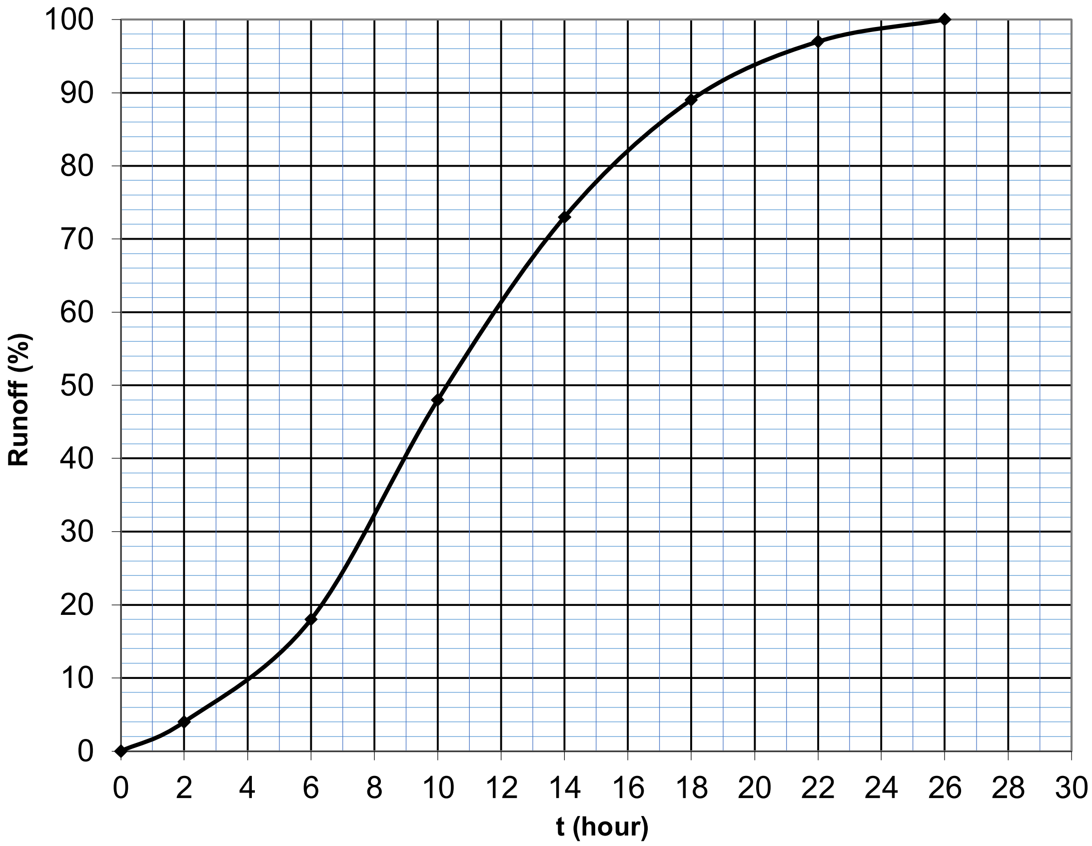
2021/2022 IIA 3D5 - Water Engin1eering Dr D. Liang

1.

(a) The rainfall excess is the amount of rainwater that is responsible for overland runoff. It is equal to the total rainfall minus the retention, infiltration and evapotranspiration.

(b) Construct the S curve

Time (hour)	0	2	6	10	14	18	22	26
Runoff (%)	0	4	18	48	73	89	97	100



Need to find the runoff proportions due to a two-hour uniform rainfall excess.

Time (h)	0.0	1.0	3.0	5.0	7.0	9.0	11.0	13.0	15.0	17.0	19.0	21.0	23.0	25.0	27.0
S-curve (%)	0.0	1.5	7.0	13.0	24.0	40.5	54.5	67.5	77.5	85.5	91.5	95.5	98.0	99.5	100.0
2-hour shifted S-curve (%)	0.0	0.0	1.5	7.0	13.0	24.0	40.5	54.5	67.5	77.5	85.5	91.5	95.5	98.0	99.5
Subtract the 2 S-curves (%)	0.0	1.5	5.5	6.0	11.0	16.5	14.0	13.0	10.0	8.0	6.0	4.0	2.5	1.5	0.5

The first and last rows in the above table gives the outflow hydrograph for 2-hour rain.

(c) The infiltration rate at the end of the first two hours is:

$$f = f_c + (f_0 - f_c)e^{-K_f t} = 5 + (20 - 5)e^{-0.5\times 2} = 10.5 \text{ mm/hr}$$

So, all rainwater infiltrates into the ground in the first two hours, with no rainfall excess.

Need to calculate the equivalent time of infiltration.

$\int_0^T f \cdot dt = f_c(T-0) - \frac{1}{K_f}(f_0 - f_c)\left(e^{-K_f T} - 1\right)$

$= 5T - \frac{1}{0.5}(20-5)(e^{-0.5T} - 1) = 10 \times 2$ mm

By trial and error, T = 1.24 hr

The total infiltration during the second two-hour period is:

$\int_{1.24}^{3.24} f \cdot dt = f_c(3.24 - 1.24) - \frac{1}{K_f}(f_0 - f_c)\left(e^{-K_f 3.24} - e^{-K_f 1.24}\right)$

$= 5 \times 2 - \frac{1}{0.5}(20-5)\left(e^{-1.62} - e^{-0.62}\right) = 20.2$ mm

The total excess rain in the first two-hour period is: 30×2-20.2 = 39.8 mm

The total volume of the excess flow in the second two-hour period is:

$39.8 \times 10^{-3} \times 10 \times 10^6 = 3.98 \times 10^5$ m^3

According to the 2-hour hydrograph obtained in (b), the peak flow rate occurs between 8 hr and 10 hr after the excess rain is produced, which is between 10 hr and 12 hr after the onset of rain.

The peak flow rate 11 hr after the rain starts is:

$\frac{3.98 \times 10^5 \times 16.5\%}{2 \times 3600} = 9.12$ m^3/s

Examiner's Comments:

Quite a few explanations for the rainfall excess were not precise. For example, some explained that it is the rainfall that ends up in the river system or it is the rainfall that continues in the hydrological cycle.

Some found it a bit difficult to calculate the infiltration in the second two-hour period.

2.

(a) A fully-development boundary layer flow occurs in a uniform open channel flow. Once the boundary layer thickness is equal to the water depth, then the boundary layer stops developing. The fully-developed boundary layer flow can be either laminar or turbulent.

(b)

Width at free surface: $B = h \times \tan 3\,0^0 \times 2 = 1.155h$

Area: $A = \frac{1}{2} \times Bh = 0.577h^2$

Wetted perimeter: $P_h = 2 \times \frac{h}{\cos 30^0} = 2.31h$

Hydraulic radius: $R_h = \frac{A}{P_h} = 0.25h$

Manning formula: $U = \frac{1}{n} R_h{}^{2/3} S_b{}^{1/2} = \frac{1}{0.013}(0.25h)^{2/3} 0.01^{1/2} = 3.05h^{2/3}$

$Q = UA = 1.76h^{8/3} = 6$

h = 1.58 m

At critical depth, Froude number is 1.

$$Fr = \frac{U}{\sqrt{gA/B}} = \frac{3.05\ ^{2/3}}{\sqrt{g0.577\ ^2/(1.155h)}} = \frac{4.14}{\sqrt{9.81 \times 1.44/\ .82}} = 1.49$$

Supercritical

(c) Gradually varied flow: $\frac{dh}{dx} = \frac{S_b - S_f}{1 - Fr^2} = \frac{S_b - \frac{n^2 \cdot U^2}{R_h{}^{4/3}}}{1 - Fr^2}$

When h = 1.4 m: $U = \frac{5.5}{2.5 \times 1.4} = 1.57$ m/s

$R_h = \frac{2.5 \times 1.4}{2.5 + 2.8} = 0.66$ m

$S_f = \frac{n^2 \cdot U^2}{R_h{}^{4/3}} = \frac{0.015^2 \times 1.57^2}{0.66^{4/3}} = 9.65 \times 10^{-4}$

$Fr = \frac{U}{\sqrt{gh}} = \frac{1.57}{\sqrt{9.81 \times 1.4}} = 0.424$

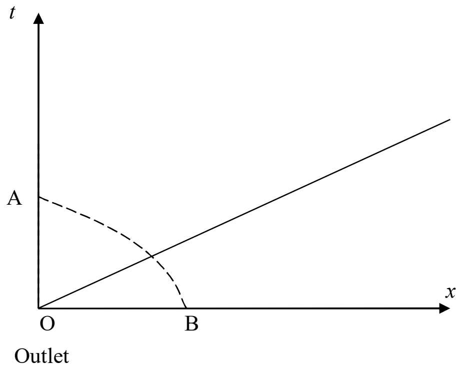
$\frac{dh}{dx} = \frac{S_b - S_f}{1 - Fr^2} = \frac{0.003 - 0.000965}{1 - 0.424^2} = 0.00248$

$\frac{\Delta h}{15} = 0.00248$

$\Delta h = 0.037$ m

Water depth is 1.437 m

(d)



The flume occupies the positive x axis, so the velocity is negative.

From any point A at the boundary, the flow is critical. So, $U_A = -\sqrt{gh_A}$

Draw a negative line from A. Write the negative relationship between A and B.

$$U_A - 2\sqrt{gh_A} = U_B - 2\sqrt{gh_B}$$

$$-\sqrt{gh_A} - 2\sqrt{gh_A} = 0 - 2\sqrt{gh}$$

$$-3\sqrt{gh_A} = -2\sqrt{gh}$$

$$h_A = \frac{4}{9}h$$

Examiner's Comments:

A few students made a silly mistake about the flow rate through a cross-section. Rather than Q=UA, they took U=QA.

A few mistakenly expressed the denominator of the Froude number to be $\sqrt{gR_h}$, rather than $\sqrt{g\,A/B}$.

In (c), some wanted to use the averaged frictional slope (S_f) in the calculations, which led to complicated algebra.

3

(a) In order to convey clear water, sediment transport is not allowed.

Given d = 1 cm, then $d_* = d \cdot \left(\frac{g(s-1)}{\nu^2}\right)^{\frac{1}{3}} = 0.01 \times \left(\frac{9.81(2.65-\ )}{10^{-12}}\right)^{\frac{1}{3}} = 253.0$

Find the critical Shields parameter.

$$\theta_c = \frac{0.30}{1+1.2d_*} + 0.055[1 - exp(-0.02d_*)] = \frac{0.30}{304.6} + 0.055[1 - 0.0063] = 0.0556$$

The critical shear stress is:

$$\tau_{bc} = \theta_c g(\rho_s - \rho)d = 0.0556 \times 9.81 \times 1650 \times 0.01 = 9.0 \text{ Pa}$$

The actual shear stress is:

$$\tau_b = \rho g R_h S_f = 9810 \frac{\left(1+1+\frac{2h}{\tan 20^o}\right)h/2}{1+2h/\sin 20^o} 0.001$$

By equating τ_b to τ_{bc}, we obtain h = 1.78 m

(b.i) Assume this is a wide channel.

$$u_* = \sqrt{ghS_b} = \sqrt{9.81 \times 0.2 \times 0.001} = 0.0443 \text{ m/s}$$

Given d = 1 mm, $d_* = d \cdot \left(\frac{g(s-1)}{\nu^2}\right)^{\frac{1}{3}} = 0.001 \times \left(\frac{9.81(2.65-1)}{10^{-12}}\right)^{\frac{1}{3}} = 25.3$

$$w_s = \frac{\nu}{d}\left[\sqrt{10.36^2 + 1.049 \cdot {d_*}^3} - 10.36\right] = \frac{10^{-6}}{0.001}\left[\sqrt{10.36^2 + 1.049 \cdot 25.3^3} - 10.36\right]$$

$$w_s = 0.12 \text{ m/s}$$

$$\frac{u_*}{w_s} = 0.37\ , \frac{u_* d}{\upsilon} = 44.3$$

Using Liu's Diagram, dunes are formed.

(b.ii) The grain-related Chezy coefficient

$$C' = 7.8\, ln\left(\frac{12.0 \cdot R_h}{k_s{}'}\right) = 7.8\, ln\left(\frac{12.0 \times 0.2}{0.001}\right) = 60.71$$

The overall Chezy coefficient

$$C = \frac{U}{\sqrt{hS_b}} = \frac{0.6}{\sqrt{0.2 \times 0.001}} = 42.43$$

The Shields parameter is

$$\theta = \frac{\tau_b}{g(\rho_s - \rho)d} = \frac{ghS_b}{g(s-1)d} = \frac{0.2 \times 0.001}{(2.65-1) \times 0.001} = 0.12$$

Meyer-Peter and Müller $$\frac{q_b}{\sqrt{g(s-1)\cdot d^3}} = 8\left[\left(\frac{C}{C'}\right)^{1.5}\theta - 0.047\right]^{1.5}$$

$$q_b = 8\left[\left(\frac{C}{C'}\right)^{1.5}\theta - 0.047\right]^{1.5}\sqrt{g(s-1)\cdot d^3}$$

$$q_b = 8\left[\left(\frac{42.43}{60.71}\right)^{1.5}0.12 - 0.047\right]^{1.5}\sqrt{9.81(2.65-1)\cdot 0.001^3} = 3.5765\times 10^{-6}$$

$q_b = 3.5765\times 10^{-6}\times 2650 = 0.0095$ kg/(m s)

(iii)

$D_{tx}+D_L = (0.15+5.86)\hbar u_* = 0.053$ m^2/s

Examiner's Comments:

The most common problem was the accuracy of the computation.

A few got wrong with the units of the bed load transport rate.

4.

(a) Diffusion transports material from a higher concentration region to a lower concentration region. Advection transports material from the upstream location to the downstream location.

(b.i)

Relative roughness height: $\frac{k_s}{D} = \frac{0.02}{200} = 0.0001$

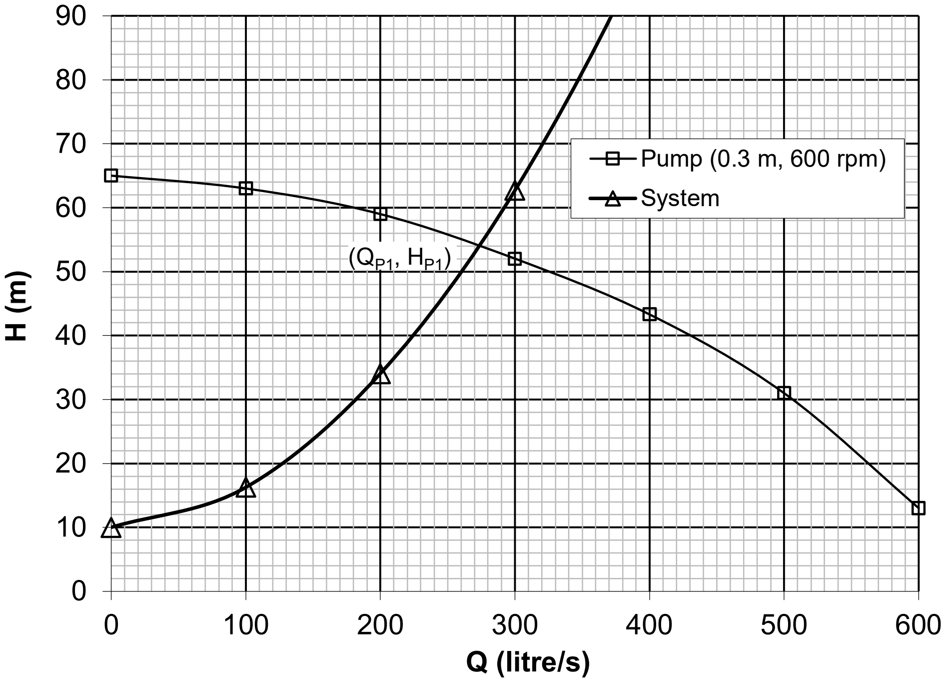
System curve: $H = 10 + \left(\lambda \frac{L}{D} + \sum \zeta\right)\frac{U^2}{2g} = 10 + \left(\lambda \frac{150}{0.2} + 1.5\right)\frac{U^2}{2\times 9.81}$

Construct the system curve using Moody diagram as:

Q (litre/s)	0	100	200	300	400	500	600
Pump (0.3 m, 600 rpm)	65	63	59	52	43.3	31	13
U (m/s)	0.00	3.18	6.37	9.55	12.74	15.92	19.11
Re	0.0E+00	6.4E+05	1.3E+06	1.9E+06	2.5E+06	3.2E+06	3.8E+06
Lamda	0.0000	0.0142	0.0135	0.0131	0.0129	0.0128	0.0127
System	10.0	16.3	34.0	62.7	102.4	153.5	215.2

Find the intersection between the system curve and pump curve.

We have the flow rate to be 273 litre/s.



(b.ii) Need to find the new pump curve with the new size and speed using dimensionless groups.

At homologous points: $\frac{Q_2}{N_2 D_2^3} = \frac{Q_1}{N_1 D_1^3}$ or $\frac{H_2}{N_2{}^2 D_2^2} = \frac{H_1}{N_1{}^2 D_1^2}$.

where N_2 = 480 rpm, D_2 = 0.4 m, N_1 = 600 rpm, D_1 = 0.3 m.

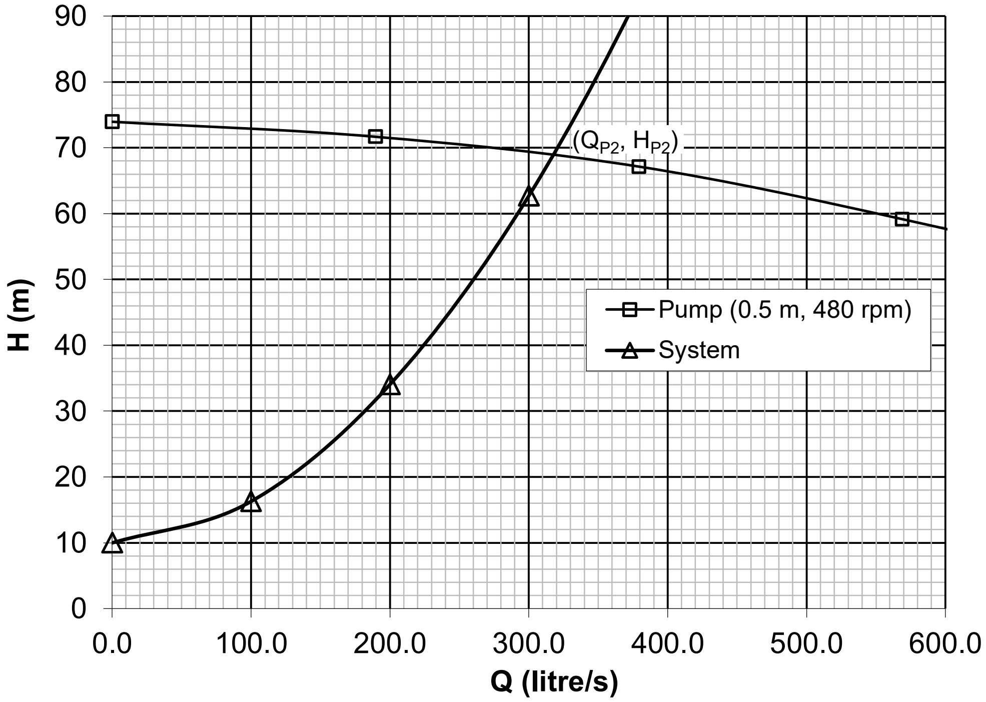
For each (Q_1, H_1) given in the table in the question, their homologous points are found.

The new pump curve is:

Q (litre^3/s)	0.0	189.6	379.3	568.9	758.5	948.1	1137.8
Pump (0.4 m, 480 rpm)	74.0	71.7	67.1	59.2	49.3	35.3	14.8

Find the intersection between the system curve and the new pump curve.

We have the flow rate to be 318 litre/s.



Examiner's Comments:

(b.ii) is a straightforward question, but some students tried to draw a parabola to find analogous points of the pump operation.