1 a)   Particle size distribution curve is a plot with the particle sizes on the abscissa and percentage finer (or % passing) on the ordinate. The percentage passing is obtained for larger sized particles like Gravel, Sands and by taking a known weight of soil and sieving them through standard sieves. The weight of the particles left on each sieve are weighed carefully obtained to calculate the percentage pass of particles passing each sized sieve. For finer particles such as fine silts and clays, a sedimentation analysis is conducted and rates of sediments are determined using Stoke's law.   [10%]

b) It is dangerous to assume soil behaviour solely based on PSD curves. For example fine particles of silt are very different to rock flour that is generated by grinding action during weathering, although both have same particle sizes. The water absorbency can be different between the particles and this leads to different soil behaviour.

Index tests such as Liquid limit, Plastic limit & shrinkage limits need to be conducted to establish the soil behaviour in addition to PSD's. The Plasticity index (PI) = LL - PL is an important parameter that is linked to water absorbency and the strength of soil.   [20%]

c) First establish the capillary rise:

$$h_c = \frac{4T}{\gamma_w D}$$ where 'D' is representative pore size.

In soils D_{10} size represents the average pore size while D_{20} is the average particle size. $D_{10} = 0.005$ mm

Capillary rise $h_c = \dfrac{4 \times 7 \times 10^{-5}}{9.81 \times 0.005 \times 10^{-3}} = 5.708$ m

This is greater than 5m ie $h_c > 5m$, therefore all the soil above the water table will remain saturated but in 'suction'.
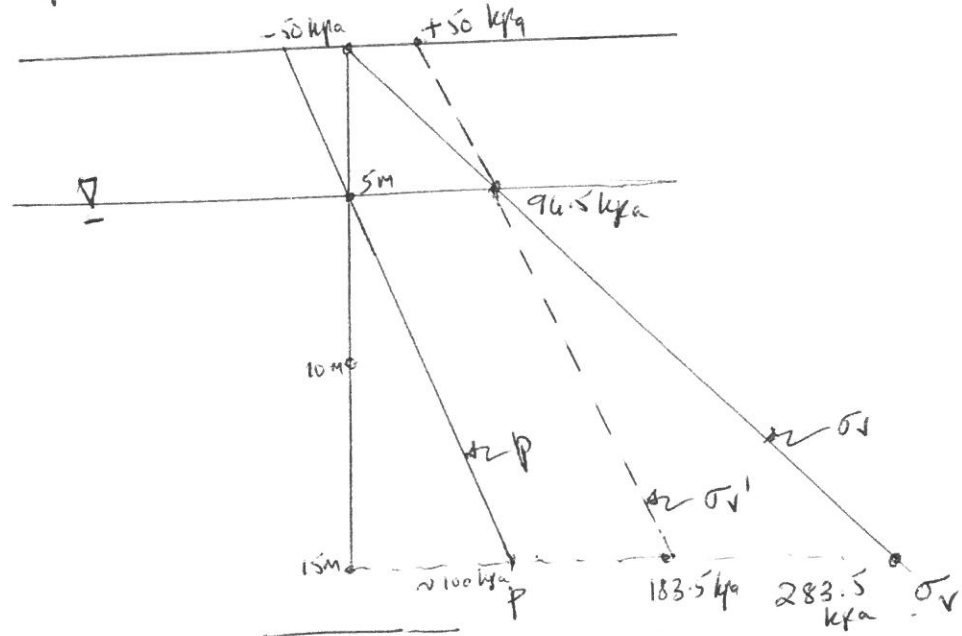
c) Cont

$$\gamma_d = 14.6 \ kN/m^3 \qquad e = 0.78$$

$$\gamma_d = \left(\frac{G_s + e \, S_r}{1+e}\right) \gamma_w = \frac{G_s \, \gamma_w}{1+e}$$

$$\therefore \ G_s = \frac{1.78 \times 14.6}{9.81} = 2.649 \approx 2.65$$

$$\gamma_{sat} = \frac{(2.65 + 0.78 \times 1)}{1 + 0.78} \times \gamma_w = 18.9 \ kN/m^3.$$

$$\sigma_v = 18.9 \, z$$
$$u = \gamma_w \, z \quad for \ z > 0 \quad \Rightarrow \sigma_v' = \sigma_v - u.$$

-50 kPa   +50 kPa
5m   94.5 kPa
10m
12 P
12 σv'
12 σv
15m ~100 kPa P   183.5 kPa   283.5 kPa σv

[20%]

1 d) i) see next page

ii) $Q = K \times \Delta \bar{h} \times N_f / N_n = 6.4 \times 10^{-7} \times 10 \times \frac{4}{10} = 2.56 \times 10^{-6} \ m^2/s$

$$L = 200 \ m. \qquad Q = 2.56 \times 10^{-6} \times 200 \quad m^3/s.$$
$$= 5.12 \times 10^{-4} \ m^3/s$$
$$\text{or } \underline{44236.8} \ litres \ per \ day$$

iii) Point A = $\bar{h}_A = 9.1 \ m \quad y_A = 4.15 \ m \Rightarrow h_A = 9.1 - 4.15 = 4.95 \ m$
$$p_A = \underline{48.56 \ kPa}$$

Point B = $\bar{h}_B = 5.1 \ m \quad y_B = 1.96 \ m \Rightarrow h_B = 3.14 \ m \Rightarrow p_B = 30.8 \ kPa$

Point C = $\bar{h}_C = 5.4 \ m \quad y_C = 6 \ m \Rightarrow h_C = -0.6 \Rightarrow p_C = -6 \ kPa$
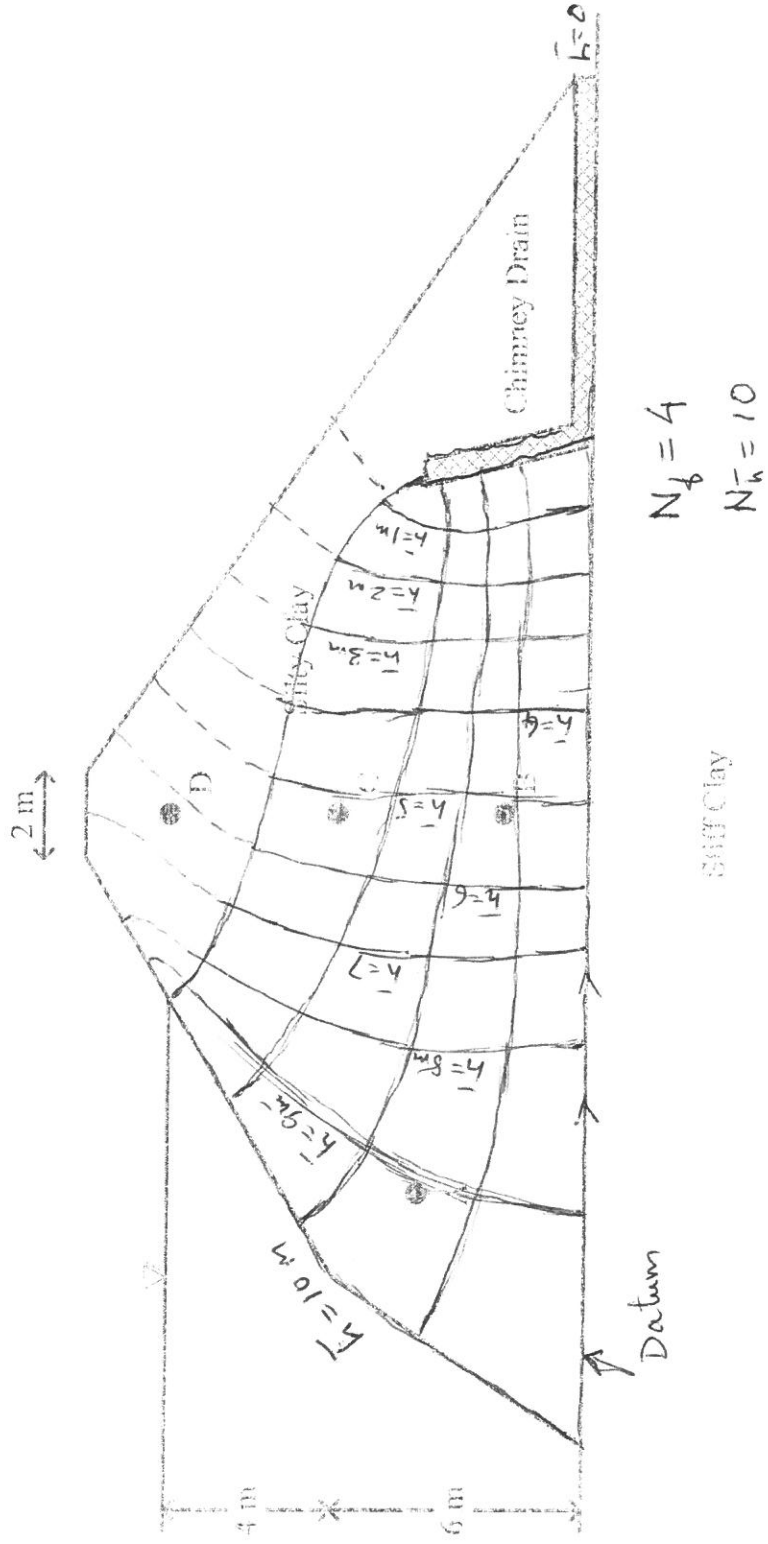
Point D = $\bar{h}_D = 6.1 \ kPa \quad y_C = 10 \ m \Rightarrow h_D = 6.1 - 10 = -3.9 \ m$
$$\therefore p_D = -38.259 \ m \approx \underline{-40 \ kPa}$$

[15%]

—2—

Candidate No:

2 m

Chimney Drain

$N_f = 4$

$N_h = 10$

Silty Clay

$h=4m$

$h=3m$

$h=2m$

$h=1m$

$h=0$

Stiff Clay

Datum

4 m

6 m

$-3-$

Extra copy of Fig 1

3D8 Building Physics & Environmental Geotechnics

Q2) Cribs

The contaminant transport mechanisms in ground water flow are: Advection, Diffusion, Mechanical dispersion and Sorption. Advection is the process by which the contaminant simply flows along with the ground water. Diffusion can occur with no ground water flow and is due to differences in concentrations and follows Fick's law. Mechanical Dispersion is transport due to mixing up of the contaminant as the ground water flows through soil and depends on the pore size, pore shape and the boundary layer formation at the interface between soil grains and the pore fluid. Sorption is when the contaminant either absorbs or adsorbs onto the soil grains.
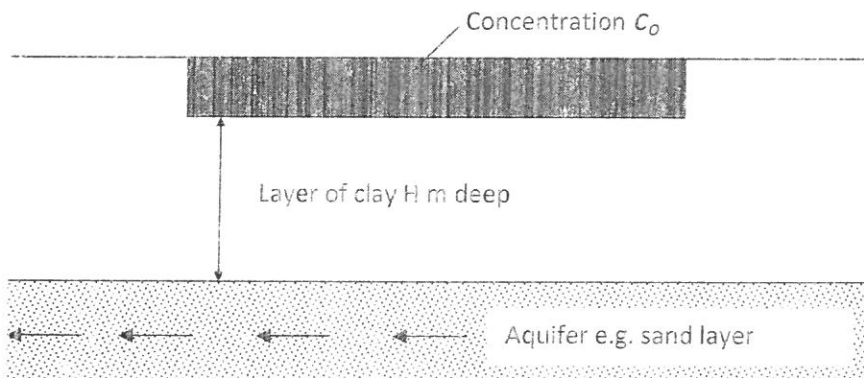
Advection will result in a step function or piston like flow causing a step change in concentration as the contaminant arrives at a given point (see Figure below). Diffusion will be more gradual change of concentration with time. Mechanical dispersion causes much more spread in concentration over time as shown below. Sorption has the effect of delaying/retarding the whole contaminant transport.
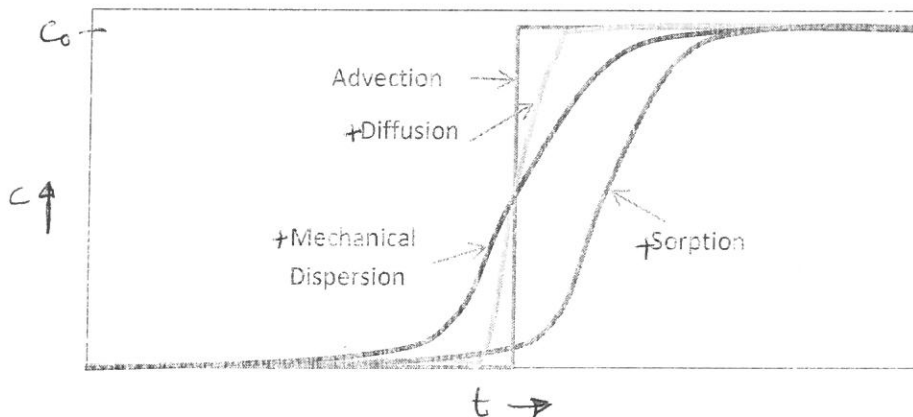
[20%]

2b) The advection dispersion equation is given as: $\dfrac{\partial c}{\partial t} = D_l \dfrac{\partial^2 c}{\partial z^2} - v_f \dfrac{\partial c}{\partial z}$

For steady state condition LHS = 0 i.e. $\dfrac{\partial c}{\partial t} = 0$.

Concentration C_0

Layer of clay H m deep

Aquifer e.g. sand layer

The advection-dispersion equation becomes:

-4-

$$D_l \frac{\partial^2 c}{\partial z^2} = v_f \frac{\partial c}{\partial z}$$

and integrating once with respect z gives:

$$\frac{D_l}{v_f} \frac{\partial c}{\partial z} = c - P$$

where P is a constant, while integrating again gives:

$$\ln(c - P) = \frac{v_f z}{D_l} + \ln Q$$

where Q is also a constant. Rearranging gives:

$$c = P + Q \exp\left(\frac{v_f z}{D_l}\right)$$

which must satisfy the boundary conditions:

for $z = 0$ $\qquad\qquad c = P + Q = c_o$

for $z = L$ $\qquad\qquad c = P + Q \exp\left(\frac{v_f L}{D_l}\right) = 0$

the solution of which gives:

$$P = \frac{-c_o \exp\left(\frac{v_f L}{D_l}\right)}{1 - \exp\left(\frac{v_f L}{D_l}\right)} \quad \text{and} \quad Q = \frac{c_o}{1 - \exp\left(\frac{v_f L}{D_l}\right)}$$

and therefore the steady state distribution is:

$$\frac{c}{c_o} = \frac{\exp\left(\frac{v_f z}{D_l}\right) - \exp\left(\frac{v_f L}{D_l}\right)}{1 - \exp\left(\frac{v_f L}{D_l}\right)}$$

If the mean linear water velocity is very small, then the exponentials can be approximated by power series:

$$\frac{c}{c_o} = \frac{\left(1 + \frac{v_f z}{D_l}\right) - \left(1 + \frac{v_f L}{D_l}\right)}{1 - \left(1 + \frac{v_f L}{D_l}\right)} = \frac{L - z}{L}$$

i.e. if advection is negligible and diffusion therefore dominates then the steady state distribution is linear from source to sink. This result could have also been reached by integrating the diffusion equation directly.

[25%]

Q3) c) The relevant equation for GSHP is;

$$\frac{E}{2\pi B} \, ln \left(\frac{r_2}{r_1}\right) = -\lambda \, (T_2 - T_1)$$

Substituting values and allowing for heat abstraction;

$$\frac{2.5 \times 10^3}{2\pi \, 50} \, ln \left(\frac{2.4}{0.4}\right) = 2.6 \, (T_2 - 1)$$

Therefore at distance of 2m from the bore well wall, the temperature will be = 6.484 °C.

The radius of influence can be estimated as when the temperature T_2 approaches the ambient ground temperature of 10 °C.

$$\frac{2.5 \times 10^3}{2\pi \, 50} \, ln \left(\frac{R}{0.4}\right) = 2.6 \, (10 - 1)$$

Therefore the radius of influence of the GSHP is R= 7.57 m.   [25%]

3) d) For maximum energy output we want to pack the GSHP's as closely as possible, but without the Radius of influence overlapping. We can fit 26 wells in the first row as shown below, which is the plan view of site 401.5m × 200m. Similarly we can fit 15 wells in each column.
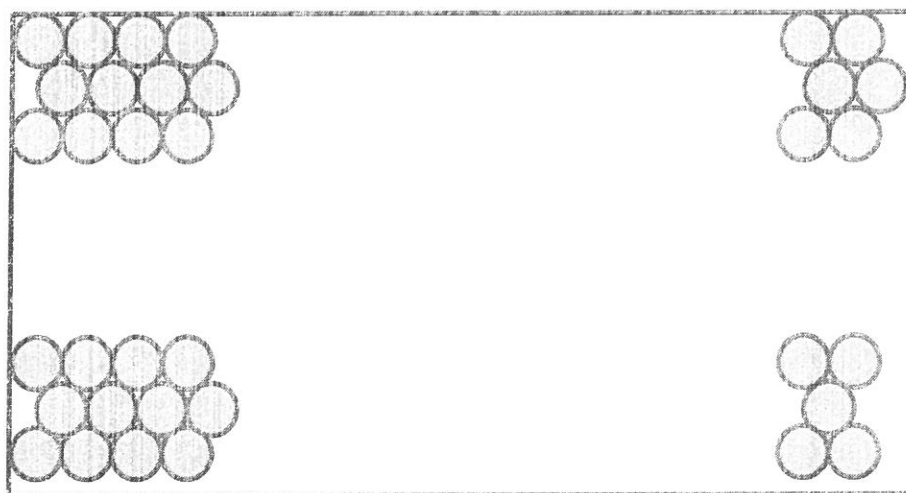
So total number of wells we can have on this site = 26 × 13 = 390 wells.

Total energy output = 390 × 2.5 kW = 975 kW (i.e. approximately 1 MW).

[30%]

III

III. (b) $\hat{T}_c(x,t) = \hat{T}(x)\, e^{2\pi i t/t_P}$ —— ①

$\hat{T}(x) = T_1\, e^{-(1+i)x/d_P}$ —— ②

Insert ② in ①

(i) $\hat{T}_c(x,t) = T_1\, e^{-x/d_P} \cdot e^{i\left(2\pi t/t_P - x/d_P\right)}$

Ⓐ —— $T(x,t) = T_i\, e^{-x/d_P} \sin\left(\dfrac{2\pi t}{t_P} - \dfrac{x}{d_P}\right)$

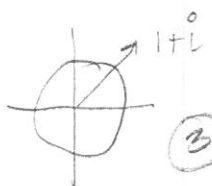(ii) $\hat{Q}(x) = \lambda A\, \dfrac{\partial \hat{T}}{\partial x}$ (heat conduction)

$\hat{Q}(x,t) = \lambda A\, \dfrac{\partial \hat{T}(x,t)}{\partial x}$

$= \lambda A\, \dfrac{\partial\left[T_1\, e^{-(1+i)x/d_P}\, e^{2\pi i t/t_P}\right]}{\partial x}$

$\Rightarrow \quad \dfrac{\lambda A}{d_P} T_1 \sqrt{2}\, e^{-x/d_P}\, e^{i\left[\pi/4 - x/d_P + 2\pi t/t_P\right]}$

$(1+i) = \sqrt{2}\, e^{i\pi/4}$ Ⓑ $\Rightarrow \dfrac{\lambda A}{d_P} T_1 \sqrt{2}\, e^{-x/d_P} \sin\left(\dfrac{2\pi t}{t_P} - \dfrac{x}{d_P} + \dfrac{\pi}{4}\right)$

(iii)

③ From Ⓐ $\quad$ Max T $\sim \dfrac{2\pi t}{t_P} - \dfrac{x}{d_P} = \dfrac{\pi}{2}$

④ From Ⓑ $\quad$ Max Q, $\sim \dfrac{2\pi t}{t_P} - \dfrac{x}{d_P} + \dfrac{\pi}{4} = \dfrac{\pi}{2}$

III (b) (iii) (5) - (3)

$$\frac{2\pi}{t_P}(t_T - t_Q) - \frac{\pi}{4} = 0$$

$$\boxed{t_t - t_Q = \frac{t_P}{8}}$$

III (a)   diffusivity (thermal) $= \frac{\lambda}{\rho c}$

$$a = \frac{\lambda}{\rho c} = \frac{3 \cdot 2}{2 \times 10^6} = 1.6 \times 10^{-6} \ m^2/s$$

* defines $\wedge$ rate at which heat flows thru a material

* measures the material's ability to conduct heat relative to storing it.

* characterizes 'unsteady' heat conduction

IV (c)   For 27°C dbT & 55 RH,
            Dewpoint is at 17°C

From Psychrometic chart ⟸ humidity at 17°C ~ 0.012 kg/kg

            air is cooled to 15°C

from psychrometric chart ⟸ humidity at 15°C ~ 0.0106 kg/kg

            Moisture condensed out : 0.012 - 0.0106
                                    = 0.0014 kg/kg

IV (a) Geometry of the Room

$l = 6m$

$w = 4m$ (ext. wall)

$h = 3m$

Window $= 2.5 \times 2.2\ m$

Properties: $n = 0.5$ ACH

$U_{ext\,wall} = 0.6\ W/m^2K$

$U_{int\,wall} = 1.6\ W/m^2K$

$U_{window} = 2.8\ W/m^2K$

Environment:

$T_0 = -2°C \qquad T_{ad} = 19°C$

(i) Indoor Room Temperature

(a) heat Gains from internal walls

$U_{int} \times Area_{int} \times (T_{ad} - T_i)$

$\Rightarrow 1.6 \times [2 \times (6 \times 4) + 2 \times (6 \times 3) + (4 \times 3)] \times ...$

$...\times (19 - T_i)$

$\Rightarrow 153.6 (19 - T_i) \quad — ①$

(b) heat losses:

$[U_{win} \times Area_{win} + U_{ext\,wall} \times Area_{ext\,wall}] \times (T_i + 2) + ...$

$... + [\rho_{air}\, C_{air} \times \dfrac{nV}{3600}] \times (T_i + 2)$

ρair & Cair from data book

$\Rightarrow [15.4 + 3.9^2 + 12] \times (T_i + 2) \quad — ②$

Heat Balance: $① = ②$

$153.6 (19 - T_i) = 31.3 (T_i + 2)$

$\Rightarrow 4.9 (19 - T_i) = T_i + 2$

$\Rightarrow \boxed{15.45°C}$

(ii) Heating Demand to maintain 23°C

$$\begin{cases} UA_{int} = 153.6 \ W/K \ (\text{from (a)}) \\ UA_{ext} = 31.3 \ W/K \end{cases}$$

heating demand $\sim 153.6 \ (23-19) + 31.3 \ (23+2)$

$$\sim \boxed{1397 \ Watts}$$

(iii) Area of ceiling $\sim$ area of radiator

$$\sim 6 \times 4 = 24 \ m^2$$

Area of all other surfaces $= 84 \ m^2$

$\epsilon_1 = 0.9$ (for all surfaces)

$T_{surface} = 23°C \quad (T_2)$

$T_{ceiling} = 34°C \quad (T_1)$

$Q_{12} = h_r A_1 \cdot (T_1 - T_2)$

$$h_r = \frac{4 \ \sigma \ T_{12}^3}{\dfrac{1-\epsilon_1}{\epsilon_1} + \dfrac{1}{F_{12}} + \dfrac{1-\epsilon_2}{\epsilon_2} \dfrac{A_1}{A_2}}$$

$\sigma = 5.67 \times 10^{-8} \ W \, m^2 \, K^+ \ (\text{data book})$

$T_{12} = \dfrac{T_1 + T_2}{2} \ K; \qquad F_{12} = 1 \quad (\text{view factor})$

$T_{12} = 301.65 \ K$

$h_r = 5.44 \ W/m^2 K$

$Q_{12} = 5.44 \times 24 \times (34 - 23)$

$$= \boxed{1438 \ W}$$

<u>Yes sufficient for (ii)</u>

IV

(b) $\lambda_{wall} = 0.60 \ W/mK$ $\qquad d_{wall} = 0.250 \ m$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad d_{clay} = 0.2 \ m$

$\lambda_{clay} = 0.09 \ W/mK$ $\qquad R_{so} = 0.04$

$\delta_{clay} = 25 \times 10^{-6}$ $\qquad\qquad R_{si} = 0.13$

$U\text{-Value } grd = 0.317 \ W/m^2 K$

$R_{ceiling} = 2.5 \ m^2 K/W$

$n = 0.5 \ ACH$

$T_{ext} = 8.5°C \qquad RH_{ext} = 80\%$

$T_{int} = 22°C \qquad RH_{int} = 48\%$

$Area = 10 \times 16 \ m^2$

$height = 1 m$

$T_i \ (22°C)$

T_{ext}

$K_{vent} \qquad K_{ceiling}$

$K_{wall} \qquad T_{basement}$

T_{ext}

K_{ground}

T_e
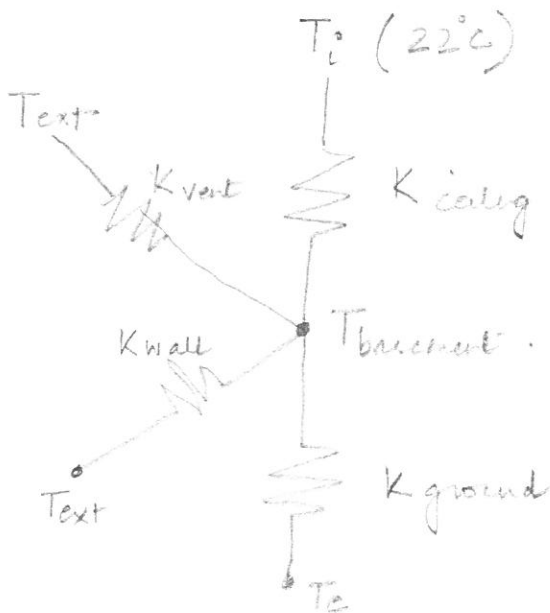
$Area \ wall = 2(10 + 16) \times 1$

$\qquad\qquad\qquad = 52 \ m^2$

$Area \ floor = 10 \times 16 = 160 \ m^2$

$Volume = 10 \times 16 \times 1 = 160 \ m^3$

$K_{ceiling} = \dfrac{1}{R_{ceiling}} \times Area \ ceiling \Rightarrow 64 \ W/K$

$K_{out} \sim K_{ground} + K_{vent} + K_{wall} \Rightarrow$

$\qquad U_{ground} \times Area \ ground + \dfrac{nV}{3600} \ \rho_{air} \ c_{air} + U_{wall} \ A_{wall}$

$=$

①

$$R_{wall} = R_{so} + R_{si} + \frac{d_{wall}}{\lambda_{wall}}$$

$$= 0.13 + 0.04 + \frac{0.25}{0.60} = 0.59 \, m^2K/W$$

$$U_{wall} = \frac{1}{R_{wall}} = 1.7 \, W/m^2K$$

Inserting in ①

$$0.317 \times 160 + \frac{0.5 \times 160}{3600} \times 1.2 \times 10^3 + 1.7 \times 160$$

$$\Rightarrow 166.79 \, W/K$$
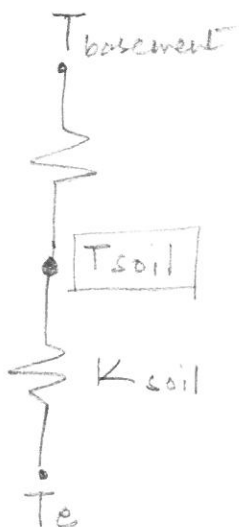
$$T_{basement} = \frac{K_{out} \, T_{ext} + K_{ceiling} \, T_{int}}{K_{out} + K_{ceiling}}$$

$$\boxed{\Rightarrow 12.26 \, °C}$$

Moisture Calculation

Temperature at Soil surface

$T_{basement}$

T_{soil}

K_{soil}

T_e

$$R_{clay} = R_{si} + \frac{d_{clay}}{\lambda_{clay}}$$

$$= 0.13 + \frac{0.2}{0.09}$$
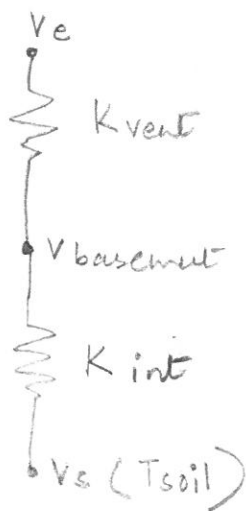
$$= 2.35 \, m^2/WK$$

$$K_{clay} = \frac{1}{R_{clay}} = 0.426$$

$$K_{soil} = \left[\frac{1}{U_{soil}} - \frac{0.20}{0.09} - 0.13 \right]^{-1}$$

$$= 1.25 \, W/K$$

$$T_{soil} = \frac{K_{clay}\, T_{basement} + K_{soil}\, T_e}{K_{clay} + K_{soil}}$$

$$= 9.5\,°C$$

$$V_s(T_{soil}) = 9.11\ g/m^3$$

V_e

K_{vent}

$V_{basement}$

K_{int}

$V_s(T_{soil})$

$$K_{in} = \frac{\varepsilon_{clay}}{d_{clay}} \cdot Area =$$

$$K_{vent} = \frac{n}{3600} \times Volume$$

$$V_{basement} = \frac{K_{in}\, V_s(T_{soil}) + K_{vent}\, V_s(T_e)\, RH}{K_{in} + K_{vent}}$$

$$= 7.91\ g/m^3$$

Relative Vapour Content

$$RH = \frac{7.91}{V_s(T_{basement})} \times 100$$

$$= \frac{7.91}{10.87} \times 100 = 73\%$$

i.e. no problem w/ moisture