2017 IIA 3D5 - Water Engin1eering Dr D. Liang

1.

(a)

The total infiltration during the storm is:

$$\int_{t_1}^{t_2} f \cdot dt = f_c(t_2 - t_1) - \frac{1}{K_f}(f_0 - f_c)\left(e^{-K_f t_2} - e^{-K_f t_1}\right)$$

$$= 2(1-0) - \frac{1}{1}(10-2)\left(e^{-1} - e^{0}\right) = 7.057 \text{ mm}$$

The excess rain is: 20-7.057 = 12.943 mm

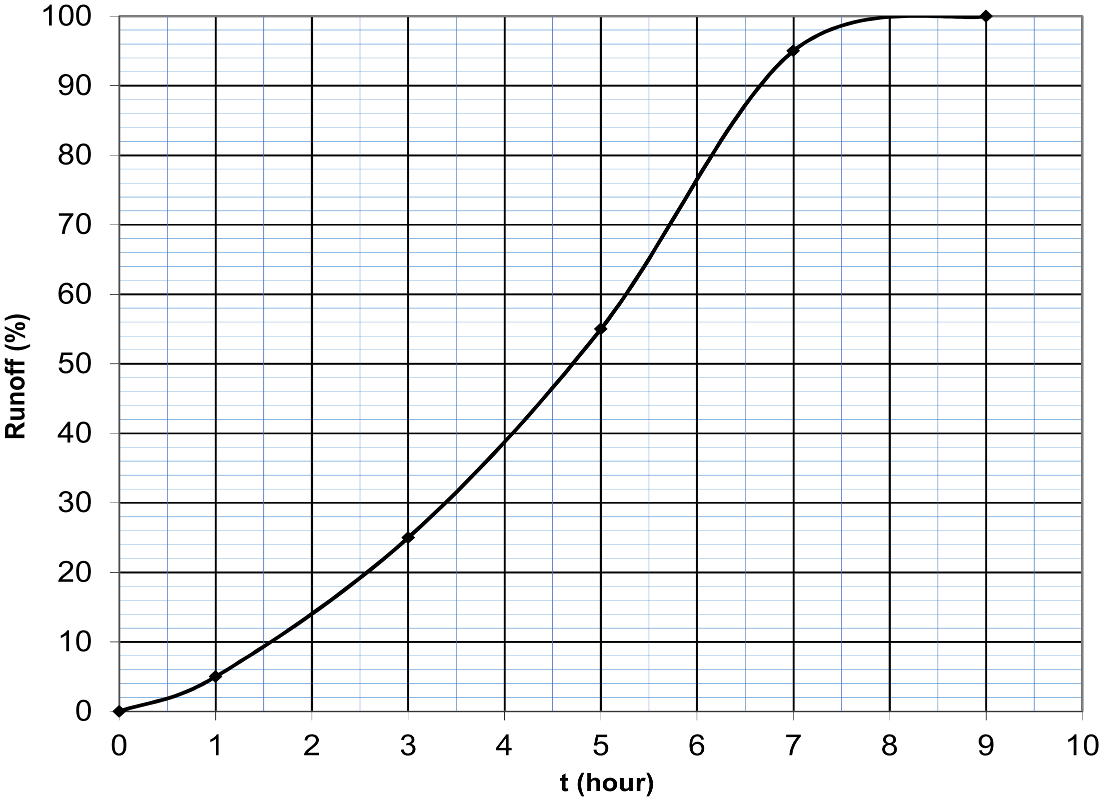
The total volume of the excess flow is: $12.943 \times 10^{-3} \times 1 \times 10^{4} = 129.43 \text{ m}^3$

It is necessary to change the time step of the unit hydrograph.

The S-curve (2-hour periods) is:

Time (h)	0	1	3	5	7	9	11
Percentage	0	5	25	55	95	100	100

Plot the S-curve:



Read its values with a 1-hour period; construct the 1-hour unit hydrograph; calculate the flow rate contribution by each hour of the rainfall:

Time (h)	0.0	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	8.5	9.5	10.5
S-curve (%)	0.0	2.0	9.0	19.0	31.0	46.0	65.0	87.0	98.5	100.	100	100
1-hour shifted S-curve (%)	0.0	0.0	2.0	9.0	19.0	31.0	46.0	65.0	87.0	98.5	100	100
Subtract the 2 S-curves (%)	0.0	2.0	7.0	10.0	12.0	15.0	19.0	22.0	11.5	1.5	0.0	0.0
Flow volume of the storm (m^3)	0.00	2.59	9.06	12.94	15.53	19.41	24.59	28.47	14.88	1.94	0.00	0.00
Flow rate (l/s)	0.00	0.72	2.52	3.60	4.31	5.39	6.83	7.91	4.13	0.54	0.00	0.00

Hence, the maximum flow rate is 7.91 l/s, which occurred 6 hours into the storm.

(b) Other losses include surface retention, evaporation and transpiration.

(c) The main assumption is the linearity. For example, doubling the excess rainfall doubles the runoff, without changing the shape of the hydrograph. It also assumes the rainfall to be uniform in space and time over the catchment.

(d.i) First, calculate h_1 according to the uniform flow theory.
According to the Manning formula:

$$U = C\sqrt{R_{h1}S_b} = \frac{1}{n}R_{h1}^{\ 2/3}S_b^{\ 1/2}$$

$$19 = 3h_1\frac{1}{0.013}\left(\frac{3h_1}{3+2h_1}\right)^{2/3}0.01^{1/2}$$

1.11 m is a solution to the above equation.

(d.ii) According to the energy equation between cross section 1 and cross section 2:

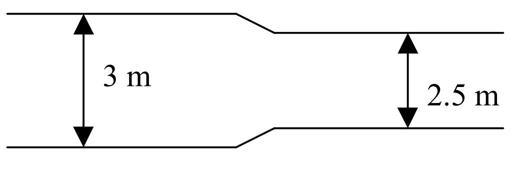
$$h_1 + \frac{U_1^{\ 2}}{2\times 9.81} = h_2 + \frac{U_2^{\ 2}}{2\times 9.81}$$

$$U_1 = \frac{19}{3\times 1.11} = 5.706 \text{ m/s}, \qquad U_2 = \frac{19}{2.5\times h_2}$$

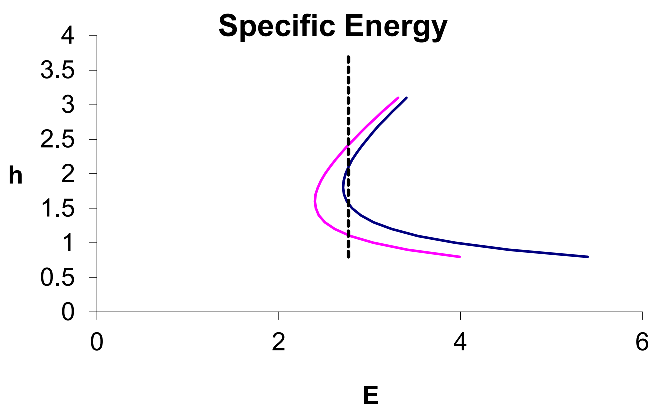
So, $$1.11 + \frac{5.706^2}{2\times 9.81} = h_2 + \frac{19^2}{(2.5\times h_2)^2\times 2\times 9.81}$$

$$h_2 + \frac{2.944}{h_2^{\ 2}} = 2.77$$

Solve this cubic equation h_2 = 1.56 m.
It can be easily proved that the flow is supercritical (Fr>1) in both section 1 and section 2.



Plan view



2

(a.i)

$$q = \frac{Q}{B} = \frac{50}{10} = 5 \text{ m}^3\text{/s/m}$$

$$\therefore \text{Critical depth: } h_c = \sqrt[3]{\frac{q^2}{g}} = 1.366 \text{ m}$$

(a.ii) $U = \frac{50}{3 \times 10} = 1.667 \text{ m/s}$

$R_h = \frac{3 \times 10}{10 + 2 \times 3} = 1.875 \text{ m}$

$U = C\sqrt{R_{h1} S_b}$, so $1.667 = C\sqrt{1.875 \times 0.0005}$

$C = 54.44$

According to $C = 7.8 \ln\left(\frac{12.0 \cdot R_h}{k_s}\right)$

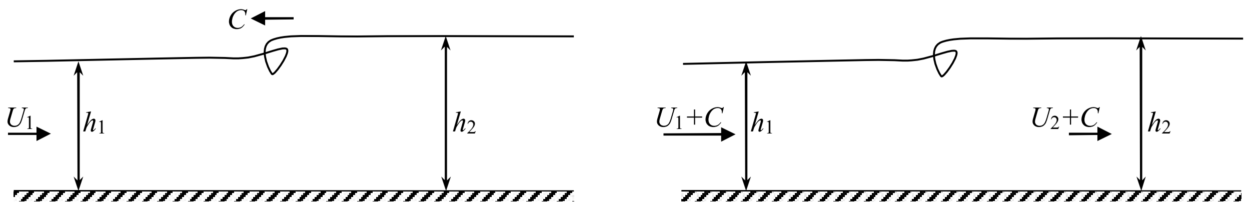
$$54.44 = 7.8 \ln\left(\frac{12.0 \cdot 1.875}{k_s}\right)$$

$$\ln\left(\frac{12.0 \cdot 1.875}{k_s}\right) = 6.98$$

$$\frac{12.0 \cdot 1.875}{k_s} = 1074.4$$

$k_s = 0.021 \text{ m}$

(b)



Stationary reference frame

Moving reference frame

Continuity eq. $h_1(C + 0.5) = h_2(C + U_2)$

Momentum eq. $\frac{gh_1^2}{2} - \frac{gh_2^2}{2} = h_2(C + U_2)^2 - h_1(C + U_1)^2$

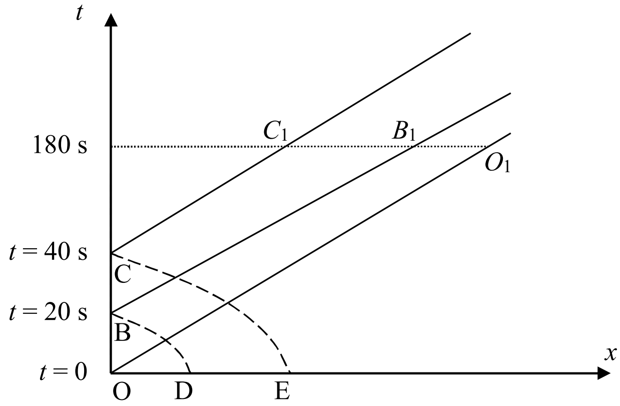
$$\frac{g}{2}\left(h_1^2 - h_2^2\right) = h_2\left(\frac{h_1}{h_2}\right)^2 (C + 0.5)^2 - h_1(C + 0.5)^2$$

$$\frac{9.81}{2}(9 - 16) = 4\left(\frac{3}{4}\right)^2 (C + 0.5)^2 - 3(C + 0.5)^2$$

$C = 6.266$ m/s

If using the symbolic derivation, we have $C = (gh_2)^{\frac{1}{2}}\left[\frac{1 + h_2 / h_1}{2}\right]^{\frac{1}{2}} - U_1$.

(c)



The positive characteristic starting from O divides the affected/unaffected regions.

Positive line OO_1 is straight: $\frac{dx}{dt} = U_O + \sqrt{gh_O}$

$$\frac{x_{O1} - 0}{180} = -1 + \sqrt{9.81 \times 4.0} \quad => \quad x_{o1} = 947.55 \text{ m}$$

Draw negative line through line B, according the –ve relationship:

$$U_B - 2\sqrt{9.81 \cdot 4.05} = -1 - 2\sqrt{9.81 \times 4} \quad => U_B = -0.922 \text{ m/s}$$

Positive line BB_1 is straight: $\frac{dx}{dt} = U_B + \sqrt{gh_B}$

$$\frac{x_{B1} - 0}{180 - 20} = -0.922 + \sqrt{9.81 \times 4.05} \quad => \quad x_{B1} = 860.99 \text{ m}$$

Draw negative line through line C, according the –ve relationship:

$$U_C - 2\sqrt{9.81 \cdot 4} = -1 - 2\sqrt{9.81 \times 4} \quad => U_C = -1 \text{ m/s}$$

Positive line CC_1 is straight: $\frac{dx}{dt} = U_C + \sqrt{gh_C}$

$$\frac{x_{C1} - 0}{180 - 40} = -1 + \sqrt{9.81 \times 4.} \quad => \quad x_{C1} = 736.99 \text{ m}$$

In the direction of the flow (in the negative x axis direction),
the water depth rises from O_1 to B_1 over a length of:

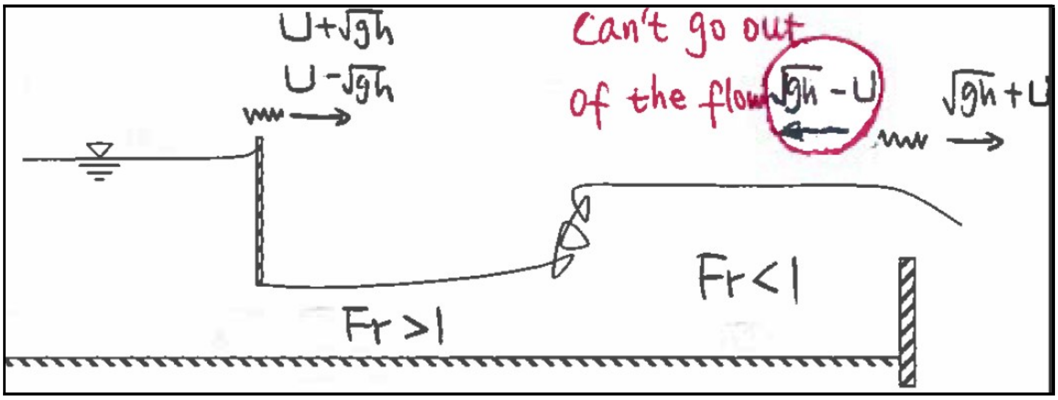
947.55 – 860.99 = 86.56 m

the water depth drops from B_1 to C_1 over a length of:

860.99 – 736.99 = 124 m

3 (a)

The disturbance propagates at speed $\sqrt{g\frac{A}{B}}$. Based on the analyses shown in the figure, the disturbance generated downstream cannot propagate upstream into the supercritical region. Hence, the disturbance will accumulate into a finite-amplitude hydraulic jump.



(b) The two axes of the Shields Diagram are particle Reynolds number and the Shields parameters. For conditions above the curve, sediment movement occurs. For conditions below the curve, sediment movement does not happen. The Shields curve describes the critical condition for the sediment movement to occur.

(c.i)

Given $d = 0.2$ mm, then $d_* = d\cdot\left(\frac{g(s-1)}{\nu^2}\right)^{1/3} = 0.0002\times\left(\frac{9.81(2.65-1)}{10^{-12}}\right)^{1/3} = 5.06$

$$w_s = \frac{\nu}{d}\left[\sqrt{10.36^2+1.049\cdot d_*^{\,3}}-10.36\right] = \frac{10^{-6}}{0.0002}\left[\sqrt{10.36^2+1.049\cdot 5.06^3}-10.36\right]$$

$$w_s = 0.0262 \text{ m/s}$$

According to the Chezy formula:

$$U = C\sqrt{R_{h1}S_b}$$
$$2 = C\sqrt{3\times 0.0004}$$
$$C = 57.74$$

The bed shear stress is: $\tau_b = \rho g\frac{U^2}{C^2} = 9810\frac{2^2}{57.74^2} = 11.77 \text{ Pa}$

The shear velocity is: $u_* = \sqrt{g}\frac{U}{C} = \sqrt{9.81}\frac{2}{57.74} = 0.108 \text{ m/s}$

Based on the formula: $\frac{\bar{c}(z)}{\bar{c}(a)} = \left(\frac{h-z}{z}\cdot\frac{a}{h-a}\right)^{\frac{w_s}{\kappa u_*}}$, with $z = 1$ m and $a = 0.01$ m

$$\frac{\bar{c}(1)}{\bar{c}(0.01)} = \left(\frac{3-1}{1}\cdot\frac{0.01}{3-0.01}\right)^{\frac{0.0262}{0.4\times0.108}}$$

$$\frac{\bar{c}(1)}{\bar{c}(0.01)} = 0.00669^{0.606} = 0.048$$

$$\bar{c}(1) = 200\times 0.048 = 9.62 \text{ kg/m}^3$$

(c.ii)

From the parameters: a/h = 0.01, $\frac{w_s}{\kappa u_*} = \frac{0.0262}{0.4 \times 0.108} = 0.606$

$I_1 = 2.174$, $I_2 = -4.254$

Also need the bed roughness height in calculating the suspended load.

$$C = 7.8 \ln\left(\frac{12.0 \cdot R_h}{k_s}\right) = 7.8 \ln\left(\frac{12.0 \cdot 3}{\mathrm{k_s}}\right) = 57.74$$

$k_s = 0.022$ m

$$q_s = \int_a^h \bar{c}(z)\bar{u}(z)dz = 11.6 \cdot u_* \cdot \bar{c}(a) \cdot a \cdot \left[I_1 \ln\left(\frac{30h}{k_s}\right) + I_2 \right]$$

$$q_s = 11.6 \cdot 0.108 \cdot 200 \cdot 0.01 \cdot \left[2.174 \ln\left(\frac{30 \times 1}{0.022}\right) - 4.254 \right] = 28.66 \ \text{kg/(m s)}$$

(c.iii) This is a 2-D continuous release problem.

$$D_x = D_L + D_{tx} = (5.86 + 0.15)hu_* = 6.01 \times 3 \times 0.108 = 1.947 \ \text{m}^2\text{/s}$$

$$D_y = D_{ty} = 0.15hu_* = 0.15 \times 3 \times 0.108 = 0.0486 \ \text{m}^2\text{/s}$$

$$\bar{c}(x,y) = \frac{\dot{M}/h}{U\sqrt{4\pi \frac{x}{U} D_y}} \exp\left(-\frac{y^2}{4D_y x/U}\right)$$

Need to consider the image source.

For the real source, x = 100 m, y = 1.9 m;

For the image source, x = 100 m, y = 2.1 m.

$$0.01 = \frac{\dot{M}/3}{2\sqrt{4\pi \frac{100}{2} 0.0486}} \exp\left(-\frac{1.9^2}{4 \times 0.0486 \times 100/2}\right) + \frac{\dot{M}/3}{2\sqrt{4\pi \frac{100}{2} 0.0486}} \exp\left(-\frac{2.1^2}{4 \times 0.0486 \times 100/2}\right)$$

$$0.01 = \frac{\dot{M}/3}{2\sqrt{4\pi \frac{100}{2} 0.0486}} [0.690 + 0.635]$$

$$\dot{M} = 0.250 \ \text{kg/s}$$

4.

(a)

Assume S_f is uniform in the section and can be calculated using Manning's equation with a water depth h = (0.6+0.67)/2 = 0.635 m:

$$R_h = \frac{3\times 0.635}{3+2\times 0.635} = 0.446$$

$$S_f = \frac{n^2 \cdot U^2}{R_h^{4/3}} = \frac{0.015^2 \cdot q^2}{0.446^{4/3}\times 0.635^2} = 0.00164q^2$$

From the gradually varied flow equation: $\frac{d}{dx}\left(h+\frac{U^2}{2g}\right) = S_b - S_f$

$$\frac{\left(0.67+\frac{q^2}{2g\cdot 0.67^2}\right)-\left(0.6+\frac{q^2}{2g\cdot 0.6^2}\right)}{60} = 0.002 - 0.00164q^2$$

From there we can solve for q =0.843 m^2/s

and we have Q = 0.843 × 3 = 2.53 m^3/s

(b) If rivers are narrow, then the horizontal shear caused by the river bank also contribute to the longitudinal dispersion.

If there are vegetations, then they increase the velocity non-uniformity and thus increase the longitudinal dispersion coefficient.

Rivers are often not straight, and there are secondary flows. The secondary currents increase the mixing of pollutant.

(c.i) Static lift = 10 m.

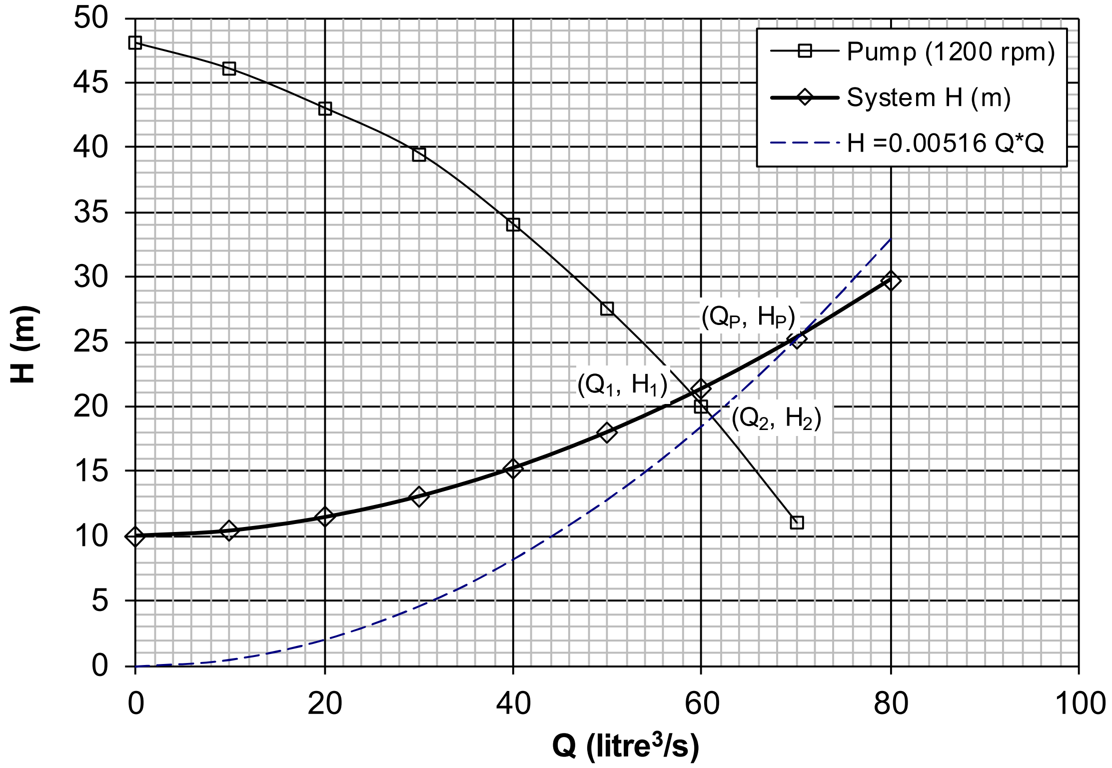
Roughness height: $\frac{k_s}{D} = 0.0005$

The head of the system is: $10+\lambda\frac{L}{D}\frac{U^2}{2g} = 10+\lambda\frac{5000}{0.3}\frac{U^2}{2\times 9.81}$

Q (litre/s)	0	10	20	30	40	50	60	70	80
Pump (1200 rpm)	48	46	43	39.5	34	27.5	20	11	
U (m/s)	0	0.14	0.28	0.42	0.57	0.71	0.85	0.99	1.13
Re	0	4.2E+4	8.5E+4	1.3E+5	1.7E+5	2.1E+5	2.5E+5	3.0E+5	3.4E+5
λ	0	0.0235	0.0210	0.0198	0.0192	0.0188	0.0185	0.0183	0.0181
System H (m)	10	10.4	11.4	13.0	15.2	18.0	21.3	25.3	29.7

Plot the pump curve and system curve, which cross at the duty point:

Q_1 = 59 l/s



(c.ii)

Plot the parabola through the origin and (Q_P = 70 l/s, H_P = 25.3 m). It crosses the pump curve (1200 rpm) at:

Q_2 = 61 l/s, H_2 = 19.2 m

According to either $\frac{Q_p}{N_p} = \frac{Q_2}{N_2}$ or $\frac{H_p}{N_p^{\,2}} = \frac{H_2}{N_2^{\,2}}$, where N_2 = 1200 rpm, we get N_p.

$$\frac{70}{N_p} = \frac{61}{1200} \quad \Rightarrow \quad N_p = 1377 \text{ rpm}$$

$$\frac{25.3}{N_p^{\,2}} = \frac{19.2}{1200^2} \quad \Rightarrow \quad N_p = 1377.5 \text{ rpm}$$

Comments on Questions

Q1 Hydrology and steady open channel flows

For the rapidly open channel flow question (d.ii), a lot of candidates only showed that the given water depth (h_2 = 1.56 m) satisfied the energy equation. However, another water depth (h_2=2.11) also satisfies the energy balance. Strictly speaking, it needs to show that the flow is supercritical, which is the reason for the water level rise at the channel contraction.

Q2 Uniform flow and unsteady flow

A lot of candidates found the tidal bore question (b) challenging. Although the question hinted "treating the bore as a moving hydraulic jump", some candidates still solved the question in the stationary reference frame and applied the energy principle. Most candidates grasped the method of characteristics, but quite a few did not read part (c) carefully. Instead of the streamwise lengths for the water level rise and drop, they only calculated the distances of the peak water level and the nearby still water level from the river mouth.

Q3 Sediment transport and pollutant transport

Most of the candidates could not explain why hydraulic jumps occur. Quite a few made futile attempts by drawing the specific energy vs. water depth curve, which was used in the lecture to explain the water level change over a hump. Candidates generally knew that the Shields curve describes the threshold motion of sediment, but very few remembered the horizontal axis of the Shields diagram. Several confused this diagram with Liu's diagram. Some candidates calculated the suspended sediment transport over the entire water column, rather than between the specified levels. They then wrongly assumed that sediment transport was negligible between 1 m above the bed and the free surface. In this question, the sediment transport rate between 1 m and 3 m above the bed is slightly larger than that between 1 cm and 1 m above the bed. Almost all the candidates were aware of the method of images in the pollutant transport question, but some did not notice that the image source and the real source did not contribute equally to the concentration at the point of measurement.

Q4 Gradually varied flow and pipe-pump system

Almost all the candidates were able to apply the correct equation to solve the gradually varied flow question, but some made the calculation over complicated and time-consuming. In plotting the system curve, some candidates assumed the rough turbulent flow regime to calculate the pipe friction coefficient, which was unnecessary and inaccurate. No one was totally correct in finding the new pump speed to achieve an increased flow rate. They were aware of the non-dimensional groups for pumps, but had difficulty to find the homologous (analogous) operating points.

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