2023/2024 IIA 3D5 - Water Engin1eering Dr D. Liang

1.

(a) Students should mention the following catchment characteristics. One point shall be given for each catchment characteristic identified, for a possible total of 3 points.

- Size: More intense rainfalls are generally distributed over a relatively smaller area; a stream collecting water from a small catchment area is likely to give greater runoff intensity per unit area (a hydrograph with a big peak early on)
- Slope: If the surface slope is steep, water will flow quickly, and absorption and evaporation losses will be less, resulting in greater runoff.
- Storage: more storage along the catchment (reservoirs, natural lakes,wetlands), less pronounced the hydrograph peak, because surface runoff travelling downstream is slowed down by the storage
- Roughness: greater roughness will lead to lower runoff and a flatter hydrograph.
- Density: greater channel density can contribute to greater surface runoff as water finds an easy way to travel fast downstream. Low channel density typically means that water travels downstream more slowly, and thus the hydrograph will also be flatter.
- Geology: catchments underlain by highly permeable rocks (sandstone) tend to have flatter hydrographs, as more water infilitrates into the groundwater and contributes to baseflow, which is less variable than surface runoff. In catchments with impermeable soils (clays), we typically get hydrographs with more pronounced peaks in response to rainfall.

Two points shall be given to answers that correctly identify processes altering catchment characteristics, such as:

- Human processes that alter these characteristics include land use change, construction on in-stream hydraulic structures, operation of reservoirs.

(b) Definitions of El Nino should contain the following. Correct definitions are worth 2 points.

- The most prominent pattern of inter-annual climate variability, and is known to influence river flow and flooding at the global scale
- Involves changes in the temperature of waters in the central and eastern tropical Pacific Ocean.
- Occurs on periods ranging from about three to seven years
- El Niño involves a warming of the ocean surface, or above-average sea surface temperatures, in the central and eastern tropical Pacific Ocean

Explanation of why El Nino causes fisheries decline in the Pacific and drought in Australia should contain the following. Correct explanation is worth 3 points.

Decline in fisheries in the Pacific [2 points]

- El Niño also has a strong effect on marine life off the Pacific coast. During normal conditions, upwelling brings water from the depths to the surface; this water is cold and nutrient rich.

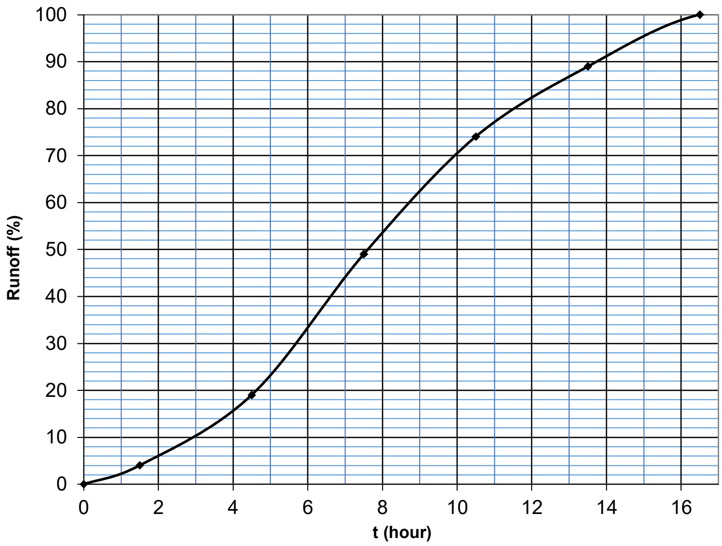
- During El Niño, upwelling weakens or stops altogether. Without the nutrients from the deep, there are fewer phytoplankton off the coast and therefore fish (like anchovies) have fewer nutrients, leading to fisheries collapse.

Wheat production in Australia [1 point]

- El Nino means that there is less warm water off the coast of Australia and thus less wet and unsettled weather over Australia.
- This leads to drought conditions in Australia, impacting wheat production in the country.

(c) Construct the S curve

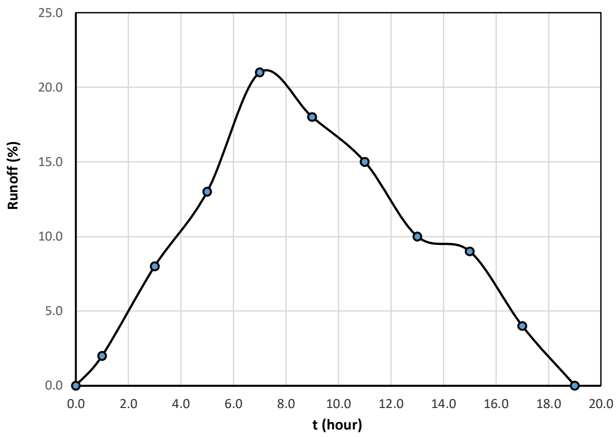
Time (hour)	0	1.5	4.5	7.5	10.5	13.5	16.5	19.5
Runoff (%)	0	4	19	49	74	89	100	100



Need to find the runoff proportions due to a two-hour uniform rainfall excess.

Time (h)	0.0	1.0	3.0	5.0	7.0	9.0	11.0	13.0	15.0	17.0	19.0
S-curve (%)	0.0	2.0	10.0	23.0	44.0	62.0	77.0	87.0	96.0	100.0	100.0
2-hour shifted S-curve (%)	0.0	0.0	2.0	10.0	23.0	44.0	62.0	77.0	87.0	96.0	100.0
Subtract the 2 S-curves (%)	0.0	2.0	8.0	13.0	21.0	18.0	15.0	10.0	9.0	4.0	0.0

The first and last rows in the above table gives the outflow hydrograph for 1-hour rain.



2.

(a)

Cross-section area: $A = 3 \times 2 + 2\sqrt{3} \times 2 = 12.93$ m

Flow velocity: $U = \frac{Q}{A} = \frac{50}{12.93} = 3.87$ m/s

Width at water level: $B = 3 + 2 \times 2 \times \sqrt{3} = 9.93$ m

Froude Number: $Fr = \frac{3.09}{\sqrt{9.81 \times \frac{12.93}{9.93}}} = \frac{3.87}{3.57} = 1.08$

Flow is supercritical.

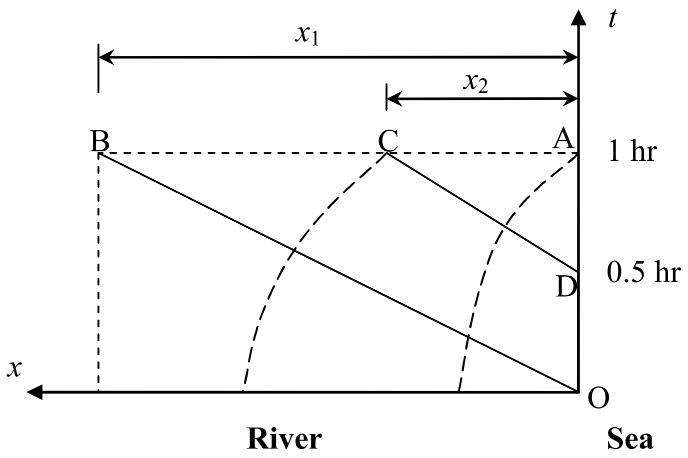
Wetted perimeter: $P_h = 3 + 2 \times 2 \times 2 = 11$ m

Hydraulic radius: $R_h = \frac{12.93}{11} = 1.18$ m

Manning formula: $U = \frac{1}{n} {R_h}^{2/3} {S_b}^{1/2}$

$$S_b = \left(\frac{Un}{{R_h}^{2/3}}\right)^2 = \left(\frac{3.87 \times 0.015}{1.18^{2/3}}\right)^2 = 0.0027$$

(b)



Suppose it is point C, so $h_C = 3.5$ m.

Along –ve line from C: $U_C - 2\sqrt{3.5g} = 0 - 2\sqrt{4g}$. So $U_C = -0.81$ m/s

Along +ve line CD, $U_D = U_C = -0.81$ m/s and $h_D = h_C = 3.5$ m.

This corresponds to a time of 0.5 hour at D. Consider the slope of CD:

$$\frac{x_2}{1800} = -0.81 + \sqrt{3.5g}$$

So x_2 = 9091 m

3 (i)

Total shear: $\tau_b = \rho g R_h S_b = 1000 \times 9.81 \times 2 \times 4 \times 10^{-4} = 7.848$ Pa

Grain-related roughness: $k_s' = 2d = 2 \times 0.3 \times 10^{-3} = 6 \times 10^{-4}$

Grain-related Chezy: $C' = 7.8 \ln\left(\frac{12h}{k_s'}\right) = 7.8 \ln\left(\frac{12 \times 2}{6 \times 10^{-4}}\right) = 82.65$ m$^{1/2}$ s^{-1}

Grain-related shear: $\tau_b' = \frac{g}{C'^2} \rho \cdot U^2 = \frac{9.81}{82.65^2} \cdot 1000 \cdot 1^2 = 1.436$ Pa

Form-related shear: $\tau_b'' = \tau_b - \tau_b' = 7.848 - 1.436 = 6.412$ Pa

(ii)

For uniform flow: $U = C\sqrt{R_h S_b}$

$$C = \frac{U}{\sqrt{R_h S_b}} = \frac{1}{\sqrt{2 \times 4 \times 10^{-4}}} = 35.355 \text{ m}^{1/2}\text{ s}^{-1}$$

$C = 7.8 \ln\left(\frac{12h}{k_s}\right)$ => $k_s = 0.258$ m

$$k_s'' = k_s - k_s' \approx k_s$$

(iii)

$$u_* = \sqrt{\frac{\tau_b}{\rho}} = \sqrt{g R_h S_f} = \sqrt{9.81 \times 2 \times 4 \times 10^{-4}} = 0.0886 \text{ m/s}$$

$$u_*' = \sqrt{\frac{\tau_b'}{\rho}} = \sqrt{\frac{1.436}{1000}} = 0.0379 \text{ m/s}$$ (This value is not needed.)

$$d_* = d \cdot \left(\frac{g(s-1)}{\nu^2}\right)^{1/3} = 0.3 \times 10^{-3} \times \left(\frac{9.81 \times (2.65-1)}{10^{-12}}\right)^{1/3} = 7.589$$

$$w_s = \frac{\nu}{d}\left[\sqrt{10.36^2 + 1.049 \cdot d_*^3} - 10.36\right]$$

$$w_s = \frac{10^{-6}}{0.3 \times 10^{-3}}\left[\sqrt{10.36^2 + 1.049 \times 7.589^3} - 10.36\right] = 0.0448 \text{ m/s}$$

$$\theta' = \frac{\tau_b'}{g(\rho_s - \rho)d} = \frac{1.436}{9.81 \times (2650 - 1000) \times 0.3 \times 10^{-3}} = 0.296$$

$$\bar{c}(2d) = \frac{0.331 \cdot (\theta' - 0.045)^{1.75}}{1 + 0.72 \cdot (\theta' - 0.045)^{1.75}} = \frac{0.331 \cdot (0.296 - 0.045)^{1.75}}{1 + 0.72 \cdot (0.296 - 0.045)^{1.75}} = 0.0277$$

This is the volume concentration.

According to $$\frac{\bar{c}(z)}{\bar{c}(a)} = \left(\frac{h-z}{z} \cdot \frac{a}{h-a}\right)^{\frac{w_s}{\kappa u_*}}$$

Take a = 2d = 0.6×10^{-3} m and z = 1 m.

$$\frac{\bar{c}(1)}{\bar{c}(0.6\times10^{-3})} = \left(\frac{2-1}{1} \cdot \frac{0.6\times10^{-3}}{2-0.6\times10^{-3}}\right)^{\frac{0.0448}{0.4\times0.0886}} = 3.52 \times 10^{-5}$$

So, the concentration at mid-depth is:

$$\bar{c}(1) = 3.52 \times 10^{-5} \times 0.0277 \times 2650 = 0.00258 \text{ kg/m}^3$$

(iv)

This is a two-dimensional continuous-release problem.

$$D_{ty} = 0.15hu_* = 0.15 \times 2 \times 0.0886 = 0.02658 \text{ m}^2\text{/s}$$

Considering the image of one channel bank:

$$\bar{c}(x,y) = 2 \times \frac{\dot{M}/h}{U\sqrt{4\pi \frac{x}{U} D_y}} \exp\left(-\frac{y^2}{4D_y\, x/U}\right)$$

Knowing $\bar{c}(x=50, y=1) = 10$ mg/l = 0.01 kg/m^3:

$$0.01 = 2 \times \frac{\dot{M}/2}{1 \times \sqrt{4 \times 3.14 \times \frac{50}{1} \times 0.02658}} \exp\left(-\frac{1^2}{4 \times 0.02658 \times 50/1}\right)$$

$$\dot{M} = 0.049 \text{ kg/s}$$

The total mass discharged in a day is: 0.049×24×3600 = 4233.6 kg = 4.2 ton

4.

(a) Given in the lecture notes. Chezy formula can be derived from either momentum or energy principle.

(b)

(b.i) Plot the pump curve (750rpm) and system curve ($20+0.02Q^2$).

The discharge and head at the duty point are: Q_1=17.6 litre/s, H_1=26.1m, η_1=82%.

The pump input power is: $P_1 = \frac{\rho g Q_1 H_1}{\eta_1} = \frac{1000\times9.81\times0.0176\times26.1}{0.82} = 5496$ W

(b.ii)

If the speed is still 750rpm, the discharge and head at the duty point are:

Q_2=16.2 litre/s, H_2=28m, η_2=83%.

The pump input power is: $P_2 = \frac{\rho g Q_2 H_2}{\eta_2} = \frac{1000\times9.81\times0.0162\times28}{0.83} = 5361$ W

The new system curve is H = ($20+0.03Q^2$).

When the flow rate is 17.6 litre/s, the new head is $20+0.03\text{X}17.6^2$=29.3 m

Plot a parabola between origin and new duty point (Q_2=17.6 litre/s, H_2=29.3m).

The parabola intersects the pump curve (750rpm) at: Q_0=16.9 litre/s, H_0=27.1m, η_0=82.7%

This is the equivalent operating point of the pump at 750 rpm to the desired new duty point.

These two points are analogous points (so $\eta_2 = \eta_0$).

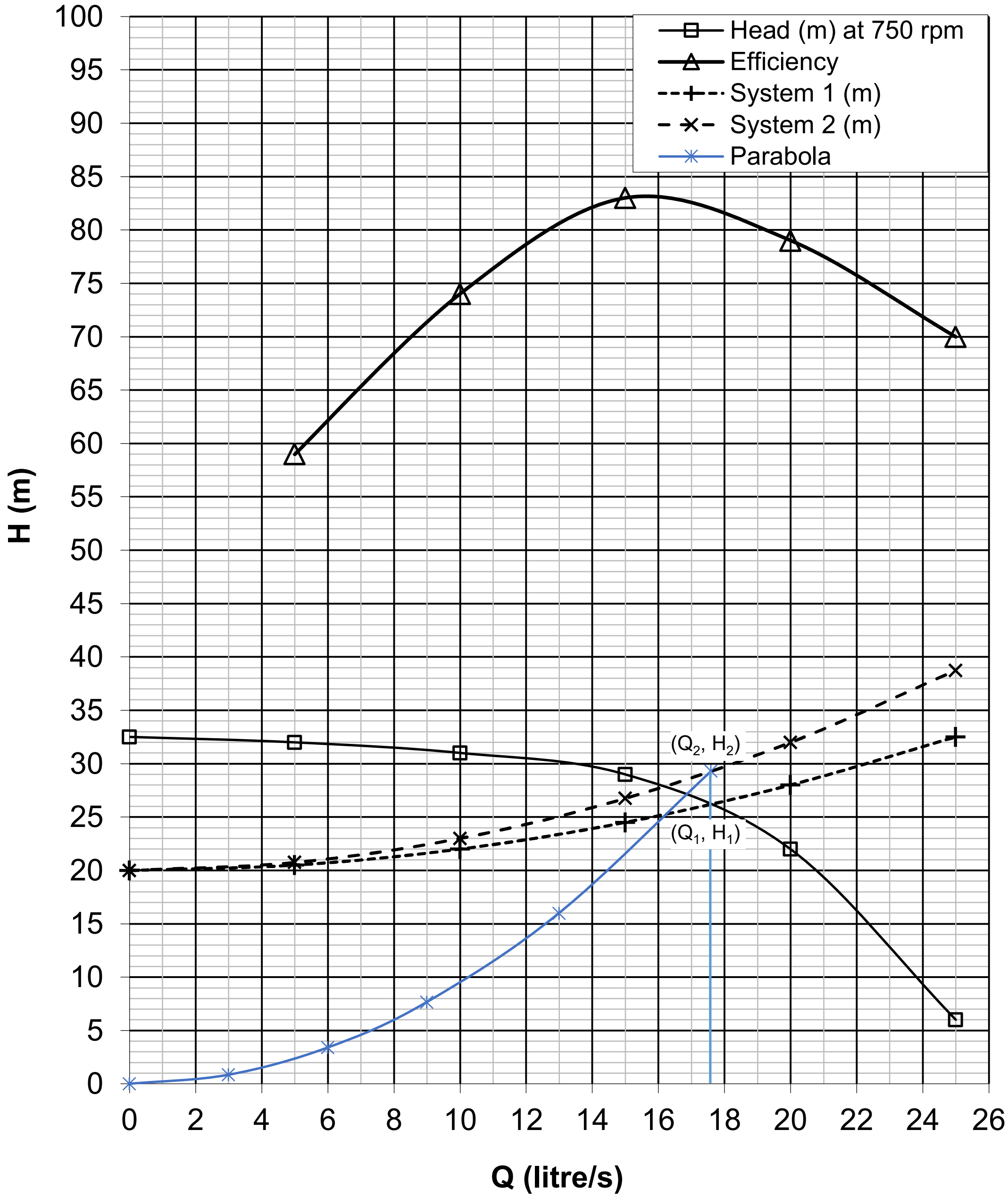
From $\frac{Q_2}{Q_0} = \frac{N_2}{N_0}$, $\frac{17.6}{16.8} = \frac{N_2}{750}$, N_2 = 781 rpm.

From $\frac{H_2}{H_0} = \left(\frac{N_2}{N_0}\right)^2$, $\frac{29.3}{27.1} = \left(\frac{N_2}{750}\right)^2$, N_2 = 780 rpm.

This is the speed that the pump needs to be run to maintain the same discharge.

The pump input power is: $P_2 = \frac{\rho g Q_2 H_2}{\eta_2} = \frac{1000\times9.81\times0.0176\times29.3}{0.827} = 6117$ W

Q (litre/s)	0	5	10	15	20	25
Head (m) at 750 rpm	32.5	32.0	31.0	29.0	22.0	6.0
Efficiency		59	74	83	79	70
System 1 (m)	20.0	20.5	22.0	24.5	28.0	32.5
System 2 (m)	20.0	20.8	23.0	26.8	32.0	38.8


Head (m) at 750 rpm
Efficiency
System 1 (m)
System 2 (m)
Parabola
H (m)
Q (litre/s)
(Q2, H2)
(Q1, H1)
100
95
90
85
80
75
70
65
60
55
50
45
40
35
30
25
20
15
10
5
0
0
2
4
6
8
10
12
14
16
18
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26