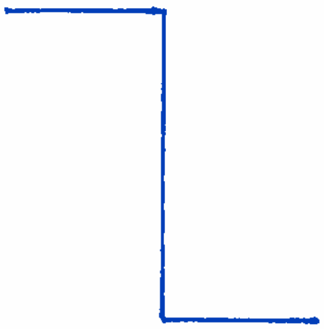
1a)



i) $J = \frac{2}{3} b t^3 + \frac{1}{3} b t^3 = \frac{5}{3} b t^3$

ii) $\Gamma = \frac{(3b)^2}{4} I_{yy}$

$$I_{yy} = 2\left[\frac{b^3 t}{12} + bt\left(\frac{b}{2}\right)^2\right] = \frac{1}{6} b^3 t + \frac{1}{2} b^3 t = \frac{2}{3} b^3 t$$

$$\Longrightarrow \quad \Gamma = \frac{9}{4} b^2 \, \frac{2}{3} b^3 t = \frac{3}{2} b^5 t$$

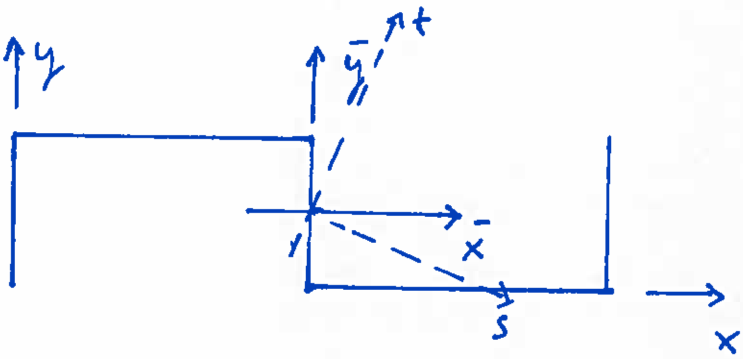
$$\lambda^2 = \frac{E}{G} \, \frac{3}{2} b^5 t \, \frac{3}{5} b^3 t \quad \Longrightarrow \quad \lambda = \frac{3b^2}{t}\sqrt{\frac{E}{10G}}$$

$$T = GJ\phi' \quad \Longrightarrow \quad \phi' = \frac{T}{GJ}$$

$$\phi = (L-\lambda)\frac{T}{GJ} = \left(L - \frac{3b^2}{t}\sqrt{\frac{E}{10G}}\right)\frac{3Tb}{5bt^3G}$$

$$= 0{,}6\,\frac{T}{Gbt^3}\left(L - 0.949\,\frac{b^2}{t}\sqrt{\frac{E}{G}}\right)$$

1b)

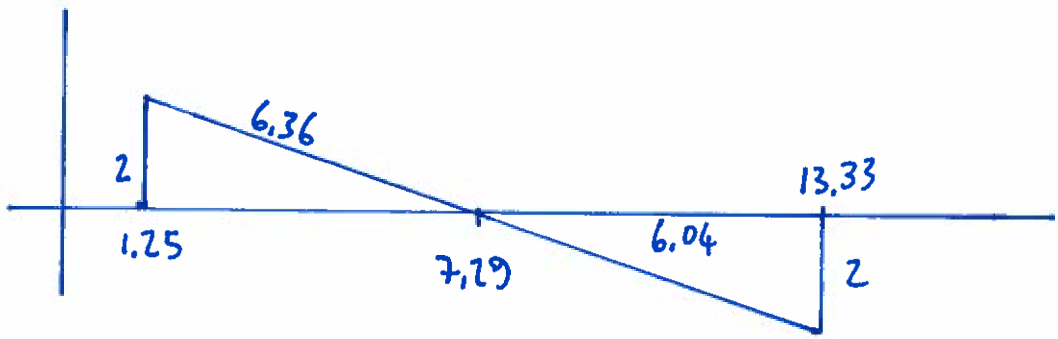


Centroid: $x_s = 2b \quad y_s = \frac{b}{2}$

$$I_{xx} = 3\frac{\ell^3 t}{12} + 2\left(\frac{\ell}{2}\right)^2 2\ell t = \frac{5}{4}\ell^3 t$$

$$I_{yy} = 2\left(\ell t\cdot(2\ell)^2\right) + 2\left(\frac{8\ell^3}{12}t + 2\ell t\,\ell^2\right) = \frac{40}{3}\ell^3 t$$

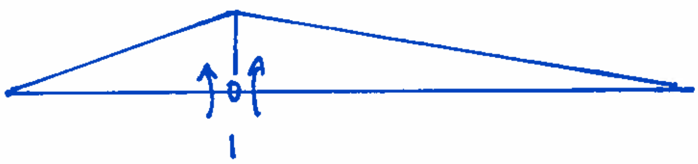
$$I_{xy} = 2\ell t(-\ell)\frac{\ell}{2} + 2\ell t(\ell)\left(-\frac{\ell}{2}\right) = -2\ell^3 t$$



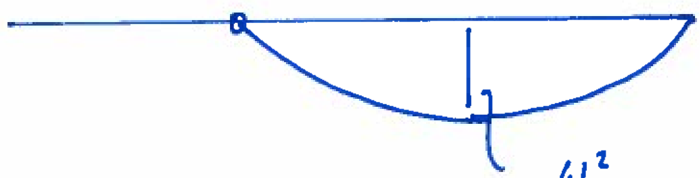
$$I_{ss} = 13.65\,\ell^3 t \qquad I_{tt} = 0{,}928\,\ell^3 t$$

$$\tan(2\theta) = \frac{2}{6{,}04} \implies \theta = 9{,}16^\circ$$

2a)



$$EI\delta_{11} = \frac{1}{3}L + \frac{2}{3}L = L$$



$$EI\delta_{10} = \frac{2L}{3}L^2 w = \frac{L^3 w}{3}$$

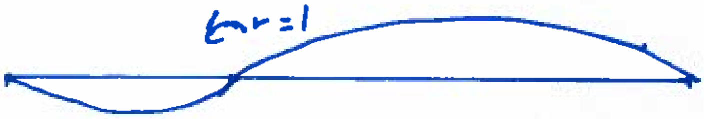
$$w\frac{4L^2}{8} = \frac{L^2}{2}w$$

$$X\delta_{11} = \delta_{10}$$

$$\Longrightarrow X = \frac{\delta_{10}}{\delta_{11}} = \frac{L^3 w}{3L} = \underline{\frac{L^2 w}{3}}$$

2b)

$$\frac{w(2L)^2}{8} = \frac{wL^2}{2}$$



$$\frac{3EI}{L}$$

$$\frac{3EI}{2L}$$

$$\left(\frac{3}{2}\frac{EI}{L} + \frac{3EI}{L}\right)r = \frac{wL^2}{2} \Longrightarrow EIr = \frac{wL^3}{9}$$

$$\Longrightarrow M_B = \frac{3EI}{L}\frac{wL^3}{9EI} = \underline{\frac{L^2 w}{3}}$$

2c)

$$-EIw'' = -R_A z - R_B \{z-L\} + w\frac{\{z-L\}^2}{2}$$

$$-EIw' = -R_A\frac{z^2}{2} - \frac{R_B}{2}\left\{\frac{z-L}{2}\right\}^2 + \frac{w}{6}\{z-L\}^3 + C_1$$

$$-EIw = -R_A\frac{z^3}{6} - \frac{R_B}{6}\left\{\frac{z-L}{2}\right\}^3 + \frac{w}{24}\{z-L\}^4 + C_1 x + C_2$$

Five unknowns; need five equations

$w=0$ at $z=0$, $z=L$ and $z=3L$

$R_A + R_B + R_C = 2Lw$ vertical equilibrium

$3R_C L + R_B L = 2wL(2L) = w4L^2$ moment equilibrium about $z=0$

$w(0)=0 \implies C_2 = 0$

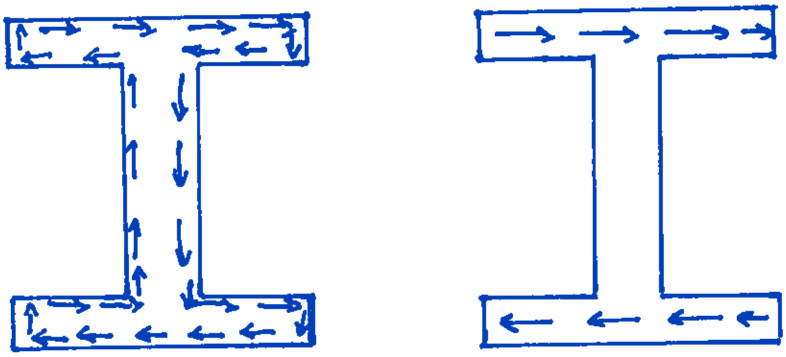
$w(L)=0 \quad -R_A\frac{L^3}{6} + C_1 L = 0 \implies C_1 = R_A\frac{L^2}{6}$

$w(3L)=0 \quad -R_A\frac{9L^3}{2} - R_B\frac{L^3}{6} + w\frac{2}{3}L^4 + R_A\frac{L^3}{2} = 0$

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1/3 & 1 \\ 4 & 1/6 & 0 \end{pmatrix}\begin{pmatrix} R_A \\ R_B \\ R_C \end{pmatrix} = \begin{pmatrix} 2Lw \\ \frac{4wL}{3} \\ \frac{2wL}{3} \end{pmatrix}$$

2d) $-EIw' = -R_A\frac{z^2}{2} - R_B\frac{1}{2}\left\{\frac{z-L}{2}\right\}^2 + 1.\left\{z-\frac{L}{2}\right\}^0 + C_1$

3a)



3b) $M_{LT} = \frac{\pi}{L}\sqrt{GJEI_{min}}\left(1+\frac{\pi^2 E\Gamma}{L^2 GJ}\right)^{\frac{1}{2}}$

$L = 6\ m$

$E = 210 \cdot 10^9\ N/m^2 \qquad G = E/2.6 \quad (\nu = 0.3)$

$I_{min} = 2692 \cdot 10^{-8}\ m^4$

$J = 101 \cdot 10^{-8}\ m^4$

$D_f = 536.7 - 17.4 = 519.3\ mm = 0.5193\ m$

$\Gamma = I_{min}\frac{D_f^2}{4} = \frac{2692 \cdot 10^{-8}}{4}(0.5193)^2 = 181.5\ m^6 \cdot 10^{-8}$

$1 + \frac{\pi^2 E\Gamma}{L^2 GJ} = 1 + \frac{\pi^2}{36} \cdot 2.6\ \frac{181.5}{101} = 2.28$

$M_{basic} = \frac{\pi}{L}\sqrt{GJEI_{min}} = \frac{\pi}{L}E\sqrt{\frac{I_{min}J}{2.6}} =$

$= \frac{\pi}{6}\ 210 \cdot 10^9\sqrt{\frac{101 \cdot 2692}{2.6} \cdot 10^{-8}} = 355.6\ kNm$

$M_{LT} = 355.6\sqrt{2.28} = \underline{536.9\ kNm}$

3c)

$$\begin{bmatrix} M_A \\ M_B \end{bmatrix} = \frac{EI}{L} \begin{bmatrix} s & sc \\ sc & s \end{bmatrix} \begin{bmatrix} \theta_A \\ \theta_B \end{bmatrix}$$

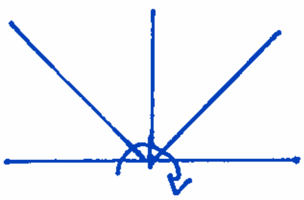
$M_B = 0 \quad \Rightarrow \quad sc\,\theta_A + s\,\theta_B = 0 \quad \Rightarrow \quad \theta_B = -c\,\theta_A$

$$M_A = \frac{EI}{L}(s\,\theta_A + sc\,\theta_B) = \frac{EI}{L}(s\,\theta_A - sc^2\theta_A)$$

$$= \underline{\frac{EI}{L}\, s(1-c^2)\,\theta_A}$$

if $P=0 \quad s=4 \quad c=\frac{1}{2} \Rightarrow s(1-c^2) = 4(1-\frac{1}{4}) = 3$

For the framework



$M_{applied} = 0$

$$M_{applied} = \frac{EI}{L}\left(2s(1-c^2) + 9\right) = 0$$

Buckling condition is $\quad 2s(1-c^2) + 9 = 0$

$$s(1-c^2) = -4.5$$

P/P_E	s	c	$s(1-c^2)$
1.6	1.224	2.4348	-6.032
1.4	1.6782	1.6557	-2.922

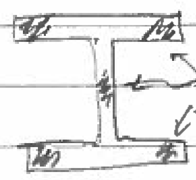
$\Rightarrow \quad P/P_E = 1.502$

$$P = \frac{\pi^2 EI}{L^2} \cdot 1.502$$

$$= 14.82\,\frac{EI}{L^2}$$

3D4 Q4. a)

Residual stresses.



compressive stresses

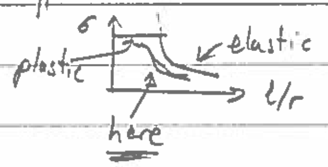
Rest in tension, to self-equilibrate
(zero net axial force)

(Due to exposed areas solidifying first, and then as rest cools + contracts, it pulls these into compression).

Can affect buckling behaviour, particularly the flange outstands.

→ can precipitate local buckling of flange outstands due to preexisting compression there

→ can affect overall buckling, particularly in elastic-plastic region

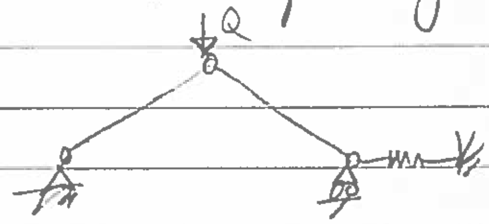


, because as the member starts to buckle, it encounters plastic behaviour sooner than it would if no residual stresses.

b). – clear demarcation between "axial" and "lateral" loads
– no "lateral" loads
– axial end shortening $\propto$ (lateral deflection)^2
– no lateral deflection when axial load is zero

These do not hold for snap-through buckling

eg



(c) Rayleigh-Ritz.

TOT. POT. ENERGY Π = strain energy − ext. W.D.

$$\Pi = \frac{1}{2}\int EI(w'')^2 dx - \frac{P}{2}\int (w')^2 dx$$

$$w = A\phi(x)$$

$$\Pi = \frac{1}{2}A^2\int EI(\phi'')^2 dx - \frac{P}{2}A^2\int(\phi')^2 dx.$$

$$= \frac{1}{2}kA^2 \quad \text{with } k = \int EI(\phi'')^2 dx - P\int(\phi')^2 dx$$

So buckles when stiffness $k \to 0$

$$\Rightarrow \quad P = \frac{\int EI(\phi'')^2 dx}{\int(\phi')^2 dx}$$

Let $\phi = x(L-x) = xL - x^2$

$\phi' = L - 2x$

$\phi'' = -2$

$$\int EI(\phi'')^2 dx = EI(4)L.$$

$$\int(\phi')^2 dx = \int_0^L (L-2x)^2 dx$$

$$= \int_0^L L^2 - 4xL + 4x^2 \, dx$$

$$= \left[L^2x - \frac{4x^2L}{2} + \frac{4x^3}{3}\right]_0^L = L^3 - 2L^3 + \frac{4L^3}{3} = \frac{L^3}{3}$$

$$\therefore \quad P_{cr} = \frac{4EIL}{L^3/3} = \frac{12EI}{L^2}$$

This is greater than Euler load $\left(\frac{\pi^2 EI}{L^2}\right)$ as $12 > \pi^2 \approx 10$.

It is higher because Rayleigh-Ritz always overestimates (or is correct). It is correct if ϕ is an eigenvector. In this case, eigenvector is half-sine wave (not quadratic). Total potential energy goes flat in eigendirection first, then other directions later (hence over estimate).

3D4 2016

Q4 d)

Perry-Robertson deals with inelasticity by avoiding it altogether.

As soon as any part of the cross-section yields, that is taken as "failure", and determines the "design" load.

This avoids all the philosophical complications associated with the path-dependent nature of inelastic behaviour, as exemplified by say Shanley's analysis which resolved the column paradox.