1.

(a) Excess rainfall is the total rainfall minus infiltration, evaporation, evapotranspiration, etc. It is the part that contributes to the overland runoff.

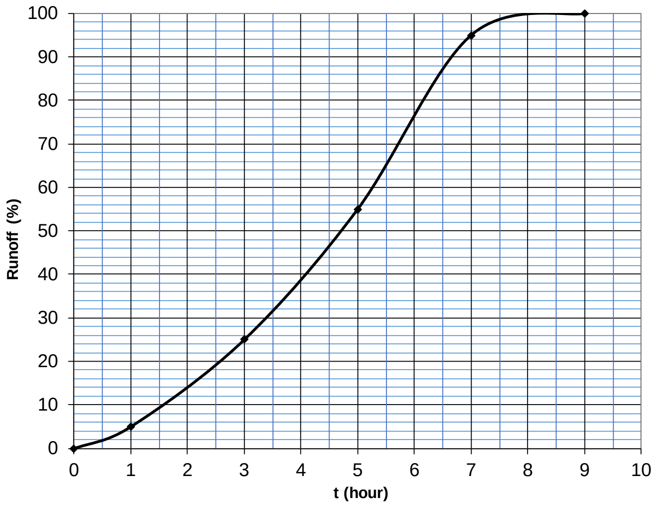
During the rainfall, rainwater lands over the entire catchment area. Hence, infiltration takes place over the whole catchment area. After the rainfall stops, we assume that water is not present over the whole catchment area but only limited to the rivers and streams. Because of the relatively small area over which infiltration takes place, the infiltration is much less important after the rainfall stops. This assumption will not be valid if there are still large areas of puddles after the rainfall stops.

(b) Assumptions include uniform rainfall in space and time over the catchment and linearity.

Rainfall and infiltration may not be uniformed distributed over the catchment.

The main problematic assumption is the linearity. In reality, if there are more runoff, it may travel faster and so the flood peak may arrive sooner. Alternatively, if there is much more runoff, the flow may spill onto the floodplain, thereby attenuating the peak.

(c) First, it is necessary to change the time step of the unit hydrograph.



The total volume of rainwater in the 1st hour is

$$10 \times 10^{-3} \times 10^{7} = 10^{5}\ \text{m}^3$$

The total volume of rainwater in the 2nd hour is

$$10 \times 10^{-3} \times 10^{7} = 10^{5}\ \text{m}^3$$

The total volume of rainwater in the 1st hour is

$$20 \times 10^{-3} \times 10^{7} = 2 \times 10^{5}\ \text{m}^3$$

Time (h)	0.00	0.50	1.50	2.50	3.50	4.50	5.50	6.50	7.50	8.50	9.50	10.50
S-curve (%)	0.00	2.00	9.00	19.00	31.00	46.00	65.00	87.00	98.50	100.00	100.00	100.00
1-hour shifted S-curve (%)	0.00	0.00	2.00	9.00	19.00	31.00	46.00	65.00	87.00	98.50	100.00	100.00
Subtract the two S-curves (%)	0.00	2.00	7.00	10.00	12.00	15.00	19.00	22.00	11.50	1.50	0.00	0.00
Rainwater distribution in 1st hour (m^3)	0.00E+00	2.00E+03	7.00E+03	1.00E+04	1.20E+04	1.50E+04	1.90E+04	2.20E+04	1.15E+04	1.50E+03	0.00E+00	0.00E+00
Rainwater distribution in 2nd hour (m^3)	0.00E+00	0.00E+00	2.00E+03	7.00E+03	1.00E+04	1.20E+04	1.50E+04	1.90E+04	2.20E+04	1.15E+04	1.50E+03	0.00E+00
Rainwater distribution in 3rd hour (m^3)	0.00E+00	0.00E+00	0.00E+00	4.00E+03	1.40E+04	2.00E+04	2.40E+04	3.00E+04	3.80E+04	4.40E+04	2.30E+04	3.00E+03
Flow rate (m^3/s)	0.00	0.56	2.50	5.83	10.00	13.06	16.11	19.72	19.86	15.83	6.81	0.83
Flow rater inluding base flow (m^3/s)	10.00	10.56	12.50	15.83	20.00	23.06	26.11	29.72	29.86	25.83	16.81	10.83

The maximum flow rate is 29.86 m^3/s in 6-7 hours.

2

(a) $$\frac{dh}{dx}=\frac{S_b-S_f}{1-Fr^2}=\frac{S_b}{1-\dfrac{U^2}{gh}}=\frac{S_b}{1-\dfrac{q^2}{gh^3}}$$

$$\left(1-\frac{q^2}{gh^3}\right)dh=S_b dx$$

$$h+\frac{q^2}{2gh^2}=S_b x+K_1$$

$$x=\frac{1}{S_b}\left(h+\frac{q^2}{2gh^2}\right)+K$$

This proof will be straightforward if starting with the following steady flow equation:

$$\frac{d}{dx}\left(h+\frac{U^2}{2g}\right)=S_b-S_f$$

(b) This is a two-dimensional instant release problem.

$$u_*=\sqrt{ghS_b}=\sqrt{9.81\times 2.0\times 0.0005}=0.099 \text{ m/s}$$

$$D_x=D_L+D_{tx}=(5.86+0.15)hu_*=6.01\times 2.0\times 0.099=1.19 \text{ m}^2\text{/s}$$

$$D_y=D_{ty}=0.15hu_*=0.15\times 2.0\times 0.099=0.03 \text{ m}^2\text{/s}$$

$$\bar{c}(x,y,t)=\frac{M/h}{4\pi t\sqrt{D_xD_y}}\exp\left(-\frac{(x-Ut)^2}{4D_xt}-\frac{y^2}{4D_yt}\right)$$

The contribution from the real source:

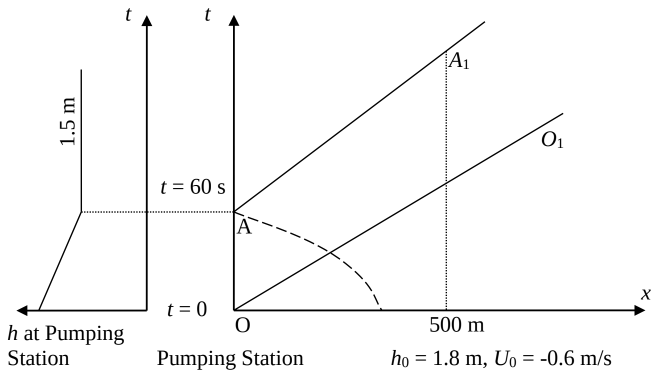
$$\bar{c}(1000,3,1000)=\frac{100/2}{4\pi\times 1000\times\sqrt{1.19\times 0.03}}\exp\left(-0-\frac{3^2}{4\times 0.03\times 1000}\right)=0.0205 \text{ kg/m}^3$$

The contribution from the image source:

$$\bar{c}(1000,7,1000)=\frac{100/2}{4\pi\times 1000\times\sqrt{1.19\times 0.03}}\exp\left(-0-\frac{7^2}{4\times 0.03\times 1000}\right)=0.0140 \text{ kg/m}^3$$

The concentration is: 0.035 kg/m^3.

(c)



h_A = 1.5 m/s. Draw a negative line through line A, according the –ve relationship:

$$U_A-2\sqrt{9.81\times 1.5}=-0.6-2\sqrt{9.81\times 1.8} \;\Rightarrow\; U_A=-1.33 \text{ m/s}$$

Positive line AA_1 is straight: $\frac{dx}{dt} = U_A + \sqrt{gh_A}$

$$\frac{500-0}{t_{A1}-60} = -1.33 + \sqrt{9.81\times 1.5} \quad => \quad t_{A1} = 259.5 \text{ s}$$

3

(a) The bedform pattern changes from plane, ripples, dune, plane, standing waves and anti-dunes.

(b) Because the flow is subcritical when dunes are present while the flow is supercritical when dunes are supercritical. Please refer to the Lecture Note (page 17).

(c.i) The first term represents the sediment flux due to the vertical advection; the second term represents the sediment diffusive flux due to the concentration gradient.

(c.ii)
$$w_s\bar{c} + D_{tz}\frac{d\bar{c}}{dz} = 0$$
$$D_{tz}\frac{d\bar{c}}{dz} = -w_s\bar{c}$$
$$\frac{d\bar{c}}{\bar{c}} = -\frac{w_s}{D_{tz}}dz$$
$$\ln\bar{c} = -\frac{w_s}{D_{tz}}z + K_1$$
$\bar{c} = K\exp\left(-\frac{w_s}{D_{tz}}z\right)$, with K being a constant of integration.

(c.iii) D_{tz} is a parabolic function.

(d) Given d = 0.2 mm, then $d_* = d\cdot\left(\frac{g(s-1)}{\nu^2}\right)^{1/3} = 0.0002\times\left(\frac{9.81(2.65-1)}{10^{-12}}\right)^{1/3} = 5.06$

$$w_s = \frac{\nu}{d}\left[\sqrt{10.36^2 + 1.049\cdot d_*^{\,3}} - 10.36\right] = \frac{10^{-6}}{0.0002}\left[\sqrt{10.36^2 + 1.049\cdot 5.06^3} - 10.36\right]$$
$$w_s = 0.026 \text{ m/s}$$

Chezy coefficient is: $C = 7.8\ln\left(\frac{12.0\times 1}{0.01}\right) = 55.30$

Shear stress is: $\tau_b = \rho g\frac{U^2}{C^2} = 9810\times\frac{1.7^2}{55.30^2} = 9.27$ Pa

Shear velocity is: $u_* = \sqrt{\frac{\tau_b}{\rho}} = \sqrt{\frac{9.27}{1000}} = 0.096$ m/s

$a/h = \frac{1\times 10^{-2}}{1} = 0.01$, $w_s/(\kappa u_*) = \frac{0.026}{0.4\times 0.096} = 0.677$

From the Table in the data sheet,
$$I_1 = 2.174 + (0.788 - 2.174)\frac{0.677-0.6}{1.0-0.6} = 1.907$$
$$-I_2 = 4.254 + (2.107 - 4.254)\frac{0.677-0.6}{1.0-0.6} = 3.841$$

$$q_s = \int_a^h \bar{c}(z)\bar{u}(z)dz = 11.6\cdot u_*\cdot\bar{c}(a)\cdot a\cdot\left[I_1\ln\left(\frac{30h}{k_s}\right) + I_2\right]$$

$$q_s = 11.6\times 0.096\times 9.9\times 0.01\times\left[1.907\times\ln\left(\frac{30\times 1}{0.01}\right) \text{-} 3.841\right] = 1.26 \text{ kg/(m s)}$$

4.

(a.i) By knowing the critical depth of 1.5 m, we can calculate the flow rate.

$q = h_c\sqrt{gh_c} = 5.75$ m^2/s

h_c = 1.5 m

$U_c = \sqrt{gh_c} = \sqrt{9.81\times1.5} = 3.836$ m/s

When the bed slope is 0.0025, the uniform flow depth is h_{01}.

$$U = C\sqrt{h_{01}S_{b1}} = \frac{1}{n}h_{01}{}^{2/3}S_{b1}{}^{1/2}$$

$$q = \frac{1}{n}h_{01}{}^{5/3}S_{b1}{}^{1/2}$$

$$5.75 = \frac{1}{0.02}\times h_{01}{}^{5/3}\times 0.0025^{1/2}$$

$$h_{01} = 1.648 \text{ m}$$

When the bed slope is 0.005, the uniform flow depth is h_{02}.

$$q = \frac{1}{n}h_{02}{}^{5/3}S_{b2}{}^{1/2}$$

$$5.75 = \frac{1}{0.02}\times h_{02}{}^{5/3}\times 0.005^{1/2}$$

$$h_{02} = 1.339 \text{ m}$$

Hence, the water depth has to decrease from 1.648 m upstream of the slope change to 1.339 m downstream of the slope change (dh/dx < 0 over the transition) The flow velocity increases as the water depth decreases, causing the bed friction and energy loss rate to increase. We have $S_f = S_{b1}$ =0.0025 at 1.648 m water depth and $S_f = S_{b2}$ = 0.005 at 1.339 m water depth. When the water depth is between 1.339 m and 1.648 m, the friction loss rate is

$$S_{b1} < S_f < S_{b2}$$

Along the upstream mild slope, $S_{b1} - S_f < 0$.

Along the downstream steep slope, $S_{b2} - S_f > 0$

$$\frac{dh}{dx} = \frac{S_b - S_f}{1 - Fr^2}$$

Along the upstream slope, we need the flow to be subcritical $Fr < 1$ to get $\frac{dh}{dx} < 0$.

Along the downstream slope, we need the flow to be supercritical to obtain $\frac{dh}{dx} < 0$.

Therefore, the critical flow has to occur exactly at the break in the slope.

(a.ii)

$$\frac{d}{dx}\left(h + \frac{U^2}{2g}\right) = S_b - S_f$$

When water depth is 1.6 m, U = 3.594 m/s

$$\left(h + \frac{U^2}{2g}\right) = 2.258 \text{ m}$$

$$S_f = \frac{n^2\cdot U^2}{h^{4/3}} = 0.00276$$

When water depth is 1.5 m, U = 3.836 m/s

$$\left(h+\frac{U^2}{2g}\right)=2.250 \text{ m}$$

$$S_f=\frac{n^2\cdot U^2}{h^{4/3}}=0.00343$$

When water depth is 1.4 m, U = 4.107 m/s

$$\left(h+\frac{U^2}{2g}\right)=2.260 \text{ m}$$

$$S_f=\frac{n^2\cdot U^2}{h^{4/3}}=0.00431$$

When the water depth changes from 1.6 m to 1.5 m, S_b = 0.0025

$$\frac{2.250-2.258}{\Delta x}=0.0025-\frac{0.00276+0.00343}{2}$$

$$\Delta x=13 \text{ m}$$

When the water depth changes from 1.5 m to 1.4 m, S_b = 0.005

$$\frac{2.260-2.250}{\Delta x}=0.0050-\frac{0.00343+0.00431}{2}$$

$$\Delta x=8.85 \text{ m}$$

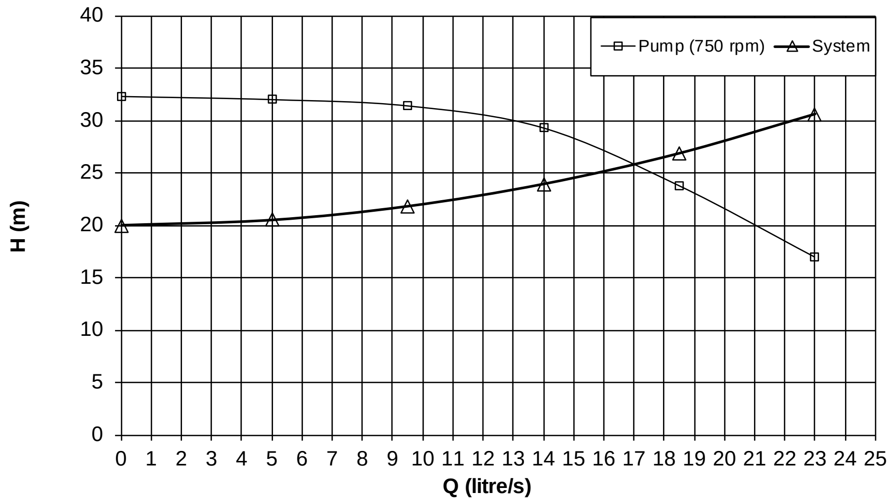
(b.i)

Static lift = 20 m.

The system curve is: $H = 20+0.02Q^2$

Q (litre/s)	0	5	9.5	14	18.5	23
Pump (750 rpm)	32.3	32	31.4	29.3	23.8	17
System	20	20.5	21.805	23.92	26.845	30.58

Plot the pump curve and system curve, which cross at the duty point: Q = 17 l/s



(b.ii)

The new duty point is Q_2 = 17 l/s, $H_2 = 20+0.03Q^2 = 28.67$ m.

The parabolic line through the origin and this new duty point is

$$H = \frac{H_2}{{Q_2}^2} Q^2 = \frac{28.67}{17^2} Q^2 = 0.0992 Q^2$$

It crosses the pump curve (750 rpm) at:

$Q_1 = 16.3$ l/s, $H_1 = 26.7$ m

According to either $\frac{Q_2}{N_2} = \frac{Q_1}{N_1}$ or $\frac{H_2}{{N_2}^2} = \frac{H_1}{{N_1}^2}$, where $N_1 = 750$ rpm, we get N_2.

$$\frac{17}{N_2} = \frac{16.3}{750} \quad \Rightarrow \quad N_2 = 782 \text{ rpm}$$

$$\frac{28.67}{{N_2}^2} = \frac{26.7}{750^2} \quad \Rightarrow \quad N_2 = 777 \text{ rpm}$$