2016 IIA 3D5 - Water Engin1eering Dr D. Liang

1.

(a)

The total volume of water fell on the catchment is: $40 \times 10^{-3} \times 7 \times 10^{6} = 280 \times 10^{3}\ \text{m}^3$

The total volume of the excess flow is:

$$(3+8+5+2+1) \times 2 \times 3600 = 136800\ \text{m}^3$$

The total volume of water infiltrated to the soil is: 280,000 – 136,800 = 143,200 m^3

The infiltration depth into the soil is: $\dfrac{143200}{7 \times 10^6} = 0.02046\ \text{m} = 20.46\ \text{mm}$

According to:

$$\int_0^2 f \cdot dt = f_c(2-0) - \frac{1}{0.6}(15 - f_c)\left(e^{-0.6\times 2} - 1\right) = 2f_c + 17.470 - 1.165 f_c = 20.46$$

$f_c = 3.58\ \text{mm/h}$

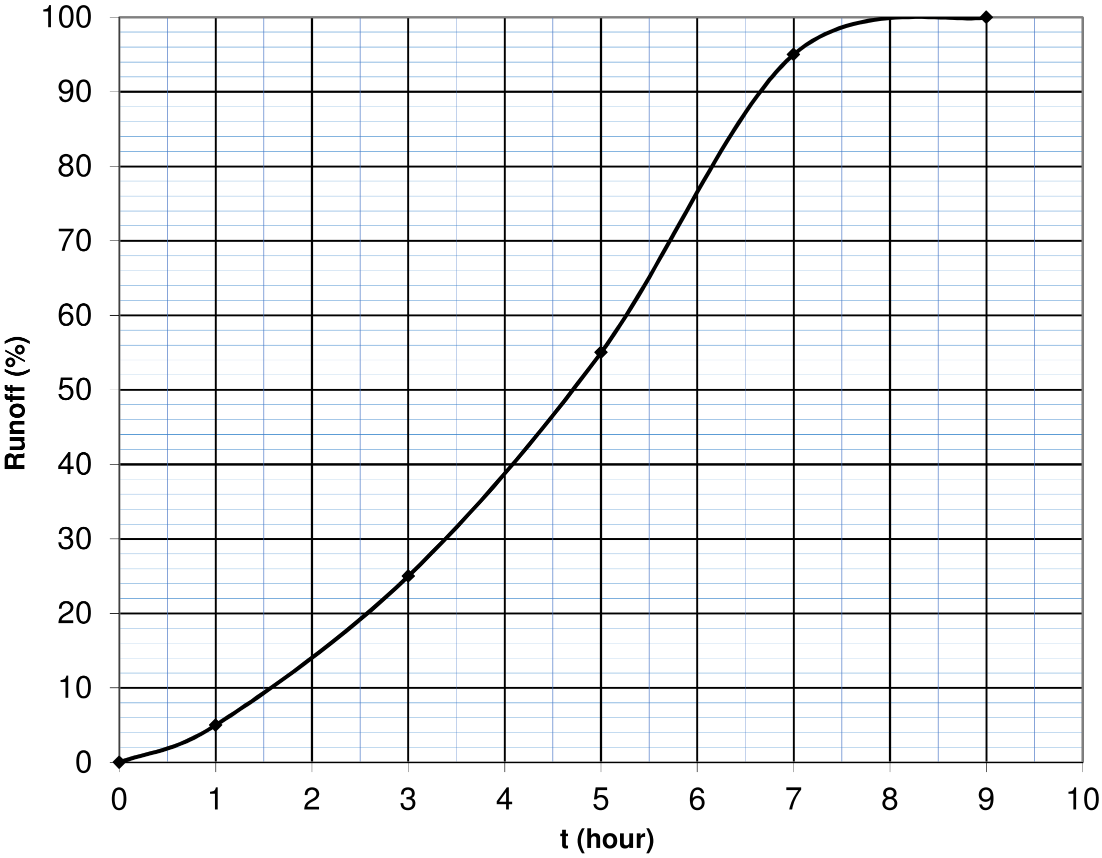
(b)

It is necessary to change the time step of the unit hydrograph.

The S-curve (2-hour periods) is:

Time (h)	0	1	3	5	7	9	11
Percentage	0	5	25	55	95	100	100

Plot the S-curve:



Read its values with a 1-hour period, construct the 1-hour unit hydrograph, calculate the flow rate contribution by each hour of the rainfall:

Time (h)	0.0	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	8.5	9.5	10.5	11.5
S-curve (%)	0.0	2.0	9.0	19.0	31.0	46.0	65.0	87.0	98.5	100.	100	100	100
1-hour shifted S-curve (%)	0.00	0.00	2.00	9.00	19.00	31.00	46.00	65.00	87.00	98.50	100	100	100
Subtract the 2 S-curves (%)	0.00	2.00	7.00	10.00	12.00	15.00	19.00	22.00	11.50	1.50	0.00	0.00	0.00
Contribution of 20mm rain in the 1st hour (m^3/s)	0.00	0.11	0.39	0.56	0.67	0.83	1.06	1.22	0.64	0.08	0.00	0.00	0.00
Contribution of 10mm rain in the 2nd hour (m^3/s)	0.00	0.00	0.06	0.19	0.28	0.33	0.42	0.53	0.61	0.32	0.04	0.00	0.00
Contribution of 10mm rain in the 3rd hour (m^3/s)	0.00	0.00	0.00	0.06	0.19	0.28	0.33	0.42	0.53	0.61	0.32	0.04	0.00
Total flow rate by the 3-hour rainfall (m^3/s)	0.00	0.11	0.44	0.81	1.14	1.44	1.81	2.17	1.78	1.01	0.36	0.04	0.00

Hence, the maximum flow rate is 2.17 m^3/s.

(c)

Rapidly varied flow – The depth changes abruptly over a comparatively short distance. The rapidly varied flow is known as a local phenomenon. Examples are the flow over a hump and the hydraulic jump.

Gradually varied flow – The depth changes over a long distance. The bed friction has to be considered in the analysis.

2

(a.i) Bed shear stress: $\tau_b = \rho g H S_b$

$10 = 1000 \times 9.81 \times 1.5 \times S_b$, so $S_b = 0.00068$

Manning equation: $U = C\sqrt{HS_b} = \dfrac{1}{n} H^{2/3} S_b^{1/2}$

$$U = \frac{1}{0.025} \times 1.5^{2/3} \times 0.00068^{1/2} = 1.37 \text{ m/s}$$

(a.ii) Shear velocity: $u_* = \sqrt{\dfrac{\tau_b}{\rho}} = \sqrt{\dfrac{10}{1000}} = 0.1 \text{ m/s}$

Chezy coefficient: $C = \dfrac{1}{n} \cdot H^{1/6} = 7.8 \ln\left(\dfrac{12.0 \cdot H}{k_s}\right)$

$$\frac{1}{0.025} \times 1.5^{1/6} = 7.8 \ln\left(\frac{12.0 \times 1.5}{k_s}\right)$$

$k_s = 0.07454$ m

Based on $\dfrac{\bar{u}(z)}{u_*} = \dfrac{1}{\kappa} \ln\left(\dfrac{30.0z}{k_s}\right)$, with $z = H$

$$\frac{\bar{u}(H)}{0.1} = \frac{1}{0.4} \ln\left(\frac{30.0 \times 1.5}{0.07454}\right)$$

$\bar{u}(H) = 1.6$ m/s

(a.iii) Take the datum as the near-horizontal bed level without the hump. Neglect the small bed slope owing to the short distance in the analysis. Total energy head in front of the hump:

$$1.5 + \frac{1.37^2}{2 \times 9.81} = 1.596 \text{ m}$$

Total energy head above the hump shall be equal to this value:

$$1.5 - 0.05 + \frac{U^2}{2 \times 9.81} = 1.596$$

$U = 1.69$ m/s

According to the continuity equation:

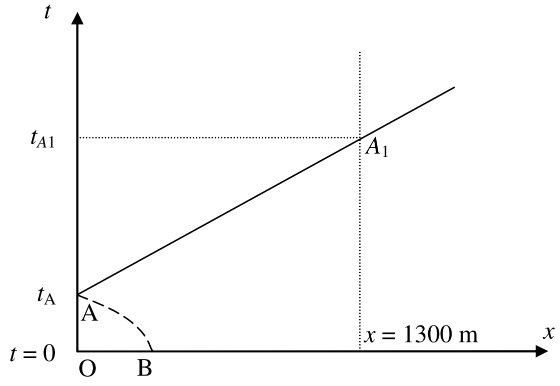
$1.5 \times 1.37 = 1.69H$

$H = 1.216$ m

So, $\Delta z = 1.5 - 0.050 - 1.216 = 0.234$ m

(b.i) See lecture notes.

(b.ii)



Line AA_1 is straight. The water depth is 2.2 + 0.3 = 2.5 m.

From the boundary condition: $t_A = \dfrac{0.3}{0.5} = 0.6$ h

Along –ve line AB

$$U_A - 2\sqrt{9.81 \times 2.5} = -0.6 - 2\sqrt{9.81 \times 2.2} \Rightarrow U_A = 0.013 \text{ m/s}$$

U_A and U_0 have opposite signs.

Positive line AA_1 is straight: $\dfrac{dx}{dt} = U_A + \sqrt{gh_A}$

$$\frac{1300 - 0}{t_{A1} - 0.6 \times 3600} = 0.013 + \sqrt{9.81 \times 2.5}$$

$$t_b = 2160 + 261.8 = 2421.8 \text{ s}$$

3

(a) Shields parameter is a nondimensional number used to calculate the initiation of motion of sediment in a fluid flow. It is proportional to the ratio of fluid shear force on the particle to the weight of the particle.

(b.i) The bed shear stress is: $\tau_b = \rho g R_h S_f = 1000 \times 9.81 \times 3 \times 2 \times 10^{-4} = 5.886 \text{ Pa}$

The shear velocity is: $u_* = \sqrt{g R_h S_f} = \sqrt{9.81 \times 3 \times 2 \times 10^{-4}} = 0.0767 \text{ m/s}$

Based on the formula: $\dfrac{\bar{c}(z)}{\bar{c}(a)} = \left(\dfrac{h-z}{z} \cdot \dfrac{a}{h-a} \right)^{\frac{w_s}{\kappa u_*}}$, with $z = 1$ and $a = 0.1$

$$0.1 = \left(\frac{3-1}{1} \cdot \frac{0.1}{3-0.1} \right)^{\frac{w_s}{0.4 \times 0.0767}}$$

$$0.1 = 0.069^{\frac{w_s}{0.03068}}$$

$$\frac{w_s}{0.03068} = \frac{\ln(0.1)}{\ln(0.069)} = 0.861$$

$w_s = 0.0264$ m/s

(b.ii) Taking $d = 0.2$ mm, then $d_* = d \cdot \left(\dfrac{g(s-1)}{\nu^2} \right)^{1/3} = 0.0002 \times \left(\dfrac{9.81(2.65-1)}{10^{-12}} \right)^{1/3} = 5.06$

$$w_s = \frac{\nu}{d}\left[\sqrt{10.36^2 + 1.049 \cdot d_*^{\,3}} - 10.36 \right] = \frac{10^{-6}}{0.0002}\left[\sqrt{10.36^2 + 1.049 \cdot 5.06^3} - 10.36 \right]$$

$w_s = 0.0262$ m/s is close to 0.0264 m/s

(b.iii) $\dfrac{u_* d}{\nu} = \dfrac{0.0767 \times 0.0002}{10^{-6}} = 15.3$

$$\frac{u_*}{w_s} = \frac{0.0767}{0.0264} = 2.9$$

From Liu's diagram, the bedform is in the transition regime between dunes and antidunes.

(b.iv) $\theta_c = \dfrac{0.30}{1+1.2 d_*} + 0.055\left[1 - \exp(-0.02 d_*)\right] = \dfrac{0.30}{1+1.2 \times 5.06} + 0.055\left[1 - \exp(-0.02 \times 5.06)\right]$

$$\theta_c = 0.0424 + 0.0053 = 0.0477$$

Calculate the flow velocity

$$C = 7.8 \ln\left(\frac{12.0 \cdot R_h}{k_s} \right) = 7.8 \ln\left(\frac{12.0 \cdot 3}{0.1} \right) = 45.9$$

$$U = C\sqrt{R_h S_b} = 45.9\sqrt{3 \times 2 \times 10^{-4}} = 1.12 \text{ m/s}$$

$$C' = 7.8 \ln\left(\frac{12.0 \cdot R_h}{k_s'} \right) = 7.8 \ln\left(\frac{12.0 \cdot 3}{3 \times 0.0002} \right) = 85.8$$

$$\tau_b' = \rho g \frac{U^2}{C'^2} = 1000 \times 9.81 \times \frac{1.12^2}{85.8^2} = 1.67 \text{ Pa}$$

$$\theta' = \frac{\tau_b'}{g(\rho_s - \rho)d} = \frac{1.67}{9.81(2650-1000)0.0002} = 0.516$$

$$T = \frac{\theta' - \theta_c}{\theta_c} = \frac{0.516 - 0.0477}{0.0477} = 9.82$$

$$\frac{q_b}{\sqrt{g(s-1)\cdot d^3}} = 0.053\frac{T^{2.1}}{d_*^{0.3}}$$

$$\frac{q_b}{\sqrt{9.81(2.65-1)\cdot 0.0002^3}} = 0.053\frac{9.82^{2.1}}{5.06^{0.3}} = 3.949$$

$$q_b = 4.49\times10^{-5}\ \text{m}^3/(\text{m}\cdot\text{s}) = 0.12\ \text{kg}/(\text{m}\cdot\text{s})$$

(b.v) This is a 2-D instantaneous release problem.

$D_x = D_L + D_{tx} = (5.86+0.15)hu_* = 6.01\times3\times0.0767 = 1.383\ \text{m}^2/\text{s}$

$D_y = D_{ty} = 0.15hu_* = 0.15\times3\times0.0767 = 0.0345\ \text{m}^2/\text{s}$

The concentration field of the dye cloud is $\bar{c}(x,y,t) = \dfrac{M/h}{4\pi t\sqrt{D_x D_y}}\exp\left(-\dfrac{(x-Ut)^2}{4D_x t} - \dfrac{y^2}{4D_y t}\right)$.

The maximum concentration is:

$$\frac{M/h}{4\pi t\sqrt{D_x D_y}} = \frac{10/3}{4\pi\times600\sqrt{1.383\times0.0345}} = 2.1\times10^{-3}\ \text{kg/m}^3$$

4.

(a) For an irregular channel, the critical flow occurs when $Fr = \dfrac{U}{\sqrt{g\dfrac{A}{B}}} = 1$

With the water level shown in Fig. 2, the width at the water surface level is:

$$B = 2\times\frac{D}{2}\sin(\pi-\beta) = D\sin\beta$$

The area of the cross section is:

$$A = 2\cdot\frac{D^2\beta}{8} + \frac{1}{2}\cdot D\sin\beta\cdot\frac{D}{2}\cos(\pi-\beta) = \frac{D^2\beta}{4} - \frac{1}{4}\cdot D^2\sin\beta\cdot\cos\beta = \frac{D^2}{4}(\beta - \sin\beta\cdot\cos\beta)$$

At critical condition:

$$\frac{Q}{A} = \sqrt{g\frac{A}{B}}$$

$$\frac{Q^2}{A^2} = g\frac{A}{B}$$

$$Q^2 = g\frac{A^3}{B} = g\frac{\frac{D^6}{64}(\beta - \sin\beta\cdot\cos\beta)^3}{D\sin\beta} = g\frac{D^5}{64}\frac{(\beta - \sin\beta\cdot\cos\beta)^3}{\sin\beta}$$

The above formulae are also applicable to situations with $0 < \beta < \pi$.

(b.i) Static lift = 10 m.

Flow velocity: $U = \dfrac{0.07}{\dfrac{\pi\times0.3^2}{4}} = 0.99$ m/s

Reynolds number: $\mathrm{Re} = \dfrac{0.99\times0.3}{10^{-6}} = 2.97\times10^5$

Roughness height: $\dfrac{k_s}{D} = 0.0005$

From Moody diagram: $\lambda = 0.0185$

The head of the duty point is: $10 + \lambda\dfrac{L}{D}\dfrac{U^2}{2g} = 10 + 0.0185\dfrac{5000}{0.3}\dfrac{0.99^2}{2\times9.81} = 25.4$ m

(b.ii)

Convert the pump characteristic curve into a quasi-dimensionless form.

In the pump equation, use $\dfrac{Q_p}{N_p}$ and $\dfrac{H_p}{N_p^{\,2}}$ as two independent variables.

$$\left(\frac{H_p}{N_p^{\,2}}\right)\times1200^2 = 47 - 0.03\times1200\times\left(\frac{Q_p}{N_p}\right) - 0.007\times1200^2\times\left(\frac{Q_p}{N_p}\right)^2$$

where H_p is in meters, Q_p is in l s^{-1}, and N_p is in rpm.
This equation applies to pumps of all speeds (with the pump diameter unchanged).
Plug in H_p = 25.4 and Q_p =70, then N_p can be calculated.

$$\left(\frac{25.4}{N_p^{\ 2}}\right)\times 1200^2 = 47 - 0.03\times 1200\times\left(\frac{70}{N_p}\right) - 0.007\times 1200^2\times\left(\frac{70}{N_p}\right)^2$$

$$\frac{3.66\times 10^7}{N_p^{\ 2}} = 47 - \frac{2520}{N_p} - \frac{4.94\times 10^7}{N_p^{\ 2}}$$

$$47\cdot N_p^{\ 2} - 2520\cdot N_p - 8.6\times 10^7 = 0$$

$$N_p = \frac{2520\pm\sqrt{2520^2 + 4\times 47\times 8.6\times 10^7}}{2\times 47} = \frac{2520\pm 127178.4}{94}$$

$N_p = 1379.8\ \text{rpm}$

Comments on Questions:

1. Some candidates made mistakes with the units. The Horton's model gave the depth of infiltration, whereas the excess flow was often in cubic meters. In calculating the total amount of infiltration, a few candidates mistakenly integrated the infiltration rate over the duration of the excess flow (0-10 hours), rather than the duration of the rainfall (0-2 hours). Many could not correctly identify the difference between rapidly and gradually varied flows. The difference does not lie in whether the flow is laminar or turbulent, or with or without energy losses.

2. Most candidates found it difficult to calculate the Reynolds-averaged flow velocity at the free surface. A few confused it with the shear velocity. A few used the formula for hydraulically smooth flows, without checking that the flow was actually hydraulically rough. In answering the unsteady flow question, a common mistake was the sign of the velocity.

3. Quite a few candidates did not tell the physical meaning of the Shields parameter, but explained its usage, such as an indication of the particle stability, transport capacity and bedform category. Some candidates had difficulty with the log-scale axes of the Liu's diagram. Many wrongly chose the one-dimensional or three-dimensional model in answering the pollutant transport question. The river is wide and the dye is dumped into the middle of the river, so we cannot use the one-dimensional model. After 10 min, the dye has been advected around 670 m downstream of the dumping location by the flow, while the water depth is only 3 m. Hence, we can assume that the dye is totally mixed over the depth and use the two-dimensional instantaneous-release model. If we use the three-dimensional model, then we have to consider an infinite number of images reflected by the river bed and free surface.

4. Most candidates found this question hard. The calculation of the area and width of a partially-filled circular channel was prone to errors. Because an equation was given to describe the pump characteristic curve and the flow rate of the duty point was known, the question could be answered algebraically. Many unnecessarily plotted the graph, and used the wrong "dynamically similar" points of the pump operation.

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