2015 IIA 3D5 - Water Engin1eering Dr D. Liang

1.

(a.i)

The question gives f_0 = 20 mm/hr

$$f(1) = f_c + (f_0 - f_c)e^{-K_f t} = f_c + (20 - f_c)e^{-K_f} = 10 \qquad (1)$$

$$f(2) = f_c + (f_0 - f_c)e^{-K_f t} = f_c + (20 - f_c)e^{-2K_f} = 5 \qquad (2)$$

So,

$$(20 - f_c)e^{-K_f} = 10 - f_c$$

$$(20 - f_c)e^{-2K_f} = 5 - f_c$$

Divide the above two equations on either side,

$$e^{-K_f} = \frac{5 - f_c}{10 - f_c} \qquad (3)$$

Substitute into Equation (1),

$$f_c + (20 - f_c) \times \frac{5 - f_c}{10 - f_c} = 10$$

$$f_c = 0$$

From Equation (3), $K_f = 0.693$ 1/hr

(a.ii)

There is always sufficient water supply at the top of the soil during the one-hour rainfall. Assuming the infiltration rate at the beginning of this one-hour rainfall is f_1, then

$$\int_0^1 f \cdot dt = f_c(1-0) - \frac{1}{0.693}(f_1 - f_c)\left(e^{-K_f \times 1} - 1\right) = 0 - \frac{f_1}{0.693}\left(e^{-0.693} - 1\right) = 4$$

So,

f_1=5.545 mm/hr

Calculate the time T needed for the infiltration rate to decrease from f_0 to f_1.

$$f_c + (f_0 - f_c)e^{-K_f t} = 0 + (20 - 0)e^{-0.693T} = 5.545$$

T = 1.851 hr

(T can also be directly found by solving $\int_T^{T+1} f \cdot dt = f_c \times 1 - \frac{1}{0.693}(f_0 - f_c)\left(e^{-K_f \times (T+1)} - e^{-K_f \times T}\right) = 4$)

The total infiltration in time T is:

$$\int_0^{1.851} f \cdot dt = -\frac{1}{0.693}(20 - 0)\left(e^{-0.693 \times 1.851} - 1\right) = 20.86 \text{ mm}$$

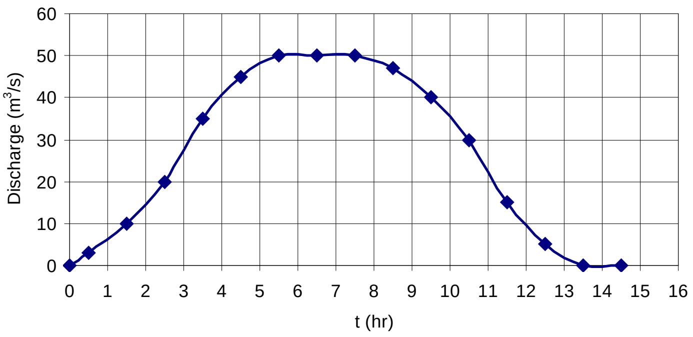
Total volume of the stored water is:

$$5 \times 10^6 \times 20.86 \times 10^{-3} = 104300 \quad \text{m}^3$$

(b.i)

From the table, the equilibrium runoff is reached after 5 hours, so the S curve is known.
Shift the S curve by 8 hours and then subtract it from the original S curve:
The complete discharge distribution of the 8-hour rainfall can be calculated.

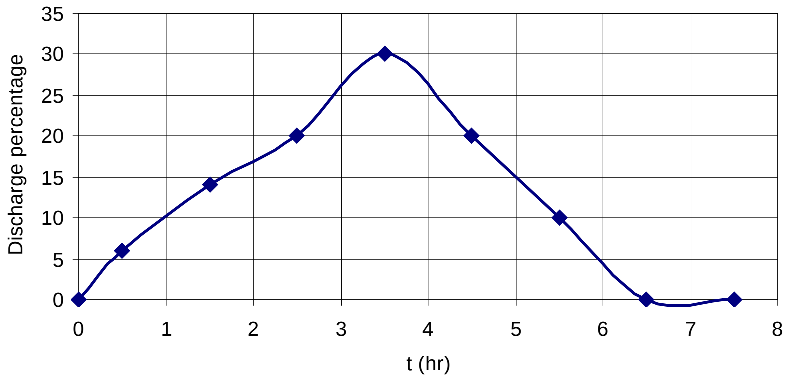
Hour	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	8.5	9.5	10.5	11.5	12.5	13.5
Duration	0-1	1-2	2-3	3-4	4-5	5-6	6-7	7-8	8-9	9-10	10-11	11-12	12-13	13-14
S curve (m^3/s)	3	10	20	35	45	50	50	50	50	50	50	50	50	50
8 hours shifted S-curve (m^3/s)									3	10	20	35	45	50
Subtract the 2 S-curves (m^3/s)	3	10	20	35	45	50	50	50	47	40	30	15	5	0



(b.ii)

Similarly to (b.i),

Hour	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5
Duration	0-1	1-2	2-3	3-4	4-5	5-6	6-7	7-8
S curve (m^3/s)	3	10	20	35	45	50	50	50
1 hour shifted S-curve (m^3/s)	0	3	10	20	35	45	50	50
Subtract the 2 S-curves (m^3/s)	3	7	10	15	10	5	0	0
%	6	14	20	30	20	10	0	0



(c)

The Biochemical Oxygen Demand (BOD) is the amount of dissolved oxygen needed by aerobic biological organisms to break down the organic material in a water body at certain temperature over a specific period. It is an indication of the amount of organic compounds in water.

2

(a.i) Energy should be conserved over the spillway.

Upstream of the spillway: $U_0 = \frac{q}{h_0} = \frac{4}{6} = 0.667 \text{ m/s}$

$$E_0 = h_0 + \frac{U_0^2}{2g} = 6 + \frac{0.667^2}{2 \times 9.81} = 6.023 \text{ m}$$

Downstream of the spillway $U_1 = \frac{q}{h_1} = \frac{4}{0.38} = 10.53 \text{ m/s}$

$$E_1 = h_1 + \frac{U_1^2}{2g} = 0.38 + \frac{10.53^2}{2 \times 9.81} = 6.031 \text{ m}$$

Hence, the given value is correct.

(a.ii) Momentum is conserved over the hydraulic jump.

Upstream of the jump:

$$M_1 = \frac{\rho g h_1^2}{2} + \rho h_1 U_1^2 = \rho\left(\frac{9.81 \times 0.38^2}{2} + 0.38 \times 10.53^2\right) = 42.84\rho$$

Downstream of the jump:

$$U_2 = \frac{q}{h_2} = \frac{4}{2.74} = 1.460 \text{ m/s}$$

$$M_2 = \frac{\rho g h_2^2}{2} + \rho h_2 U_2^2 = \rho\left(\frac{9.81 \times 2.74^2}{2} + 2.74 \times 1.46^2\right) = 42.67\rho$$

Hence, the given value is correct.

(b.i) Manning formula $U = \frac{1}{n} R_h^{2/3} S_b^{1/2}$

$$Q = UA = \frac{1}{n}\left(\frac{Bh}{B+2h}\right)^{2/3} S_b^{1/2} Bh$$

$$50 = \frac{1}{0.02} \times \left(\frac{10h}{10+2h}\right)^{2/3} \times 0.0005^{1/2} \times 10h$$

h = 2.96 m

(b.ii) At critical condition $U = \sqrt{gh}$

$$Q = UA = \sqrt{gh}Bh = \sqrt{g}Bh^{1.5}$$

$$50 = \sqrt{9.81} \times 10 \times h^{1.5}$$

h = 1.366 m

(b.iii) Upstream section:

$h_1 = 2.96 \times 1.02 = 3.02$ m

$$R_{h1} = \frac{10 \times 3.02}{10 + 3.02 \times 2} = 1.883 \text{ m}$$

$$U_1 = \frac{Q}{Bh_1} = \frac{50}{10 \times 3.02} = 1.656 \text{ m}$$

$$\left(h + \frac{U^2}{2g}\right)_1 = 3.02 + \frac{1.656^2}{2 \times 9.81} = 3.160 \text{ m}$$

$$S_{f1} = \frac{n^2 \cdot U_1^2}{R_{h1}^{4/3}} = \frac{0.02^2 \cdot 1.656^2}{1.883^{4/3}} = 0.000472$$

Downstream section:

$h_2 = 4$ m

$$R_{h2} = \frac{10\times 4}{10+4\times 2} = 2.222 \text{ m}$$

$$U_2 = \frac{Q}{Bh_2} = \frac{50}{10\times 4} = 1.25 \text{ m}$$

$$\left(h+\frac{U^2}{2g}\right)_2 = 4+\frac{1.25^2}{2\times 9.81} = 4.080 \text{ m}$$

$$S_{f2} = \frac{n^2\cdot U_2{}^2}{R_{h2}{}^{4/3}} = \frac{0.02^2\cdot 1.25^2}{2.222^{4/3}} = 0.000216$$

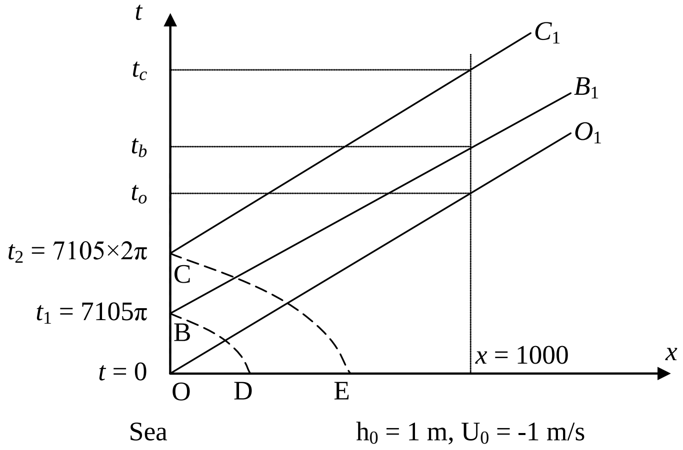
$$\frac{d}{dx}\left(h+\frac{U^2}{2g}\right) = S_b - S_f$$

$$\frac{\left(h+\frac{U^2}{2g}\right)_2 - \left(h+\frac{U^2}{2g}\right)_1}{\Delta x} = S_b - \frac{S_{f1}+S_{f2}}{2}$$

$$\frac{4.080-3.160}{\Delta x} = 0.0005 - \frac{0.000472+0.000216}{2} = 0.000156$$

$\Delta x = 5897$ m

(c)



The first high tide occurs at the mouth at $t_1 = 7105\pi = 22309.7$ s, with $h_1 = 7$ m.
Along –ve line BD

$$U_B - 2\sqrt{9.81\cdot 7} = -1-2\sqrt{9.81\times 1} \Rightarrow U_B = 9.31 \text{ m/s}$$

Positive line BB_1 is straight: $\frac{dx}{dt} = U_B + \sqrt{gh_B}$

$$\frac{1000-0}{t_b - 22309.7} = 9.31+\sqrt{9.81\times 7}$$

$$t_b = 22309.7+56.8 = 22366.5 \text{ s}$$

The following low tide occurs at the mouth at $t_2 = 7105\cdot 2\pi = 44619.4$ s, with $h_2 = 1$ m
Along –ve line CE

$$U_C - 2\sqrt{9.81\cdot 1} = -1-2\sqrt{9.81} \Rightarrow U_C = -1 \text{ m/s}$$

Positive line CC_1 is straight: $\frac{dx}{dt} = U_C + \sqrt{gh_C}$

$$\frac{1000 - 0}{t_c - 44619.4} = -1 + \sqrt{9.81 \times 1}$$

$$t_c = 44619.4 + 469.0 = 45088.4 \text{ s}$$

3

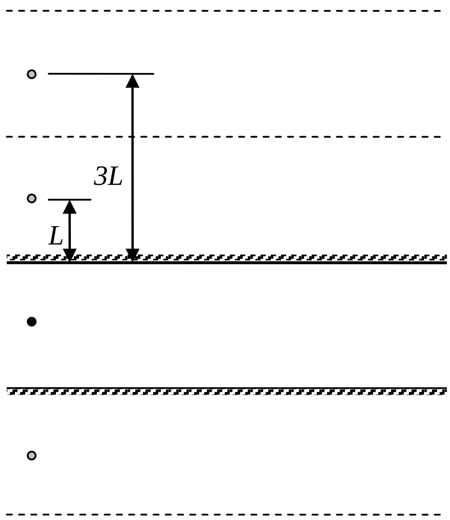
(a.i) 2D continuous release with one image. Along the edge:

$$\bar{c}(x) = \frac{2\dot{M}/h}{U\sqrt{4\pi \frac{x}{U} D_t}} \exp\left(-\frac{L^2}{4D_t\, x/U}\right)$$

From $\frac{d\bar{c}(x)}{dx} = 0$

$$x = \frac{L^2 U}{2D_t}$$

(a.ii)



$$\bar{c}(x) = \frac{2\dot{M}/h}{U\sqrt{4\pi \frac{x}{U} D_y}} \exp\left(-\frac{L^2}{4D_y\, x/U}\right) + \frac{2\dot{M}/h}{U\sqrt{4\pi \frac{x}{U} D_y}} \exp\left(-\frac{(3L)^2}{4D_y\, x/U}\right) + \frac{2\dot{M}/h}{U\sqrt{4\pi \frac{x}{U} D_y}} \exp\left(-\frac{(5L)^2}{4D_y\, x/U}\right) \cdots$$

(b.i)

$$d_* = d \cdot \left(\frac{g(s-1)}{\nu^2}\right)^{1/3} = 0.24\times 10^{-3} \times \left(\frac{9.81\times(2.65-1)}{10^{-12}}\right)^{1/3} = 6.07$$

Fall velocity

$$w_s = \frac{\nu}{d}\left[\sqrt{10.36^2 + 1.049\cdot d_*^{\,3}} - 10.36\right]$$

$$= \frac{10^{-6}}{0.24\times 10^{-3}}\left[\sqrt{10.36^2 + 1.049\times 6.07^3} - 10.36\right] = 0.034 \text{ m/s}$$

Shear velocity

$$u_* = \sqrt{\frac{\tau_b}{\rho}} = \sqrt{ghS_b} = \sqrt{9.81\times 5\times 0.001} = 0.221 \text{ m/s}$$

Rouse profile

$$\frac{\bar{c}(z)}{\bar{c}(a)} = \left(\frac{h-z}{z}\cdot\frac{a}{h-a}\right)^{\frac{w_s}{\kappa u_*}}, \quad \text{with} \quad \bar{c}(4) = 0.2 \text{ kg/m}^3$$

$$\bar{c}(z)=\bar{c}(a)\cdot\left(\frac{h-z}{z}\cdot\frac{a}{h-a}\right)^{\frac{w_s}{\kappa u_*}}=0.2\times\left(\frac{5-z}{z}\cdot\frac{4}{5-4}\right)^{\frac{0.034}{0.4\times0.221}}=0.2\times4^{0.385}\times\left(\frac{5-z}{z}\right)^{0.385}$$

Depth-averaged concentration

$$C=\frac{1}{h}\int_0^h\bar{c}(z)\cdot dz=\frac{1}{h}\times0.2\times4^{0.385}\times\int_0^h\left(\frac{5-z}{z}\right)^{0.385}dz=\frac{1}{h}\times0.341\times\frac{0.385\pi h}{\sin(0.385\pi)}$$

$= 0.44\ \text{kg/m}^3$

(b.ii)

$$C=7.8\ln\left(\frac{12.0\cdot h}{k_s}\right)=7.8\ln\left(\frac{12.0\cdot5}{0.3}\right)=41.33$$

$$U=C\sqrt{R_hS_b}=41.33\times\sqrt{5\times0.001}=2.92\ \text{m/s}$$

Can also do the integration to find U:

$$u_*=\sqrt{gR_hS_b}=\sqrt{9.81\times5\times0.001}=0.221\ \text{m/s}$$

$$\bar{u}(z)=\frac{u_*}{\kappa}\ln\left(\frac{30.0z}{k_s}\right)=\frac{0.221}{0.4}\ln\left(\frac{30.0z}{0.3}\right)$$

$$U=\frac{1}{h}\int\bar{u}(z)dz$$

$$\bar{c}(0.0005)=0.2\times4^{0.385}\times\left(\frac{5-0.0005}{0.0005}\right)^{0.385}=11.83\ \text{kg/m}^3$$

$$q_s=\int_a^h\bar{c}(z)\bar{u}(z)dz=11.6\cdot u_*\cdot\bar{c}(a)\cdot a\cdot\left[I_1\ln\left(\frac{30h}{k_s}\right)+I_2\right]$$

$$=\ 11.6\cdot0.221\cdot11.83\cdot0.0005\cdot\left[I_1\ln\left(\frac{30\times5}{0.3}\right)+I_2\right]=0.0152\cdot\left[6.21\cdot I_1+I_2\right]$$

According to a/h=0.0001 and $\frac{w_s}{\kappa u_*}=0.385$

$$I_1=363.9+\frac{0.385-0.2}{0.6-0.2}\times(16.5-363.9)=203.2$$

$$-I_2=504.9+\frac{0.385-0.2}{0.6-0.2}\times(44.53-504.9)=292.0$$

So,

$$q_s=0.0152\cdot(6.21\cdot I_1+I_2)=0.0152\cdot(6.21\cdot203.2-292)=14.7\ \text{kg/(m·s)}$$

4.

(a) The design flow rate is $Q = iA = 200 \times 200 \times \frac{50 \times 0.001}{3600} = 0.556 \text{ m}^3\text{/s}$

Darcy-Weisbach Equation $H_f = \lambda \frac{L}{D}\frac{U^2}{2g}$

$$2 = \lambda \frac{2000}{D} \frac{\left(\frac{0.556}{\pi D^2/4}\right)^2}{2g} = 51.14 \times \frac{\lambda}{D^5} \quad => \quad D = 1.91 \cdot \lambda^{0.2}$$

Need to iterate to get D.
First, guess D = 0.5 m.

Flow velocity $U = \frac{0.556}{\pi D^2/4} = 2.833 \text{ m/s}$

According to $\frac{k_s}{D} = 0.0002$ and $\frac{UD}{\nu} = 1.42 \times 10^6$, $\lambda = 0.0147$

$$D = 1.91 \cdot \lambda^{0.2} = 1.91 \cdot 0.0147^{0.2} = 0.82 \text{ m}$$

Taking this new D, $U = \frac{0.556}{\pi D^2/4} = 1.05 \text{ m/s}$

According to $\frac{k_s}{D} = 0.00012$ and $\frac{UD}{\nu} = 8.61 \times 10^5$, $\lambda = 0.0141$

$$D = 1.91 \cdot \lambda^{0.2} = 1.91 \cdot 0.0141^{0.2} = 0.81 \text{ m}$$

The minimum diameter can be estimated to be 0.815 m.

(b) Because the areas are the same, the flow rate ratio is equal to the velocity ratio

$$R_h = \frac{5.1 \times 2.6}{5.1 + 2.6 \times 2} = 1.287 \text{ m}$$

For the cut-off channel

$S_b = \Delta/400$

$$C = 7.8 \ln\left(\frac{12.0 \cdot R_h}{k_s}\right) = 7.8 \ln\left(\frac{12.0 \cdot 1.287}{0.005}\right) = 62.68$$

$$U_{cut} = C\sqrt{R_h S_b} = 62.68\sqrt{R_h \Delta / 400}$$

For the river

$S_b = \Delta/1000$

$$C = \frac{1}{n} \cdot R_h^{1/6} = \frac{1}{0.025} \cdot 1.287^{1/6} = 41.72$$

$$U_{river} = C\sqrt{R_h S_b} = 41.72\sqrt{R_h \Delta / 1000}$$

Velocity ratio

$$\frac{U_{cut}}{U_{river}} = \frac{62.68\sqrt{R_h \Delta / 400}}{41.72\sqrt{R_h \Delta / 1000}} = 2.38$$

(c.i)

When the flow rate is 410 litre/s, $U = \frac{0.41}{\pi 0.5^2/4} = 2.09 \text{ m/s}$

According to $\frac{k_s}{D} = 0.00006$ and $\frac{UD}{\nu} = 1.05 \times 10^6$, $\lambda = 0.013$

$$H_f = \lambda \frac{L}{D}\frac{U^2}{2g} = 0.013 \frac{20000}{0.5}\frac{2.09^2}{2 \times 9.81} = 115.77 \text{ m}$$

Local losses $H_l = 20\frac{U^2}{2g} = 20\frac{2.09^2}{2 \times 9.81} = 4.45 \text{ m}$

Total losses is then 120.22 m, which is about the same as the elevation difference.

(c.ii)

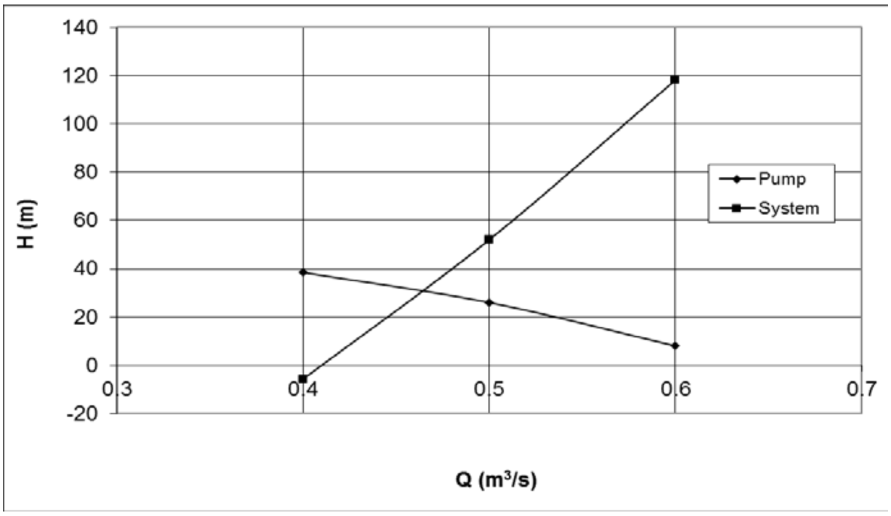
Develop the system curve.

Static head = -120 m

Losses in head = $\left(\lambda \times \frac{20000}{0.5} + 20\right) \times \frac{U^2}{2g}$

λ is determined according to Re = $\frac{UD}{\nu}$ and $k_s/D = 0.00006$

Q (m^3/s)	0.4	0.5	0.6
U (m/s)	2.04	2.55	3.06
Re	1.0E+06	1.3E+06	1.5E+06
λ	0.013	0.0125	0.012
Total Losses (m)	114.34	172.04	238.21
System head (m)	-5.66	52.04	118.21



From the diagram: Q = 0.465 m^3/s = 465 l/s, efficiency = 58%

$$P_p = \rho g Q_p H_p / \eta = 1000 \times 9.81 \times 0.465 \times 32/0.58 = 251.67 \text{ kW}$$

Comments on Questions

Q1 Hydrology and water quality

Most candidates found the infiltration question challenging. Some were confused between the instantaneous infiltration rate and the total infiltration depth over a period. The unit hydrograph question was answered better. The required S curve was given in a table, so the solutions could be derived numerically. A few still adopted the graphical approach as taught in the lecture. A lot of candidates constructed the eight-hour hydrograph based on the one-hour unit hydrograph, although the eight-hour hydrograph could be derived directly. A few candidates wrongly regarded the BOD value as an indicator of the number of bacteria in water.

Q2 Open channel flows

A straightforward question, well-answered by most candidates. A common error was the sign of the force in writing the steady flow momentum equation. Some candidates spent much time solving complicated equations. In fact, they only need to show that the given values satisfy the equations. The majority of the candidates answered the unsteady flow question correctly. A common mistake was the wrong sign of the initial velocity.

Q3 Pollutant transport and sediment transport

Generally well answered. Despite the explicit explanation of the estuary banks in the question, some candidates still neglected the boundary effects. Several candidates wrongly chose the three-dimensional model in the analysis. The suspended sediment question involved repeated use of the Rouse profile, and was prone to calculation errors.

Q4 Rational method, steady open channel flows, pipelines and pumps

It is encouraging to see that most candidates appreciated the principle of the drainage design. Almost all noticed the dependence of the friction factor on the flow rate, but some made mistakes with the units. A few candidates did not realise the change in the bed slope after the river was shortened.

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