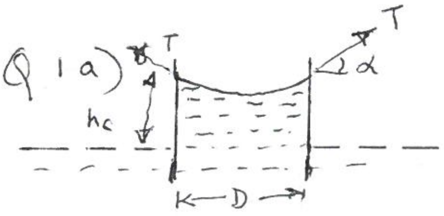
3D8 - Building Physics & Environmental Geotechnics

Cribs - 2019

Q 1 a)



Weight of the fluid $= \frac{\pi D^2}{4} \times h_c \times \gamma_w \rightarrow ①$

Surface tension T.

Force due to surface tension $= \pi D T \sin\alpha \rightarrow ②$

Equating ① & ② $\quad \frac{\pi D^2}{4} h_c \gamma_w = \pi D T \sin\alpha$.

$\therefore$ Capillary rise $= h_c = \frac{4 T \sin\alpha}{\gamma_w D}$

The average pore size in soil is characterised by D_{10}

$\therefore h_c = \frac{4 T \sin\alpha}{\gamma_w D_{10}}$ [15 %]

b) From Fig 1, the characteristic pore size $D_{10} = 0.01 m$.

Surface tension $T = 7.3 \times 10^{-5}$ kN/m $\quad \gamma_w = 9.81$ kN/m^3.

$\therefore h_c = \frac{4 \times 7.3 \times 10^{-5} \sin 65}{9.81 \times 0.01} = 2.697 \times 10^{-3}$ m

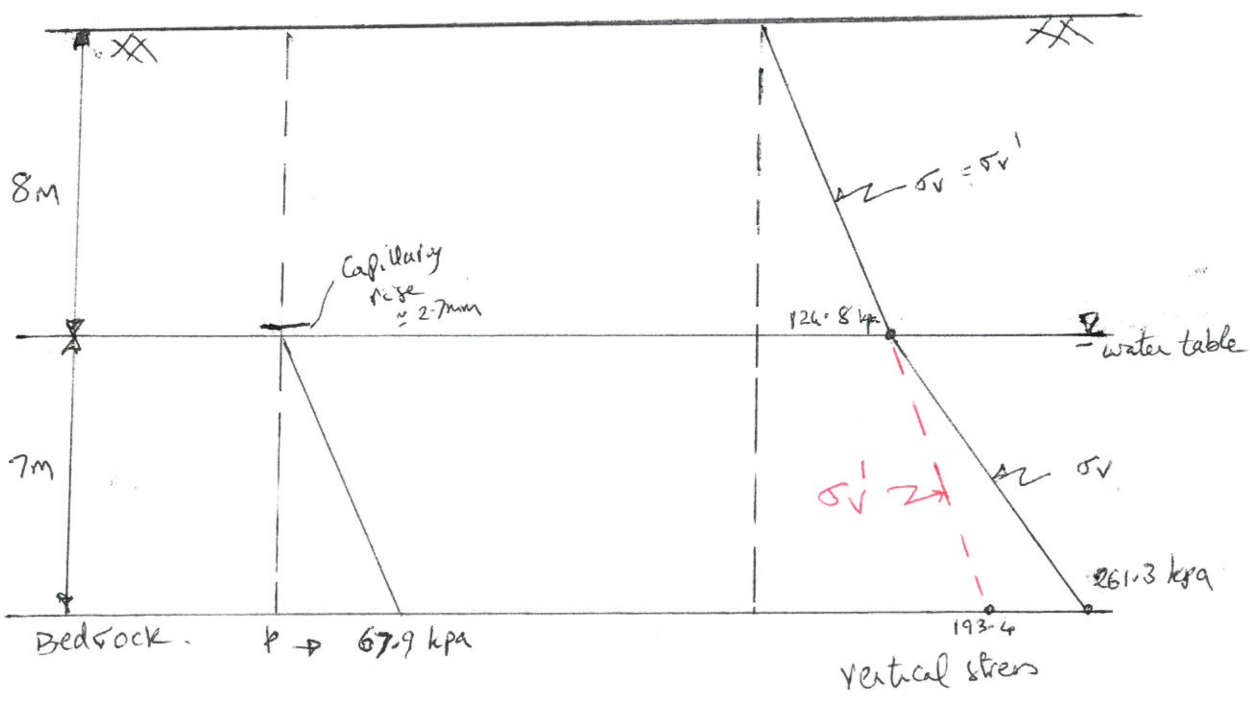
or $\simeq 2.7$ mm !

Quite small - suggests sandy soil.

Porosity $n = 0.4 = \frac{e}{1+e} \Rightarrow e = 0.67$.

$\gamma_d = \frac{G_s \gamma_w}{1+e} = \frac{2.65 \times 9.81}{1+0.67} = 15.59 \simeq 15.6$ kN/m^3.

$\gamma_{sat} = \frac{(G+e)\gamma_w}{1+e} = 19.5$ kN/m^3 $\quad \gamma'_{sat} = 9.7$ kN/m^3.

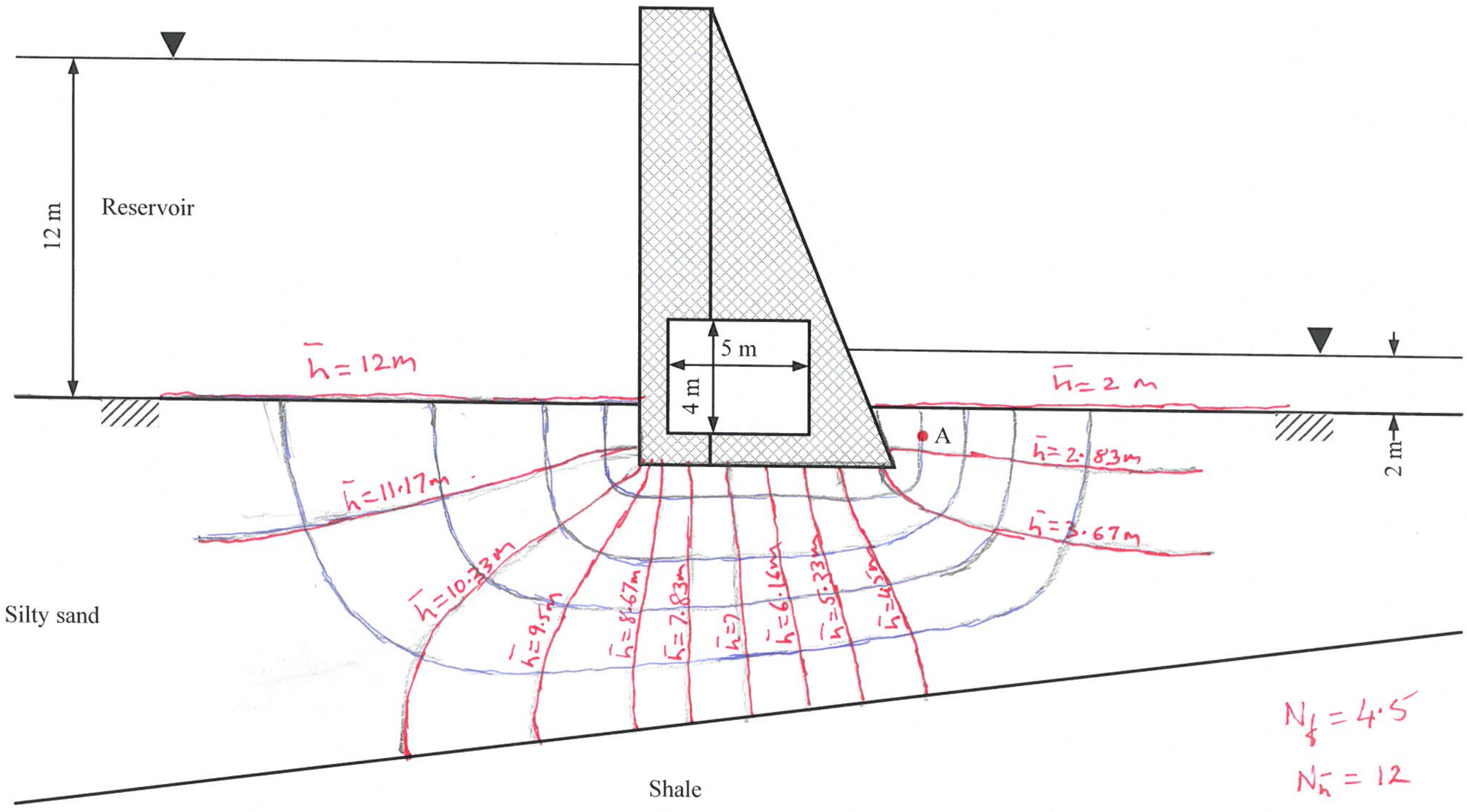


Candidate No:

ENGINEERING TRIPOS PART IIA xxxDAY xx APRIL 2019 1400 TO 1530

Q1 c) i)

DRAWN TO SCALE



Extra copy of Fig. 1

1c) ii) $q = K \Delta \bar{h} \times \frac{N_f}{N_{\bar{h}}}$ $\Delta \bar{h} = 12 - 2 = 10\,m$.

$= 2.3 \times 10^{-4} \times 10 \times \frac{4.5}{12} = 8.625 \times 10^{-4}\ m^3/s$.

For 100m long dam, leakage will be

$Q = 8.625 \times 10^{-4} \times 100 \times 1000 \times 24 \times 3600$

$= 7.452 \times 10^{6}$ litres/day

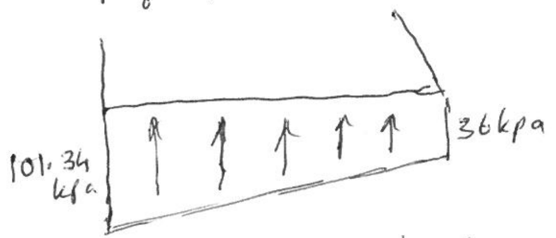
iii) FoS against sliding.

Weight of the dam $= 16 \times \left(\frac{2.5+8}{2}\right) \times 24 = 2016\ kN/m$.

less inspection gallery $= 5 \times 4 \times 24 = 480\ kN/m$.

$\therefore W_{Dam} = 1536\ kN/m$

Uplift force on the base of the dam, from flownet.



$U = \int p\,ds = 8m \times \left(\frac{101.34 + 36}{2}\right)\ kN/m$

$= 549.36\ kN/m$.

$\therefore$ Net weight of the Dam $W' = 1536 - 549.36$

$\simeq 986.64\ kN/m$.

Hydrostatic force of LHS of dam $H_1 = \frac{1}{2} \times 12 \times 117.72 = 706.32\ kN/m$

Hydrostatic force of RHS of dam $H_2 = \frac{1}{2} \times 2 \times 19.62 = 19.62\ kN/m$.

$H_1 - H_2 = 686.7\ kN/m$.

FOS against sliding $= \frac{W' \tan\phi}{H_1 - H_2} = \frac{986.64 \times \tan 35}{686.7} = 1.006$

This is quite small FoS against sliding. However the soil on the RHS side will offer some passive resistance. [15%]

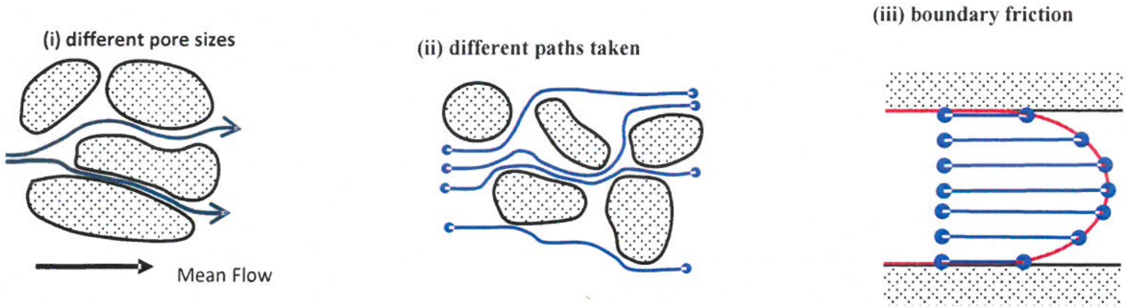
iv) Exit hydraulic gradient at 'A'.

Pore pressure at A is $\simeq 2.415 \times 9.81 = 23.7$ kPa.

upward hydraulic gradient $i = \frac{2.83 - 2}{1.2m} = 0.69 < (0.8 \sim 1.0)$

so OK. [10%]

2) a) Contaminant transport mechanism of mechanical dispersion involves mixing and flow of the contaminant as the pore fluid passes in the spaces between the grains.



Sorption on the other hand, is removal of the contaminant from the pore fluid as it either gets absorbed or adsorbed onto the soil particles. Sorption can happen as different chemical ions have different selectivity for the clay particles and can therefore displace already adsorbed ions. [10%]

b) For homogenous soils with steady state flow *n*, D_l and v_f will be constant and the one-dimensional transport equation reduces to the classic form of the advection-dispersion equation:

$$\frac{\partial c}{\partial t} = D_l \frac{\partial^2 c}{\partial x^2} - v_f \frac{\partial c}{\partial x} \pm \frac{\Phi}{n}$$

which for a particular case of linear, equilibrium adsorption becomes:

$$\frac{\partial c}{\partial t} = D_l \frac{\partial^2 c}{\partial x^2} - v_f \frac{\partial c}{\partial x} \pm \frac{\rho_b}{n} K_d \frac{\partial c}{\partial t}$$

$$\underbrace{\left[1 + \frac{\rho_b}{n} K_d\right]}_{R_d > 1} \frac{\partial c}{\partial t} = D_l \frac{\partial^2 c}{\partial x^2} - v_f \frac{\partial c}{\partial x}$$

where R_d is the retardation factor. Rearranging gives:

$$\frac{\partial c}{\partial t} = \frac{D_l}{R_d} \frac{\partial^2 c}{\partial x^2} - \frac{v_f}{R_d} \frac{\partial c}{\partial x}$$

or alternatively:

$$\frac{\partial c}{\partial \left(\frac{t}{R_d} \right)} = D_l \frac{\partial^2 c}{\partial x^2} - v_f \frac{\partial c}{\partial x}$$

so that the effect of the adsorption is to reduce the apparent dispersion coefficient and the apparent advective velocity of the contaminant. The ratio of the mean linear velocity of the groundwater, v_f, to the velocity of the $c/c_o = 0.5$ point on the concentration profile of the retarded material v_c, is therefore:

$$R_d = \frac{v_f}{v_c}$$

[20%]

2c) Fick's law states that the contaminant will spread proportional to the concentration gradient during a diffusion process.

$\therefore$ Mass flux $J_{Fick} \propto \frac{\partial c}{\partial x}$ or $J_{Fick} = -D_d^* \frac{\partial c}{\partial x}$ for a 1-d case.

Fourier's law for heat transfer states that the heat flux is proportional to the temperature gradient. For 1-D heat transfer.

$$H \propto -A \frac{\partial T}{\partial x} \quad \text{or} \quad H = -\lambda A \frac{\partial T}{\partial x}$$

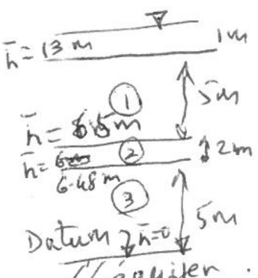
Both of these laws were based on experimental observations. The involve the gradient of the field parametre, such as concentration ~~for~~ or temperature, and a material property such as co-efficient of diffusion or thermal conductivity.

[10%]

d) Equivalent vertical permeability, for this case

i)
$$K_v = \frac{L_1 + L_2 + L_3}{\frac{L_1}{K_1} + \frac{L_2}{K_2} + \frac{L_3}{K_3}} \quad ; \quad L_3 = L_1, \; k_3 = K_1$$

$$K_v = \frac{2L_1 + L_2}{2\frac{L_1}{K_1} + \frac{L_2}{K_2}} = \frac{2 \times 5 + 2}{2 \times \frac{5}{3.2 \times 10^{-7}} + \frac{2}{6.5 \times 10^{-5}}} = 3.8362 \times 10^{-7} \text{ m/s}$$



$$i_{average} = \frac{\Delta \bar{h}}{\Delta S} = \frac{13 - 0}{12} = 1.083.$$

$$\therefore v_f = Ki = 3.8362 \times 10^{-7} \times 1.083$$
$$\simeq 4.16 \times 10^{-7} \text{ m/s}.$$

[10%]

ii) $\Delta \bar{h}_1 = v \frac{L_1}{K_1} = 4.16 \times 10^{-7} \times \frac{5}{3.2 \times 10^{-7}} = 6.5$ m.

$\therefore i_1 = \frac{6.5}{5} = 1.3$ $\qquad v_1 = 3.2 \times 10^{-7} \times 1.3 = \underline{4.16 \times 10^{-7}}$ m/s ✓

$\Delta \bar{h}_2 = v \frac{L_2}{K_2} = 4.16 \times 10^{-7} \times \frac{2}{6.5 \times 10^{-5}} = 0.0128$

$i_2 = \frac{0.0128}{2} = 6.4 \times 10^{-3}$ $\qquad v_2 = 6.4 \times 10^{-3} \times 6.5 \times 10^{-5}$
$= \underline{4.16 \times 10^{-7}}$ ✓ m/s

$\Delta \bar{h}_3 = v \frac{L_1}{K_1} \approx 6.5$ m.

$i_3 = \frac{6.5}{5} \simeq 1.3$ $\qquad v_3 = 1.3 \times 3.2 \times 10^{-7} = \underline{4.16 \times 10^{-7}}$ m/s

Flow velocity is the same in all strata

[~~10~~%]

2 d) iii) $D = \alpha_L v_f$; $\alpha_L = 0.1$

Assuming a void ratio on average for the 3 strata of 0.5

$n = \frac{e}{1+e} = 0.33$; $v_f = \frac{v}{n} = \frac{4.16 \times 10^{-8}}{0.33} \simeq 1.25 \times 10^{-7}$ m/s

$D = \alpha_L v_f$

$= 0.1 \times 1.25 \times 10^{-7} = \underline{\underline{1.25 \times 10^{-8}}}$ m²/s. [10%]

d iv) Breakthrough time for advective flow only :-

$v_f = 1.25 \times 10^{-7}$ m/s.

Layer thickness = 12 m.

Break through time $= \frac{12}{1.25 \times 10^{-7}} = 96 \times 10^{6}$ sec

$\simeq$ $\underline{\underline{3 \text{ years}}}$.

Advection + Dispersion :-

$$\frac{c(z,t)}{c_0} = \frac{1}{2}\left[\text{erfc}\left[\frac{z}{\sqrt{4 D_L t}}\right]\right]$$

For traces to appear in the aquifer $c/c_0 \simeq 0.0001$

$\therefore$ erfc$(\beta) \simeq 0$ $\Rightarrow$ $\beta = 3.0$ (From data book)

$3.0 = \frac{z}{\sqrt{4 D_L t}}$; $z = 12$ m. $t = ?$

$\sqrt{4 D_L t} = \frac{12}{3} = 4.$

$D_L t = 4$

$t = \frac{4}{D_L} = \frac{4}{1.25 \times 10^{-8}} = 320 \times 10^{6}$ sec

$\simeq$ $\underline{\underline{10.2 \text{ years}}}$.

Comment: In this case both advection & dispersion are important as both times are comparable. [20%]

III (a) Thermal conductivity is the rate at which heat passes through a specified material [λ expressed in W/mK]

Thermal diffusivity controls the time rate of temperature change as heat passes through a material [in m^2/s]

$\Rightarrow$ main difference ~ diffusivity measures the material's ability to conduct heat relative to storing it. It thus characterizes 'unsteady heat conduction'

III (b) Given:

(i)

$$\hat{T}_c(x,t) = T_1 \cdot e^{\left[\frac{-(1+i)x}{d_p}\right]} e^{\left[2\pi i t/t_p\right]}$$

$$\Rightarrow \hat{T}(x,t) = T_1 e^{-x/d_p} e^{i\left(2\pi t/t_p - x/d_p\right)}$$

$$\Rightarrow \text{(A)} \quad T(x,t) = T_1 e^{-x/d_p} \sin\left(\frac{2\pi t}{t_p} - \frac{x}{d_p}\right)$$

where $T_1 e^{-x/d_p}$ gives the amplitude (change in magnitude) of the temperature

(ii)

$$\hat{Q}(x,t) = \lambda A \frac{\partial \hat{T}(x,t)}{\partial x} \quad \text{(heat conduction)}$$

$$\Rightarrow \lambda A \frac{\partial\left[T_1 e^{-(1+i)x/d_p} e^{2\pi i t/t_p}\right]}{\partial x}$$

$$\Rightarrow \lambda A\, T_1 \sqrt{2}\, e^{-x/d_p} e^{i\left[\pi/4 - x/d_p + 2\pi t/t_p\right]}$$

$$\left\{ (1+i) = \sqrt{2}\, e^{i\pi/4} \right.$$

$$\text{(B)} \Rightarrow Q(x,t) = \frac{\lambda A}{d_p} T_1 \sqrt{2}\, e^{-x/d_p} \sin\left(\frac{2\pi t}{t_p} - \frac{x}{d_p} + \frac{\pi}{4}\right)$$

(cont.)

~ cont.

III (b) (ii) $\therefore$ The heat flow at boundary $x=0$ is :

$$\Rightarrow \quad Q(0,t) = \frac{\lambda A}{d_p} T_1 \sqrt{2} \sin\left(\frac{2\pi t}{t_p} + \frac{\pi}{4}\right)$$

(iii) From (A)

$$\text{Max } T \sim \frac{2\pi t}{t_p} - \frac{x}{d_p} = \frac{\pi}{2} \quad \text{— (C)}$$

From (B)

$$\text{Max } Q \sim \frac{2\pi t}{t_p} - \frac{x}{d_p} + \frac{\pi}{4} = \frac{\pi}{2} \quad \text{— (D)}$$

(D) − (C) will give

$$\frac{2\pi}{t_p}(t_T - t_Q) - \frac{\pi}{4} = 0$$

$$t_T - t_Q = t_p/8$$

III (c) Annual variation

$$T_{ext}(t) = T_a + T_b \sin\frac{2\pi t}{t_p}$$

$T_a = 8^\circ C$

$T_b = 10^\circ C$

will need to use eqn. (A) from 3(b)(i)

- Outdoor Minimum temp when $\sin\frac{2\pi t}{t_p} = -1$
- Calculate $d_p \sqrt{\frac{\lambda}{\rho c} \cdot \frac{t_p}{\pi}}$

$\lambda = 1.5 \ W/mK$ $\qquad t_p = 365 \times 24 \times 3600$

$\rho = 2.0 \times 10^6 \ J/m^3K$

$$\underline{d_p = 2.74 \, m}$$

cont

cont. III (c)

Using eqn A from 3(b)(i)

$$T_1\, e^{-x/dp} = 10 \cdot e^{-3/2.74} = 3.34\,^\circ C$$

Minimum ext. temp.

$$= T_a + T_b(-1) = 8 + 10(-1) = 2\,^\circ C$$

Ground temperatures:

$$T_a + T_b\, e^{-3/2.74} \cdot \sin\left(-\frac{2\pi}{4} - \frac{3}{2.74}\right)$$

$$= 8 + 3.34 \sin\left(-\frac{\pi}{2} - 1.095\right)$$

$$= \underline{\underline{6.5\,^\circ C}}$$

4

IV (a)

that outside, it will be less dense. Assuming there are no adverse wind conditions, the internal air will thus tend to rise, flowing out of upper openings, and drawing external air through lower openings.

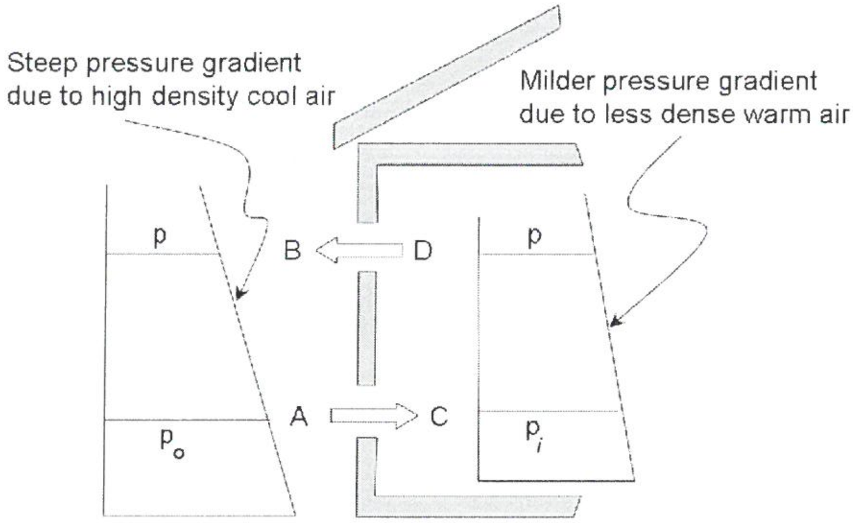


Figure 2: The buoyancy-driven stack effect (after CIBSE Guide A Figure 4.9)

The equations for stack effect can be derived by consideration of Figure 2, which shows a tall room with two openings, one at the top and one at the bottom.

The external temperature is T_o (in Kelvin) and the internal temperature is T_i everywhere through the room.

Assume that that the air is moving so slowly that kinetic energy terms (i.e. dynamic wind pressures) can be neglected everywhere except across the orifices. The pressure $p_o(H)$ outside the upper opening is thus:

$$p_o(H) = p_o - \rho_o g H \tag{12}$$

where ρ_o is the density of the external air at T_o, g is the gravitational constant and H is the distance separating the 2 openings. Similarly, inside the room, the pressure inside the upper window is:

$$p_i(H) = p_i - \rho_i g H \tag{13}$$

where ρ_i is the density of the internal air at T_i. The pressure differences across the bottom and top opening respectively are:

$$\Delta p_b = p_o - p_i$$
$$\Delta p_t = p_i(H) - p_o(H) = (p_i - \rho_i g H) - (p_o - \rho_o g H) = p_i - p_o + (\rho_o - \rho_i) g H$$

Thus, the total pressure difference between the two openings is:

$$\Delta p_s = \Delta p_b + \Delta p_t = (\rho_o - \rho_i) g H \tag{14}$$

Assuming that the air expands linearly with temperature, the density decreases as the reciprocal of temperature, thus

$$\rho \propto \frac{1}{T} \tag{15}$$

and taking conditions at say 273K as a benchmark, then

$$\frac{\rho}{\rho_{273}} = \frac{273}{T} \tag{16}$$

whence

$$\rho = \rho_{273}\frac{273}{T} \tag{17}$$

The buoyancy-driven pressure difference Δp_s driving the flow may thus be written

$$\Delta p_s = \rho_{273} g H \left(\frac{273}{T_o} - \frac{273}{T_i} \right) \tag{18}$$

Equating this to the pressure losses through the two orifices, we obtain

$$\Delta p = \frac{1}{2}\rho \frac{Q^2}{C^2_{orif1} A_1^2} + \frac{1}{2}\rho \frac{Q^2}{C^2_{orif2} A_2^2} \tag{19}$$

which rearranges to give a flow

$$\boxed{Q = \frac{A_1 A_2}{\sqrt{A_1^2 + A_2^2}} \, C_{\text{orif}} \sqrt{\frac{2\Delta p_s}{\rho}}} \tag{20}$$

with Δp_s given by Equation 18.

This equation predicts the flow rate one might obtain from a stack-driven flow through two orifices separated by a height H.

Example: Consider a room at 20°C with an external temperature of 0°C, and let there with be only two windows, one 6 metres above the other. Substitution into Equation 18 predicts a pressure deficit of 5.2 Pa. This is quite small, and is comparable to the dynamic pressure of a 2m/s wind. Or, if both openings are on the windward face, it is comparable to the wind-shear-induced pressure difference over 6m height of a mere 4m/s wind. Even moderate external winds can thus totally dominate stack effect, and stack-driven flows can thus only be expected to prevail on rather calm days.

There are several ways to improve the stack effect. One is to create sizeable "chimneys" that extend above the building height. Since the pressure deficit depends on the height differential, an increased height gives an increased effect.

IV (b) $Q = 17.5\ kW$
$H = 9\ m$
$C_{orif} = 0.6$
$dT = 5\ K$
given

data book
$\rho_{air} = 1.2\ kg/m3$
$C_{air} = 1000\ J/kgK$

(i) Calculate Ventilation Rate

$$Q = \rho\, C_p\, V\, dT$$

$$V = 17500/(1.2 \times 1000 \times 5) = \underline{\underline{2.917}}\ m^3/s$$

(ii) Effective area of each opening A^*

$$Q\,(m^3/s) = A^* \times C_{orif}\sqrt{2gH\left(\frac{273}{T_0} - \frac{273}{T_i}\right)}$$

For 5 K difference assume

$$T_0 = 15^\circ C = 288\ K$$

$$T_i = 20^\circ C = 293\ K$$

$$A^* = \frac{2.917\ m^3/s}{0.6\sqrt{2 \times 9.81 \times 9m \times \left(\frac{273}{288} - \frac{273}{293}\right)}} = 2.87\ m^2$$

~~area~~ per opening (both openings are same)

$$A^* = \frac{A_1 A_2}{\sqrt{A_1^2 + A_2^2}} = \frac{A_1^2}{\sqrt{2A_1^2}} = \frac{A_1}{\sqrt{2}}$$

$$A_1 = A^* \times \sqrt{2} = 4\ m^2$$

(c) $H = 13$ m (distance between upper & lower vents)

Ventilation Rate same for Mezzanine & lower openings.

$\therefore V = 2.917$ m²/s for lower openings [since V is now double]

$$\text{New } A^* = \frac{2.917 \text{ m}^3/\text{s}}{0.6\sqrt{2 \times 9.81 \times 13 \times \left(\frac{273}{288} - \frac{273}{293}\right)}} = 2.4 \text{ m}^2$$

Half the area of new upper vents $= 4 \text{ m}^2$

So $A_2 = 4 \text{ m}^2$

$$A^* = \frac{A_1 A_2}{\sqrt{A_1^2 + A_2^2}} = 2.4 = \frac{A_1 \times 4}{\sqrt{A_1^2 + 4^2}} = \underline{\underline{2.9 \text{ m}^2}}$$

(d) Lower & mezzanine openings/vents should be closed and just the high level openings/vents opened to use mixing ventilation.