \frac{24.89 + 6.972}{2} b^3 t = \underline{15.93\, b^3 t}$$

$$I_{\xi\xi} = (15.93 + 9.64)\, l^3 t = \underline{25.57\, l^3 t}$$

$$I_{\eta\eta} = (15.93 - 9.64)\, l^3 t = \underline{6.29\, l^3 t}$$

$$\tan 2\theta = \frac{3.556}{(24.89 - 15.93)} = 0.3968 \implies \theta = 10.82^\circ$$



$$\sigma = \frac{M_{\xi\xi}}{I_{\xi\xi}} \cdot \eta - \frac{M_{\eta\eta}}{I_{\eta\eta}} \cdot \xi$$

<u>Coordinate transformation</u>

$$\begin{pmatrix} \xi \\ \eta \end{pmatrix} = \begin{pmatrix} 0.9822 & 0.1877 \\ -0.1877 & 0.9822 \end{pmatrix} \begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix}$$

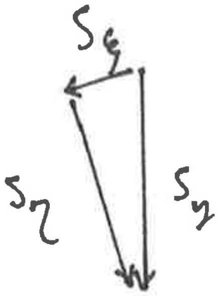
<u>Coordinates of A</u>

$$\left.\begin{array}{l} \bar{x} = 1.2778 \\ \bar{y} = 2.222 \end{array}\right\} \implies \begin{array}{l} \xi = 1.672 \\ \eta = 1.943 \end{array}$$

$$\implies \sigma = \frac{0.9822\, M_x}{25.57\, l^3 t}\, 1.943\, l + \frac{0.1877\, M_x}{6.29\, l^3 t}\, 1.672\, l$$

$$= \underline{\underline{0.1245 \frac{M_x}{l^2 t}}}$$

ii)



$$q = \frac{S_\xi}{I_{\xi\xi}} \int \eta \, dA + \frac{S_\eta}{I_{\eta\eta}} \int \xi \, dA$$

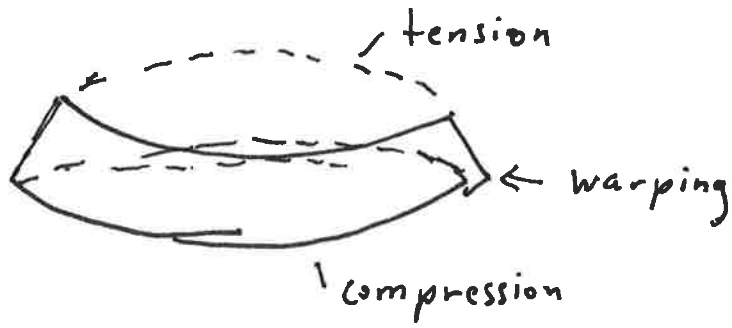
Q1. Examiner's Comment:

Overall this was a popular and well answered question. In part (a), almost all candidates were able to compute the torsion constants for both sections and demonstrated a basic understanding of warping torsion. Although most were able to compute the section properties in part (b), there were many arithmetic errors due to the length of the calculations. Few did not make use of the thin-walled assumption, which led to even lengthier calculations. Unusually high number of candidates had problems with the required coordinate transformations and decomposition of the moment vector. As expected the last question on shear flow for an unsymmetric cross-section (not treated in the course) was correctly answered by only a few.

2) a)

Warping means that plane sections do not remain plane

Lateral-torsional buckling (LTB) means compression flange wants to buckle and the tension flange does not.



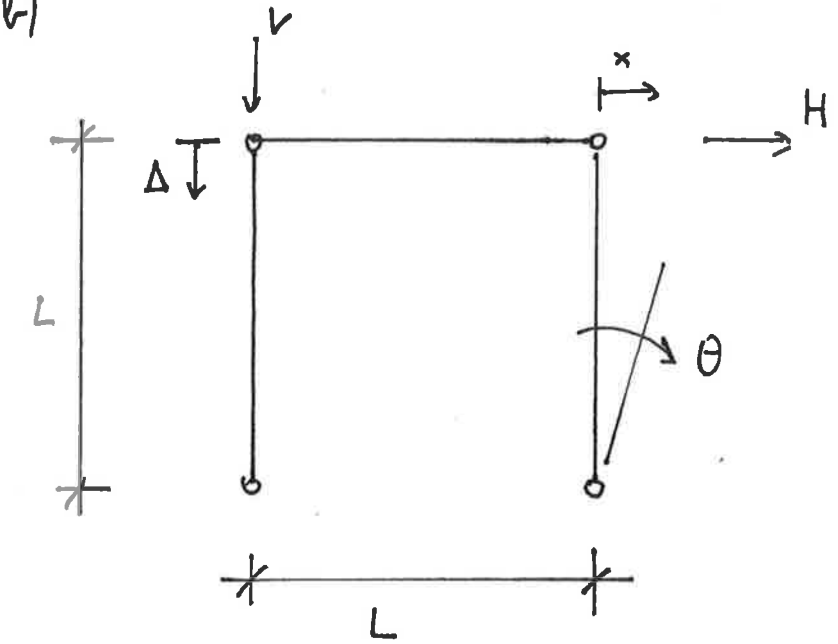
But steel beams do not want to warp. They resist it.

(Differential curvatures $\Rightarrow$ differential shear stresses $\Rightarrow$ resisting torque (warping restraint))

This resistance adds stiffness, so warping restraint means beam under-goes LTB at a higher critical moment than if are ignored warping.

$$M_{cr} = \underbrace{\frac{\pi}{L}\sqrt{GJEI_y}}_{\text{basic } M_{cr}}\ \underbrace{(1+\dots)^{1/2}}_{\text{increase due to restrained warping torsion}}$$

b)



i) Total potential energy

$$\Pi = \frac{1}{2} k z^2 - V \cdot \Delta - H x$$

Geometry: $\sin\theta = \frac{x}{L} \approx \theta$

$$\Delta = L(1-\cos\theta)$$

$$\frac{\Delta}{L} = 1-\cos\theta = 1-\left(1-\frac{\theta^2}{2}+\dots\right) \approx \frac{\theta^2}{2}$$

$$z = \sqrt{(L+x)^2+(L-\Delta)^2} - \sqrt{2}L$$

$$\frac{z}{L} = \sqrt{\left(1+\frac{x}{L}\right)^2+\left(1-\frac{\Delta}{L}\right)^2} - \sqrt{2}$$

$$\approx \sqrt{(1+\theta+\dots)^2+\left(1-\cancel{\frac{\theta^2}{2}}+\dots\right)^2} - \sqrt{2}$$

not

$$\approx \sqrt{1+2\theta+1} - \sqrt{2}$$

$$\approx \sqrt{2}\left(\sqrt{1+\theta}-1\right) \approx \sqrt{2}\left(1+\frac{\theta}{2}+\dots-1\right)$$

$$\approx \theta/\sqrt{2}$$

$$\Pi = \frac{1}{2} k \frac{\theta^2}{2} L^2 - V \frac{\theta^2}{2} L - H\theta L$$

(ii) $\Pi = \frac{1}{2} k \frac{\theta^2}{2} L^2 - V \frac{\theta^2}{2} L - H\theta L$

$$\frac{\partial \Pi}{\partial \theta} = \frac{1}{2} k L^2 \theta - V\theta L - HL$$

<u>Equilibrium</u>

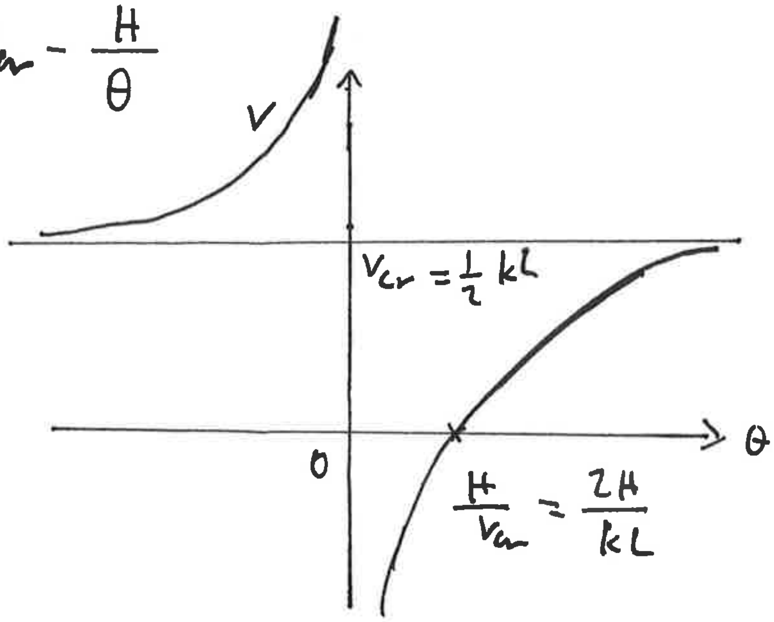
$$\frac{1}{2} k L^2 \theta - V\theta L - HL = 0$$

$$\left[\frac{1}{2} kL - V\right]\theta = H$$

$$\Longrightarrow \quad V_{cr} = \frac{1}{2} kL$$

iii) $\theta = \dfrac{H}{\frac{1}{2} kl - V} = \dfrac{H}{V_{cr} - V}$

$$V = V_{cr} - \frac{H}{\theta}$$



for $H > 0$

iv)

$V = 0.4\, kL$

$$\theta = \frac{H}{0.5kL - 0.4kL} = 10 \frac{H}{kL}$$

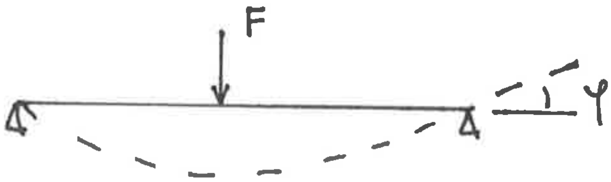
$V = -1.5\, kL$

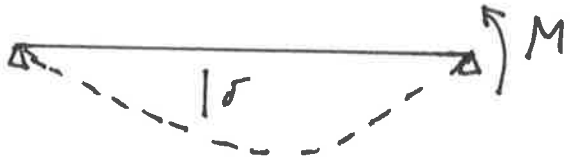
$$\theta = \frac{H}{0.5kL + 1.5kL} = \frac{H}{2kL} = 0.5 \frac{H}{kL}$$ (one twentieth of previous deflection)

Q2. Examiner's Comment:

This question appeared to have the right level of difficulty and was in general well answered. Almost all candidates had some understanding of the role of warping in lateral-torsional buckling. In part (b), most were able to write down the potential energy, but had problems with linearising using the small angle assumption. In addition, there was some confusion whether the critical load is determined using the derivative of the load with respect to the angle or the second derivative of the potential. Very few candidates mixed up the critical load with the Euler buckling load.

3 (a)





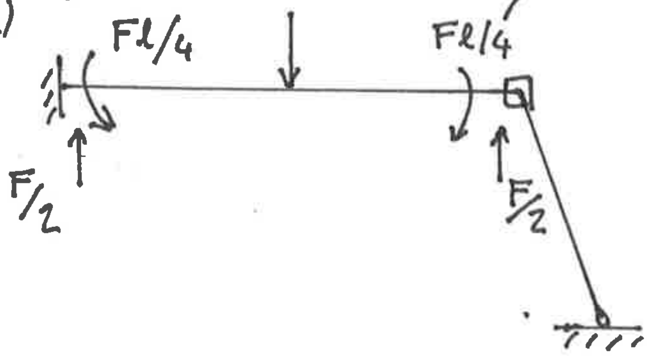
$$F \cdot \delta = M \varphi$$

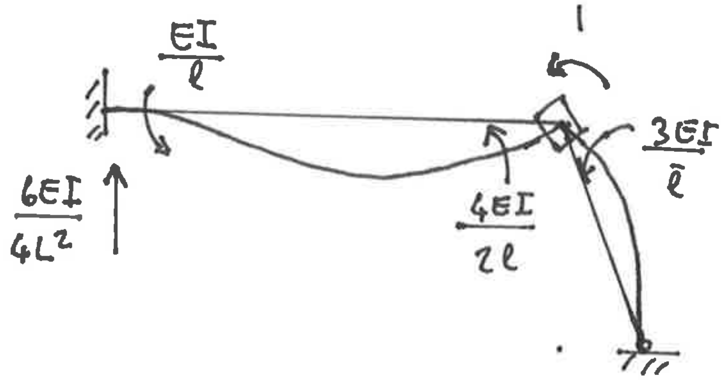
(b)

(i) kinematic indeterminacy : 1

static indeterminacy : 2

(ii)





length of inclined beam

$$\bar{l}^2 = \frac{l^2}{16} + l^2$$

$$\Rightarrow \bar{l} = 1.031 l$$

$$K_{11} = \frac{4EI}{2l} + \frac{3EI}{1.031l} = 4.91 \frac{EI}{l}$$

Rotation of connection B

$$r = \frac{Fl}{4} \cdot \frac{l}{4.91EI} = 0.0509 \frac{Fl^2}{EI}$$

iii)

Bending moment at A

$$M = \frac{F\ell}{4} + \frac{EI}{\ell}\, 0.0509\, \frac{F\ell^2}{EI} = 0.301\, F\ell$$

$$S = \frac{F}{2} + \frac{6EI}{4\ell^2} \cdot 0.0509\, \frac{F\ell^2}{EI} = 0.576\, F\ell$$

(c) Torsional moments

$$T = GJ\,\frac{r}{\ell} = 3EI\,\frac{r}{\ell}$$

$$K_{11} = 4.91\,\frac{EI}{\ell} + 3\,\frac{EI}{\ell} = 7.91\,\frac{EI}{\ell}$$

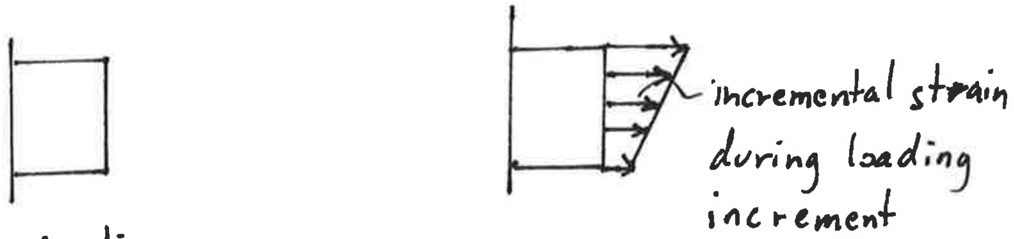
$$r = \frac{F\ell}{4} \cdot \frac{\ell}{7.91\, EI} = 0.0316\, \frac{F\ell^2}{EI}$$

Q3. Examiner's Comment:

This was the least popular, but well answered question (especially in comparison to previous years). In the introductory part (a), almost all candidates were able to write down the reciprocal theorem. In part (b), most seemed to have a good working knowledge of the displacement method and could keep it apart from the force method. The only common point of confusion was the computation of the moment and shear force at the support.

4) a)

Shanley's insight was to recognise that one could obtain curvature increases <u>without</u> strain reversal if that curvature increase occured during the loading increment.

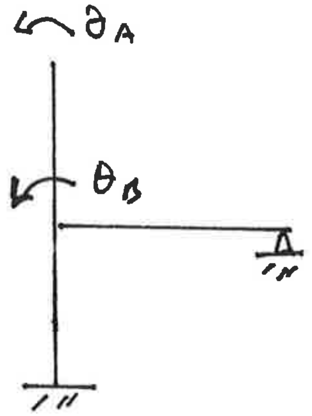


Strain diagram

Principle of Virtual Displacement assumes loads are fixed and a small virtual displacement dx (or, curvature δk, etc) is applied. Shanley's insight shows that this misses an infinite number of other possible solutions for inelastic problems (which are necessarily path dependent)

b) i)

$$\begin{bmatrix} M_A \\ M_B \end{bmatrix} = \frac{EI}{L} \begin{bmatrix} s & sc \\ sc & 2s+3 \end{bmatrix} \begin{bmatrix} \theta_A \\ \theta_B \end{bmatrix}$$



critical when det $= 0$

$$(2s+3)s - s^2c^2 = 0$$

$$2s^2 + 3s - s^2c^2 = 0$$

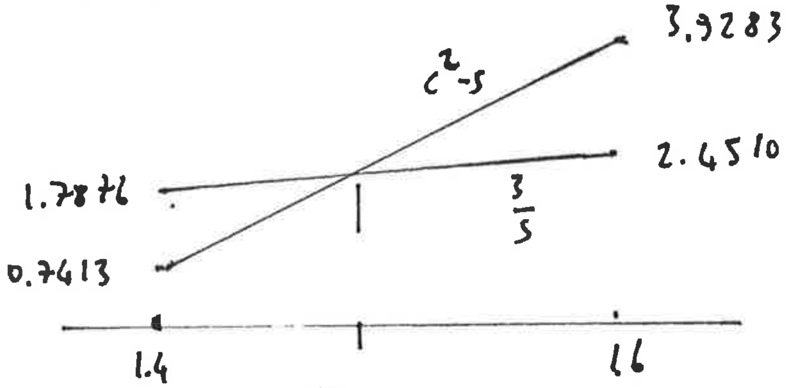
$$s^2(2-c^2) + 3s = 0$$

$$s\left[s(2-c^2) + 3\right] = 0$$

- Need $s = 0$ or $s(2-c^2)+3 = 0$

$$\Rightarrow c^2 - s = \frac{3}{s}$$

P/P_E	s	c	c^2-s	$3/s$
1.2	2.0901	1.2487		
1.4	1.6782	1.6557	0.7413	1.7876
1.6	1.2240	2.4348	3.9283	2.4510



$$\therefore \; P/P_E \approx 1.5$$

$$P_{cr} = 1.5\, P_E = 1.5\, \frac{\pi^2 EI}{L^2} = 14.8\, \frac{EI}{L^2}$$

ii) Relative rotations

$$\frac{EI}{L} \begin{bmatrix} s-\lambda & sc \\ sc & (2s+3)-\lambda \end{bmatrix} \begin{bmatrix} \theta_A \\ \theta_B \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

1st line $\quad (s-\lambda)\,\theta_A + sc\,\theta_B = 0$

$$\Rightarrow \theta_B = -\frac{(s-\lambda)}{sc}\,\theta_A$$

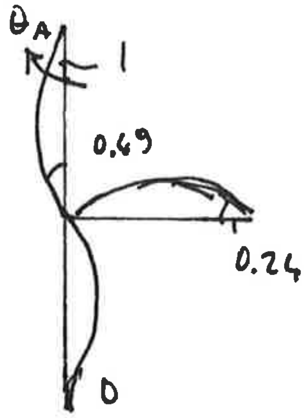
At buckling $\lambda = 0$ $\quad \Rightarrow \theta_B = -\frac{\theta_A}{c}$

$$c = \frac{1.6557 + 2.4348}{2} = 2.05$$

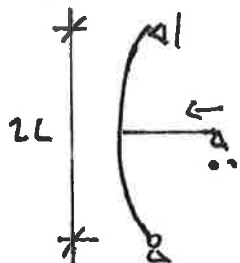
$\theta_B = -0.49\,\theta_A$

$\theta_C = 0$ fixed

$\theta_D = -\frac{1}{2}\,\theta_B = 0.24\,\theta_A$



iv) $P_{cr} = \dfrac{\pi^2 EI}{4L^2}$



Q4. Examiner's Comment:

For most parts this was a straightforward and well answered question. Quite a few candidates could not recall Shanley's resolution of the column paradox. In part (b), with few exceptions most were able to write down the stiffness matrix. All candidates knew to consider the determinant of the stiffness matrix to obtain the buckling load. It was slightly irritating to see that few did not know how to compute the requested buckling mode shape.