3D8 : Bldg Physics & Environmental Geotechniques 2016

Q1 a) Thermal conductivity of a granular media such as soils is the ability of the material to conduct heat. It depends on both the solid soil particles as well as the pore water. Expressed normally as λ it has the units of W/m/°K.

Volumetric Heat Capacity (VHC) is defined as the ability of a given volume of soil to store heat energy for given change in temperature. VHC has the units of $J/m^3/K$.

The thermal diffusivity of a material α can be viewed as the ease with which heat can flow through the soil. It is defined as the ratio of the thermal conductivity to the volumetric heat capacity. Thermal diffusivity α has the units of m^2/s. [15%]

1 b) In dry soil, much of the heat flows through conduction at the contact points between the soil grains. The air within the pore space does not conduct heat effectively. In saturated soils, heat can be transferred through conduction at contact points but also through the pore fluid present in the void space. The thermal conductivity values are therefore higher for saturated soil compared to the dry soil [2~4 W/m/°K for saturated soil verses 0.15~2.0 W/m/°K]. [10%]

1 c) ii)

Leakage rate = $q = K\,\Delta\bar{H}\,\frac{N_f}{N_h} = 4.2 \times 10^{-4} \times 6.5 \times \frac{4}{16} = 6.825 \times 10^{-4}\,\frac{m^2}{s}/m$

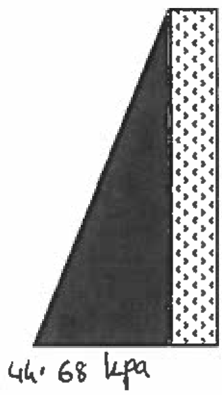
For a 100 m long tunnel this will be = $58.968\,\frac{m^2}{day} \times 100$

This is <u>58968</u> litres per day.

Pressure distribution along the base:

Pressure distribution along the left tunnel wall:

$\bar{h} = 6.5 \quad y = 6.5 \Rightarrow h = 0 \Rightarrow u = 0$ at top



$\bar{h} = 4.46875\,m \quad y = 0 \Rightarrow h = 4.46875 \Rightarrow u = h\,\gamma_w$

$u = 44.68$ kPa

[25%]

Q1 iii) weight of tunnel

$$W = (10 \times 6.5 - 8 \times 4.5 + 0.5 \times 4.5)\ 24\ kN/m^3$$

$$= 750\ kN/m$$

$$\text{Uplift} = U = \frac{1}{2} \times 44.68 \times 10 = 223.4\ kN/m$$

$$\therefore W' = W - U = 526.6\ kN/m$$

$$\text{Resisting force} = W' \tan\phi = 526.6 \times \tan 26^\circ$$

$$= 256.84\ kN/m$$

Horizontal force due to water pressure

$$= \frac{1}{2} \times 44.68 \times 6.5 = 145.21\ kN/m$$

$$\text{Soil thrust} = 80\ kN/m$$

$$\text{Total disturbing force} = 225.21$$

$$\therefore \text{FoS against sliding} = \frac{256.84}{225.21} = \underline{\underline{1.14}}$$

This is quite low and any more rain that raises water level to left of the tunnel can cause sliding failure. [15%]

Q1 IV) we can increase the weight of the tunnel by covering the tunnel roof, for example (or increasing wall thickness). It is however, better to take measures to relieve the water pressure, eg. provide drains vertically along the left tunnel wall and the base of the tunnel. Alternatively have drainage holes into tunnel and have a base drain at the tunnel invert. [10%]

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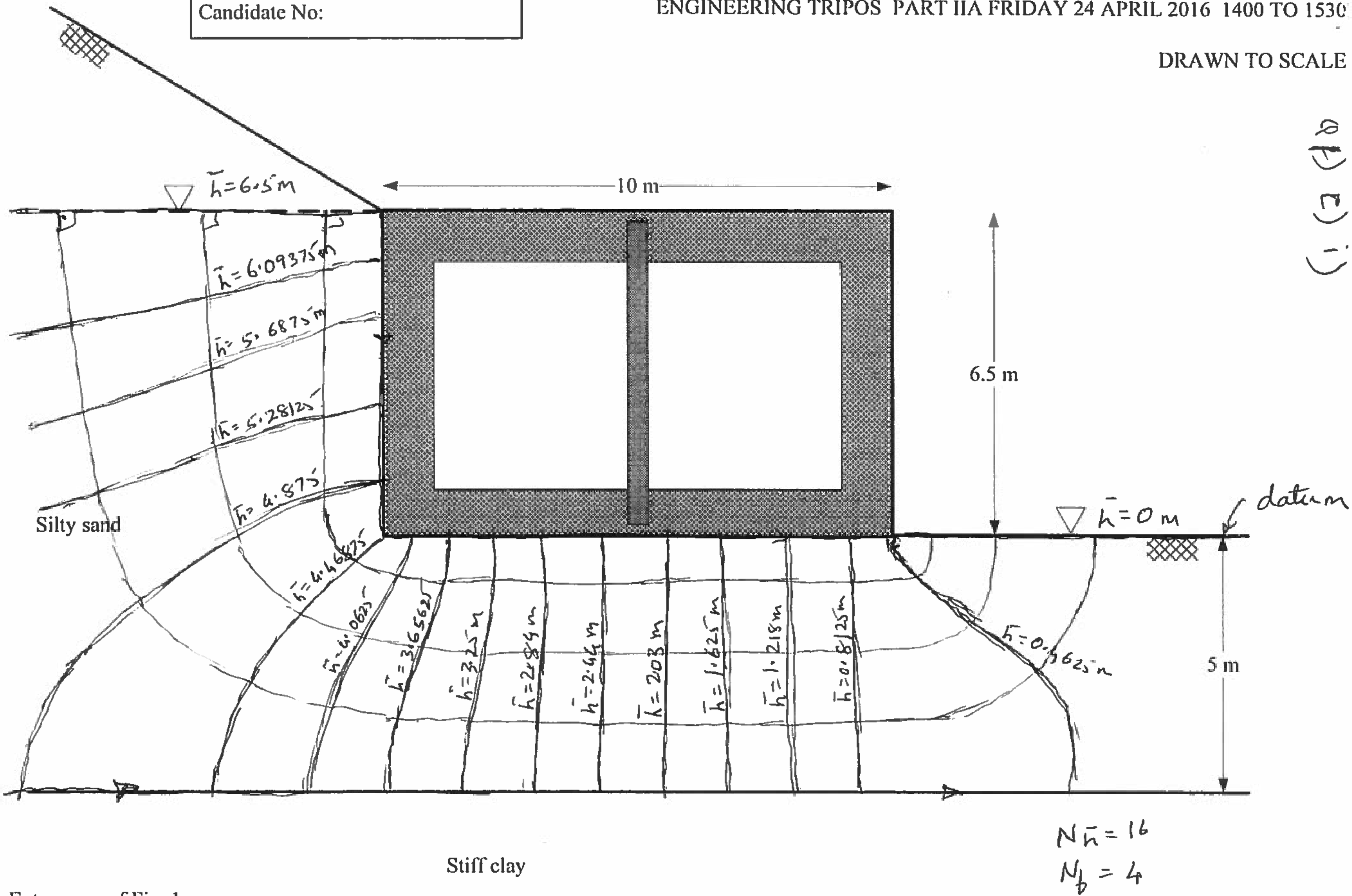
Q f) c) i)

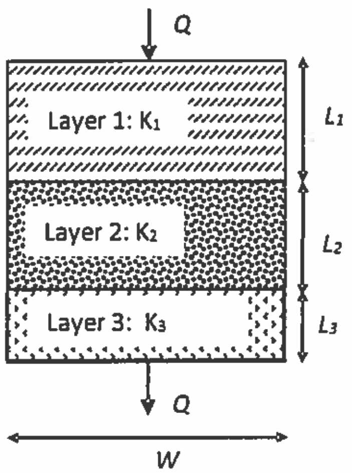


Extra copy of Fig. 1

2.2 Q1 a) When soil that is formed by rock fragments get transported by water flowing in a river into a lake or the sea, it starts to sediment. The soil gets naturally separated into layers depending on particles sizes. Also, when the flow is high larger fractions get transported and can form coarse sand layers. During winter months, when flow is low, only smaller fractions arrive forming a fine sand or clay layers. It is therefore natural to have alternating layers of coarse and fine fractions in such varved deposits.

[10%]

2b) Consider vertical flow through a soil deposit of three horizontal layers with differing hydraulic conductivity.



Assume unit length into the page. Continuity of mass flow rate requires that the volumetric flow rate of water through layer 1 be the same as that through layer 2 and so on.

$$Q = Q_1 = Q_2 = Q_3$$

$$A = A_1 = A_2 = A_3 = v \text{ constant}$$

$$\Delta\bar{h}_1 \neq \Delta\bar{h}_2 \neq \Delta\bar{h}_3 \quad \text{but} \quad \Delta\bar{h}_1 + \Delta\bar{h}_2 + \Delta\bar{h}_3 = \Delta\bar{h}$$

Darcy's Law for layer 1 gives the specific discharge:

$$v = \frac{Q}{W} = K_1 \frac{\Delta\bar{h}_1}{L_1} \text{ and } \Delta\bar{h}_1 = v\frac{L_1}{K_1}$$

where $\Delta\bar{h}_1$ is the difference in potential head between the top and bottom of layer 1. Similarly for layers 2 and 3:

$$\Delta\bar{h}_2 = v\frac{L_2}{K_2} \ \& \ \Delta\bar{h}_3 = v\frac{L_3}{K_3}$$

$$\Delta\bar{h}_1 + \Delta\bar{h}_2 = v\left[\frac{L_1}{K_1} + \frac{L_2}{K_2} + \frac{L_3}{K_3}\right]$$

or

$$v = \frac{L_1 + L_2 + L_3}{\left(\frac{L_1}{K_1} + \frac{L_2}{K_2} + \frac{L_3}{K_3}\right)}\left(\frac{\Delta\bar{h}_1 + \Delta\bar{h}_2 + \Delta\bar{h}_3}{L_1 + L_2 + L_3}\right)$$

but $\left(\frac{\Delta\bar{h}_1 + \Delta\bar{h}_2 + \Delta\bar{h}_3}{L_1 + L_2 + L_3}\right)$ is the average or effective vertical hydraulic gradient across the whole deposit, and so the effective vertical hydraulic conductivity of the deposit is:

$$K_{vertical} = \frac{L_1 + L_2 + L_3}{\left(\frac{L_1}{K_1} + \frac{L_2}{K_2} + \frac{L_3}{K_3}\right)}$$

For flow perpendicular to the layers, the specific discharge *v* must be the same in each layer.

[20%]

2c) For a falling head permeameter, the flow rate in the tube must be the same is the soil sample. If the water level changes by *–dh/dt*, then applying Darcy's law, we can write;

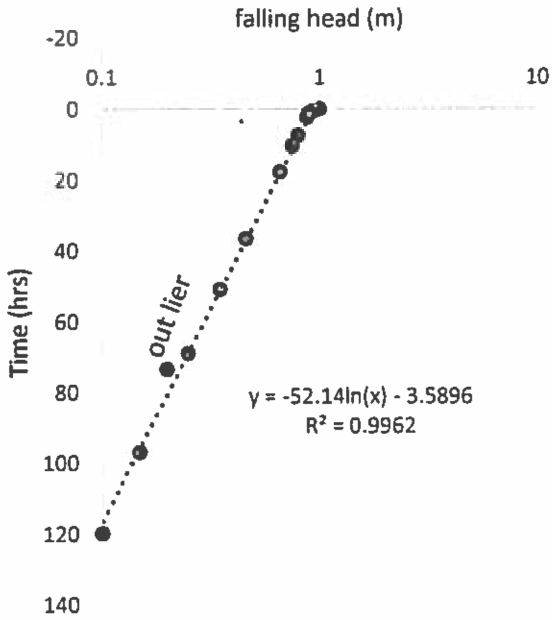
$$-a\,\frac{dh}{dt} = A\,K\,\frac{h}{L}$$

Reorganising; $$-a\int_{h1}^{h2}\frac{dh}{h} = \frac{AK}{L}\int_{t1}^{t2} dt$$

Integrating, we can write; $$K = \frac{a\,L}{A}\,\frac{\ln(\frac{h_1}{h_2})}{(t_2 - t_1)}$$

[25%]

2d) In the above equation, the first set of terms are all apparatus dependent, and the second set are results from the falling head test. We can plot the data given (avoiding initial points and outliers) on a semi-log plot and obtain the slope, as shown in the graph.



Therefore;

$$K = \frac{98.4 \times 10^{-3}}{6000} \times \frac{1}{52.14}$$

The hydraulic conductivity of the soil sample is;

$$K = 3.14538 \times 10^{-7}\ m/s$$

[25%]

2e) Using Part b) result, we can see that $K_{vertical} = \frac{L_1 + L_2 + L_3}{\left(\frac{L_1}{K_1} + \frac{L_2}{K_2} + \frac{L_3}{K_3}\right)}$

$3.14538 \times 10^{-7} = \frac{0.01+0.0884}{\frac{0.0884}{3.3\times10^{-6}} + \frac{0.01}{K_2}} \Rightarrow K_2 = 3.4959 \times 10^{-8}\ m/s$ (Hydraulic conductivity of 'rogue' silty clay layer).

Although we can work out the hydraulic conductivity, we cannot know the location of this 'rogue' silty clay layer. This 10 mm thick layer can be present anywhere within the sample and the two sandwiching silty sand layers can make up the rest of the thickness of 88.4 mm. [20%]

III (a)

- Relative Areas of surfaces;
- Geometry of the surfaces relative to one another or the "view factors"
- Emissivities of the two surfaces
- Temperatures of the two surfaces

(b)(i) Find :

$$(T_i^4 - T_j^4) - 4\bar{T}_{ij}^3 (T_i - T_j) \quad \text{——} \; ①$$

Rewriting ①

$$\Rightarrow (T_i^2 + T_j^2)(T_i + T_j)(T_i - T_j) - 4\bar{T}_{ij}^3 (T_i - T_j)$$

Remove / Ignore $(T_i - T_j)$

$$\Rightarrow (T_i^2 + T_j^2)(T_i + T_j) - 4\bar{T}_{ij}^3 \quad \text{——} \; ②$$

Given : $\bar{T}_{ij} = \frac{T_i + T_i}{2}$ and $\Delta = \frac{T_i - T_j}{2}$

So $T_i = \bar{T}_{ij} + \Delta$ and $T_j = \bar{T}_{ij} - \Delta$

Substituting with $\bar{T}_{ij}$ & Δ in ②

$$\Rightarrow [(\bar{T}_{ij} + \Delta)^2 + (\bar{T}_{ij} - \Delta)^2][(\bar{T}_{ij} + \Delta) + (\bar{T}_{ij} - \Delta)] - 4\bar{T}_{ij}^3$$

$$\Rightarrow [\bar{T}_{ij}^2 + \Delta^2 + 2\bar{T}_{ij}\Delta + \bar{T}_{ij}^2 + \Delta^2 - 2\bar{T}_{ij}\Delta][2\bar{T}_{ij}] - 4\bar{T}_{ij}^3$$

$$\Rightarrow [2\bar{T}_{ij}^2 + 2\Delta^2][2\bar{T}_{ij}] - 4\bar{T}_{ij}^3$$

$$\Rightarrow 4\bar{T}_{ij}^3\left[1 + \frac{\Delta^2}{\bar{T}_{ij}^2}\right] - 4\bar{T}_{ij}^3$$

II

(b) (ii) <u>Given</u>

$$T_i - T_j = 15^\circ C \quad \text{and} \quad \frac{T_i + T_j}{2} = \overline{T_{ij}} = 0^\circ C$$

convert temperature to Kelvin

$$T_i - T_j = 15 \text{ [illegible]} = \text{[illegible]}15 \text{ K}$$

$$\overline{T_{ij}} = 0^\circ + 273.15 = 273.15 \text{ K}$$

$$\Delta = (T_i - T_j)/2 = \text{[illegible]}7.5 \text{ K}$$

$$\text{Error} = \left(\frac{\Delta}{\overline{T_{ij}}}\right)^2 = \left(\frac{\text{[illegible]}}{273.15}\right)^2 = 0.0\text{[illegible]}\%$$

$$\left(\frac{7.5}{273.15}\right)^2 \%$$

(c) From Data Book we know that for longwave radiation exchange in air cavity

$$Q_{12} = h_r A_1 (T_1 - T_2)$$

where $h_r = 4 \epsilon_{12} \sigma \overline{T}_{12}^3$

$$\frac{1}{\epsilon_{12}} = \frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1 \quad \text{and} \quad \overline{T}_{12} = \frac{T_1 + T_2}{2}$$

$\sigma = 5.67 \times 10^{-8}$ W m^{-2} K^{-4} (Stefan – Boltzmann constant)

<u>Given Data</u>

$\epsilon_{foil} = 0.05$; $\epsilon_{s1} = \epsilon_{s2} = 0.9$; $T_1 = 15^\circ C$ (288.15 K); $T_2 = 5^\circ C$ (278.15 K)

$$T_{foil} = \frac{T_1 + T_2}{2} = 283.15 \text{ K}$$

Heat Transfer without Foil

$$\frac{1}{\epsilon_{12}} = \frac{1}{0.9} + \frac{1}{0.9} - 1 \Rightarrow 1.22 \; ; \quad \epsilon_{12} = 0.818$$

$$\overline{T}_{12} = (288.15 + 278.15)/2 \Rightarrow 283.15 \text{ K}$$

$$h_r = 4 \times 0.818 \times 5.67 \times 10^{-8} \times (283.15)^3 = 4.21 \text{ W/m}^2\text{K}$$

$$Q_{without\ foil} = 4.21 \times 10 = \underline{\underline{42.1 \text{ W/m}^2}}$$

Cont.

III

(c) Heat Transfer with foil

$$\frac{1}{\epsilon_{1,f}} = \frac{1}{0.9} + \frac{1}{0.05} - 1 \Rightarrow 20.1 \; ; \; \epsilon_{f} = 0.049$$

$$\bar{T}_{1,f} = (283.15 + 288.15)/2 \Rightarrow 285.65 \text{ K}$$

or $$\bar{T}_{2,f} = (283.15 + 278.15)/2 \Rightarrow 280.65 \text{ K}$$

$$\left[\begin{array}{l} h_r = 4 \times 0.049 \times 5.67 \times 10^{-8} \times (285.65)^3 = 0.26 \text{ W/m}^2\text{K} \\ Q = 0.26 \times 5 = \underline{1.3 \text{ W/m}^2} \end{array} \right.$$

or $$\left[\begin{array}{l} h_r = 4 \times 0.49 \times 5.67 \times 10^{-8} \times (280.65)^3 = 0.25 \text{ W/m}^2\text{K} \\ Q = 0.25 \times 5 = 1.25 \text{ W/m}^2 \end{array} \right.$$

Comment: Only 3% of original heat transfer due to longwave radiation as a result of the low-e foil. The foil reflects back most of the radiant component of heat transfer towards the inner leaf of the wall.

extra: would also help in maintaining indoor temperature longer once room heating is shut down.

(d) ϵ = Ability of surface / object to emit energy as thermal radiation

For Glass Solar Heat Gain Coefficient measures how well a glass blocks heat from sunlight. It includes fraction of solar radiation directly transmitted + that absorbed by the glass to be subsequently released inwards

Generally glass that have low-e rating also tend to have low SHGC. However this may not be the case at all.

emissivity improves the insulating properties of the glass whereas SHGC reduces direct solar radiation.

IV (a)

Stack effect or buoyancy driven ventilation is caused by temperature differences inside and outside a building. A temperature difference implies density difference. If internal air is warmer it is less dense than outside air. Internal air thus tends to rise, flowing out of upper openings and drawing external air through lower openings

or

Stack ventilation is a method of ventilation that uses pressure differences across height of the bldg to pull air through the bldg.

hot air inside rises due to buoyancy & its low pressure draws outside air.

IV (b)

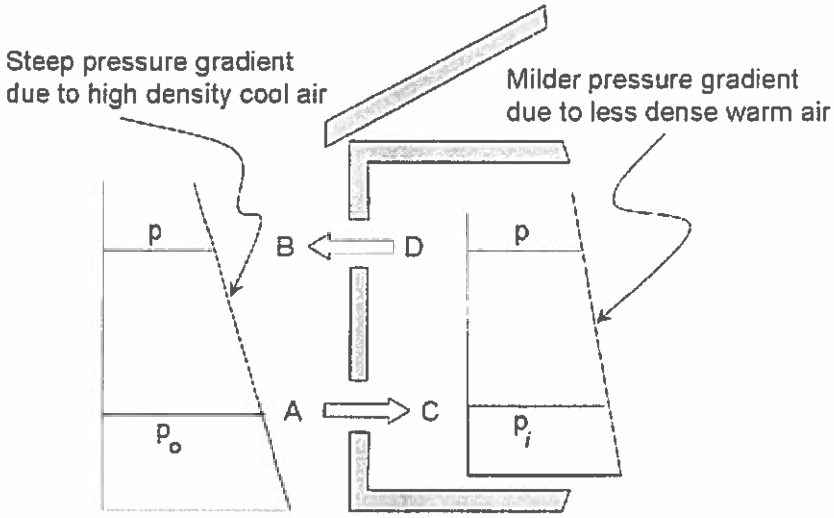


Figure 2: The buoyancy-driven stack effect (after CIBSE Guide A Figure 4.9)

The equations for stack effect can be derived by consideration of Figure 2, which shows a tall room with two openings, one at the top and one at the bottom.

The external temperature is T_o (in Kelvin) and the internal temperature is T_i everywhere through the room.

Assume that that the air is moving so slowly that kinetic energy terms (i.e. dynamic wind pressures) can be neglected everywhere except across the orifices. The pressure $p_o(H)$ outside the upper opening is thus:

$$p_o(H) = p_o - \rho_o g H \tag{12}$$

where ρ_o is the density of the external air at T_o, g is the gravitational constant and H is the distance separating the 2 openings. Similarly, inside the room, the pressure inside the upper window is:

$$p_i(H) = p_i - \rho_i g H \tag{13}$$

where ρ_i is the density of the internal air at T_i. The pressure differences across the bottom and top opening respectively are:

$$\Delta p_b = p_o - p_i$$
$$\Delta p_t = p_i(H) - p_o(H) = (p_i - \rho_i g H) - (p_o - \rho_o g H) = p_i - p_o + (\rho_o - \rho_i) g H$$

Thus, the total pressure difference between the two openings is:

$$\Delta p_s = \Delta p_b + \Delta p_t = (\rho_o - \rho_i) g H \tag{14}$$

Assuming that the air expands linearly with temperature, the density decreases as the reciprocal of temperature, thus

$$\rho \propto \frac{1}{T} \tag{15}$$

and taking conditions at say 273K as a benchmark, then

$$\frac{\rho}{\rho_{273}} = \frac{273}{T} \tag{16}$$

whence

$$\rho = \rho_{273}\frac{273}{T} \tag{17}$$

The buoyancy-driven pressure difference Δp_s driving the flow may thus be written

$$\Delta p_s = \rho_{273} g H \left(\frac{273}{T_o} - \frac{273}{T_i}\right) \tag{18}$$

Equating this to the pressure losses through the two orifices, we obtain

$$\Delta p = \frac{1}{2}\rho\,\frac{Q^2}{C_{orif1}^2 A_1^2} + \frac{1}{2}\rho\,\frac{Q^2}{C_{orif2}^2 A_2^2} \tag{19}$$

which rearranges to give a flow

$$Q = \frac{A_1 A_2}{\sqrt{A_1^2 + A_2^2}}\, C_{\mathsf{orif}} \sqrt{\frac{2\Delta p_s}{\rho}} \tag{20}$$

with Δp_s given by Equation 18.

This equation predicts the flow rate one might obtain from a stack-driven flow through two orifices separated by a height H.

Example: Consider a room at 20°C with an external temperature of 0°C and

IV (c) Given:

Bldg Area = 25 X 18 = 450 m^2

No. of floor = 5

Floor to Ceiling height = 3 m

Total air changes = 42

Cooling period = 18:00 hrs to 8:00 hrs = 14 hrs

(i) Free Cooling in kW w/ $T_i = 25°C$ and $T_e = 15°C$

Required No. of ACH (air changes per hour) = 42/14 = 3 ACH

As $Q = \frac{ACH \times Volume}{3600} = \frac{3 \times 450 \times 15}{3600} = 5.625\ m^3/s$

The mass transfer of air by natural ventilation:

$M = Q \times \rho = 5.625\ m^3/s \times 1.2\ kg/m^3$

(ρ → density of air (from databook))

$\Rightarrow 6.75\ kg/s$

Rate of cooling

$\Rightarrow M \cdot c \cdot dT \Rightarrow 6.75\ kg/s \times 1.0\ kJ/kgK \times 10$

(dT → (25°C − 10°C); c → specific heat capacity of air (from data book))

$\Rightarrow 67.5\ kJ/s$ is $67.5\ kW$

(ii) $Q = A^* C_{d} \sqrt{\frac{2 \Delta P_s}{\rho}}$

where $A^* = \frac{A_1 A_2}{\sqrt{A_1^2 + A_2^2}}$ and $\Delta P_s = \rho_{273}\, g H \left(\frac{273}{T_0} - \frac{273}{T_i}\right)$

Find A_2, given that $A_1 = 1.2\ m^2$

$H = 12\ m$

& Q (from (i)) $= 5.625\ m^3/s$

(Cont.

Continued

IV (c) (ii)

Given that Q (calculated in c(i)) = 5.625 m³/s

H = 12 m ; and A_1 = 1.2 m

$$\underline{\Delta ps} = \rho_{273}\, g\, H \left(\frac{273}{T_o} - \frac{273}{T_i}\right)$$

$$\Rightarrow 1.2 \times 9.81 \times 12 \left(\frac{273}{273.15 + 15} - \frac{273}{273.15 + 25}\right)$$

$$\Rightarrow 1.2 \times 9.81 \times 12 \left(\frac{273}{288.15} - \frac{273}{298.15}\right)$$

$$\Rightarrow 4.488$$

$$Q_1 (m^3/s) = A^* C_{oif} \sqrt{\frac{2 \Delta ps}{\rho}}$$

$$5.625 = A^* \times 0.61 \times \sqrt{\frac{2 \times 4.488}{1.2}}$$

$$\Rightarrow 5.625 = A^* \times 0.61 \times 2.735$$

$$\Rightarrow A^* = \frac{5.625}{0.61 \times 2.735} = 3.37 \ m^2$$

$$A^* = \frac{A_1 A_2}{\sqrt{A_1^2 + A_2^2}}$$

$$\Rightarrow A^{*2} (A_1^2 + A_2^2) = A_1^2 A_2^2$$

$$\Rightarrow A^{*2} A_1^2 + A_2^2 A^{*2} = A_1^2 A_2^2$$

$$\Rightarrow A_2^2 (A^{*2} - A_1^2) = - A^{*2} A_1^2$$

Given $A_1 = 1.2 \ m^2$

$A^* = 3.37 \ m^2$

$$\Rightarrow A_2^2 = \frac{-A^{*2} A_1^2}{A^{*2} - A_1^2} \Rightarrow \frac{A^{*2} A_1^2}{A_1^2 - A^{*2}}$$

$$A_2 = \frac{A^* A_1}{\sqrt{A_1^2 - A^{*2}}} = \boxed{6.26 \ m^2}$$

IV (d)

From psychrometric Chart (data book) ~ 14.6 g/kg for 30°C & 55% RH

The air saturates at 20°C

Moisture Content at 100% RH & 15°C dB ~ 10.6 g/kg

Moisture that has condensed out ~ 4 g/kg.