EGT2: IIA
ENGINEERING TRIPOS PART IIA
Module 3E10
OPERATIONS MANAGEMENT FOR ENGINEERS

## CRIB

## QUESTION I

(a) $[20 \%]$ Taichi Ohno's 7 key forms of waste are:

- Transportation - unnecessary movement of goods during distribution/handling within the factory.
- Inventory - excess inventory a cause of inefficiency and unnecessary costs.
- Motion - unnecessary material handling, product flow, distances and labour movements.
- Waiting - delays and waiting - direct wastes of time and labour/efficiency costs.
- Overproduction - finished goods inventory or stock that cannot be sold/is not demanded.
- Over processing - putting material through unnecessary steps without optimising flow or layout.
- Defects - which waste time, reduce OEE and increase quality problems.

With the rise of the service industry, these may sometimes be extended to include customer time wasted, office space (heating, water, insurance) waste and wasted potential of workers. Such factors may have some relevance to the manufacturing sector too.
(b)
(i) [10\%] A pull system uses the customer demand signal to trigger replenishment. It is an autonomous system that purely works by replenishing goods that have been consumed by the preceding process. The demand signal is the only trigger for production, not forecasts or centrally planned work orders. The demand signal is conveyed by a "kanban," which could be an empty bin, a card or electronic signal.

In a push system, production is planned centrally using a Master Production Schedule (MPS), which generally comprises of a combination of actual customer orders and forecasts. Based on the MPS and standard routing and lead-time data, work orders are centrally issued that "push" the material forward towards the customer end.

Thus, the key differences are twofold: (1) what triggers the replenishment, and (2) what information the "schedule" is based on.
(ii) [5\%] In a pull system inventory buffers are needed to convey the pull signal from a downstream process to an upstream process. Essentially the process needs some small buffers "to pull from," to convey the replenishment signal upstream. A kanban supermarket is a typical example of inventory in a pull system.
(iii) [5\%] In a push system, production is scheduled according to a Master Production Schedule (MPS). The MPS generally is a combination of actual orders and forecast orders, thus the main function of inventory in a push system is to buffer against any forecast errors.

Also, as production orders are based on fixed lead-times, WIP inventory between processes exists as actual lead-times will vary from those set in the planning system.
(c)
(i) [10\%] Since Holiday Inn is using historical demand observations, a time series method is the appropriate type of forecasting method for the firm.
(ii) [10\%] Since Standard Brands does not have any historical data for their product and would like to base their forecast on new housing starts, they can use a causal model for their prediction.
(d) $[40 \%]$

| The planned order releases for component C used to directly | assemble the end item are: |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Week | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
| Gross requirement |  |  | 80 | 90 | 100 | 110 | 100 | 120 |
| Scheduled receipts |  |  |  |  |  |  |  |  |
| Net requirement |  |  | 80 | 90 | 100 | 110 | 100 | 120 |
| Planned order release | 80 | 90 | 100 | 110 | 100 | 120 |  |  |

The planned order releases for component B are:

| Week | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Gross requirement |  |  | 160 | 180 | 200 | 220 | 200 | 240 |
| Scheduled receipts |  |  |  | 40 |  |  | 60 |  |
| Net requirement |  |  | 160 | 140 | 200 | 220 | 140 | 240 |
| Planned order release | 160 | 140 | 200 | 220 | 140 | 240 |  |  |


| The planned order releases for component C used to assemble component B are: |  |  |  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Week | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
| Gross requirement |  |  | 320 | 280 | 400 | 440 | 280 | 480 |  |  |
| Scheduled receipts |  |  |  |  |  |  |  |  |  |  |
| Net requirement |  |  | 320 | 280 | 400 | 440 | 280 | 480 |  |  |
| Planned order <br> release | 320 | 280 | 400 | 440 | 280 | 480 |  |  |  |  |

Hence, the planned order releases for component C are:

| Week | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Planned order <br> release | 320 | 280 | 480 | 530 | 380 | 590 | 100 | 120 |  |  |

## QUESTION II

(a) [10\%] The basic principles of the North West corner approach for allocating supply to demand are:

- Create a matrix of sources and destinations
- Set initial allocation from NW corner
- Calculate the change in cost of supplying one unit from each currently empty cell while preserving demand/supply rim conditions (opportunity cost)
- Reallocate the maximum possible quantity (subject to rim conditions) to the lowest cost cell, following the path evaluated.
- Solution is optimal if all the opportunity costs are zero or positive

The limitations of the algorithm are:

- Heuristic: feasible but not optimal
- Sensitive to starting point selected
- Complex for multiple item types
- No transport variations
- Fixed supply / demand
(b) $[20 \%]$ First, we should convert the mileage table into cost table. While doing that let the transportation cost for the non-connected route be equal to some large value, say 100 p .

|  | DC1 | DC2 | DC3 | Capacity <br> (in million gallons) |
| :--- | ---: | ---: | ---: | ---: |
| Refinery I | $(1.20 \mathrm{p})$ | $(1.80 \mathrm{p})$ | $(100.00 \mathrm{p})$ | $\mathbf{6}$ |
| Refinery II | $(3.00 \mathrm{p})$ | $(1.00 \mathrm{p})$ | $(0.80 \mathrm{p})$ | $\mathbf{5}$ |
| Refinery III | $(2.00 \mathrm{p})$ | $(2.50 \mathrm{p})$ | $(1.20 \mathrm{p})$ | $\mathbf{8}$ |
| Demand <br> (million gallons) | $\mathbf{4}$ | $\mathbf{8}$ |  | $\mathbf{7}$ |

Then, proceed with the heuristic as normal.

|  | DC1 | DC2 | DC3 | Capacity (in million gallons) |
| :---: | :---: | :---: | :---: | :---: |
| Refinery I | (1.20p) | (1.80p) | (100.00p) | 6 |
|  | 4 | 2 |  |  |
| Refinery II | (3.00p) | (1.00p) | (0.80p) | 5 |
|  |  | 5 |  |  |
| Refinery III | (2.00p) | (2.50p) | (1.20p) | 8 |
|  |  | 1 | 7 |  |
| Demand (million gallons) | 4 | 8 | 7 | 19 |

$$
\begin{aligned}
\text { Total Cost } & =1.2 \times 4+1.8 \times 2+1.0 \times 5+2.5 \times 1+1.2 \times 7 \\
& =4.8+3.6+5+2.5+8.4 \\
& =£ 0.243 \text { million per day }
\end{aligned}
$$

(c) [25\%] Add an extra "supply" point - Refinery IV. Set the transportation cost from refinery IV to DC2 and DC3 to the penalty cost. Indicate DC1 cannot have any shortage, i.e., refinery IV is not connected to DC1, by setting a very high transportation cost from refinery IV to DC1.

|  | DC1 | DC2 | DC3 | Capacity <br> (in million gallons) |
| :--- | ---: | ---: | ---: | ---: |
| Refinery I | $(1.20 \mathrm{p})$ | $(1.80 \mathrm{p})$ | $(100.00 \mathrm{p})$ | $\mathbf{6}$ |
| Refinery II | $(3.00 \mathrm{p})$ | $(1.00 \mathrm{p})$ | $(0.80 \mathrm{p})$ | $\mathbf{5}$ |
| Refinery III | $(2.00 \mathrm{p})$ | $(2.50 \mathrm{p})$ | $(1.20 \mathrm{p})$ | $\mathbf{6}$ |
| Refinery IV | $(100.00 \mathrm{p})$ | $(5.00 \mathrm{p})$ | $(5.00 \mathrm{p})$ | $\mathbf{2}$ |
| Demand <br> (million gallons) | $\mathbf{4}$ | $\mathbf{8}$ | $\mathbf{7}$ | $\mathbf{1 9}$ |

Then, proceed with the heuristic as normal.

|  | DC1 | DC2 | DC3 | Capacity (in million gallons) |
| :---: | :---: | :---: | :---: | :---: |
| Refinery I | (1.20p) | (1.80p) | (100.00p) | 6 |
|  | 4 | 2 |  |  |
| Refinery II | (3.00p) | (1.00p) | (0.80p) | 5 |
|  |  | 5 |  |  |
| Refinery III | (2.00p) | (2.50p) | (1.20p) | 6 |
|  |  | 1 | 5 |  |
| Refinery IV | (100.00p) | (5.00p) | (5.00p) | 2 |
|  |  |  | 2 |  |
| Demand | 4 | 8 | 7 | 19 |

$$
\begin{aligned}
\text { Total Cost } & =1.2 \times 4+1.8 \times 2+1.0 \times 5+2.5 \times 1+1.2 \times 5+5.0 \times 2 \\
& =4.8+3.6+5+2.5+6+10 \\
& =£ 0.319 \text { million per day }
\end{aligned}
$$

(d) [25\%] Add an extra "demand" point - DC4. Set the transportation cost from refinery IV to DC2 and DC3 to the penalty cost, and to DC1 to zero.

|  | DC1 | DC2 | DC3 | DC4 | Capacity <br> (in million <br> gallons) |
| :--- | ---: | ---: | ---: | ---: | :--- |
| Refinery I | $(1.20 \mathrm{p})$ | $(1.80 \mathrm{p})$ | $(100.00 \mathrm{p})$ | $(0.00 \mathrm{p})$ | $\mathbf{6}$ |
| Refinery II | $(3.00 \mathrm{p})$ | $(1.00 \mathrm{p})$ | $(0.80 \mathrm{p})$ | $(1.50 \mathrm{p})$ | $\mathbf{5}$ |
| Refinery III | $(2.00 \mathrm{p})$ | $(2.50 \mathrm{p})$ | $(1.20 \mathrm{p})$ | $(2.20 \mathrm{p})$ | $\mathbf{8}$ |
| Demand <br> (million gallons) | $\mathbf{4}$ | $\mathbf{8}$ | $\mathbf{4}$ | $\mathbf{3}$ | $\mathbf{1 9}$ |

Then, proceed with the heuristic as normal.

|  | DC1 | DC2 | DC3 | DC4 | Capacity (in million gallons) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Refinery I | (1.20p) | (1.80p) | (100.00p) | (0.00p) | 6 |
|  | 4 | 2 |  |  |  |
| Refinery II | (3.00p) | (1.00p) | (0.80p) | (1.50p) | 5 |
|  |  | 5 |  |  |  |
| Refinery III | (2.00p) | (2.50p) | (1.20p) | (2.20p) | 8 |
|  |  | 1 | 4 | 3 |  |
| Demand (million gallons) | 4 | 8 | 4 | 3 | 19 |

Total Cost $=1.2 \times 4+1.8 \times 2+1.0 \times 5+2.5 \times 1+1.2 \times 4+2.2 \times 3$
$=4.8+3.6+5+2.5+4.8+6.6$
$=£ 0.273$ million per day
(e) $[20 \%]$

|  | DC1 | DC2 | DC3 | DC4 | Capacity (in million gallons) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Refinery I | $3^{(1.20 p)}$ | (1.80p) | (100.00p) | $3^{(0.00 p)}$ | 6 |
| Refinery II | (3.00p) | $\mathbf{5}^{(1.00 p)}$ | (0.80p) | (1.50p) | 5 |
| Refinery III | $\mathbf{1}^{(2.00 \mathrm{p})}$ | $3^{(2.50 p)}$ | $4^{(1.20 p)}$ | (2.20p) | 8 |
| Demand (million gallons) | 4 | 8 | 4 | 3 | 19 |

Total Cost $=1.2 \times 3+1.0 \times 5+2.0 \times 1+2.5 \times 3+1.2 \times 4$
$=3.6+5+2+7.5+4.8$
$=£ 0.229$ million per day

## QUESTION III

(a) $[20 \%]$ The fixed time period model typically requires holding more inventory on average, since it must protect against stockout during the review period and lead time from reordering. Therefore, the fixed-order quantity model is preferred for more expensive items because average inventory is lower.

The fixed-order quantity model has no review period. Therefore, the fixed-order quantity model is more appropriate for important items such as critical repair parts because there is closer monitoring and therefore quicker response to a potential stockout.

The fixed time period model is preferred when several different items are purchased from the same vendor, and there are potential savings from ordering all these items at the same time (economies of scale).

The fixed time period model has no physical count of inventory items after an item is withdrawn. By contrast, the fixed-order quantity model requires more time and resources to maintain because every addition or withdrawal is recorded (a perpetual inventory system). Therefore, the fixed-order quantity model should be preferred only when such a monitoring is feasible. Note that advances in information technologies (point of sale computers, bar coding, RFID) have greatly reduced the cost and facilitated the use of the fixed-order quantity model.
(b) $[10 \%]$ At optimal solution, the firm's annual inventory holding cost should be equal to their annual setup cost. Since the current annual holding cost is $£ 500$ and annual setup cost is $£ 700$, we should increase $Q$ further to increase the annual holding cost and decrease the annual setup cost. Therefore, the optimal order quantity should be larger than 1,000 units.
(c)
(i) $[10 \%]$

$$
\begin{aligned}
& E O Q_{i}=\sqrt{2 K_{i} \lambda_{i} / h_{i}} \\
& T C_{i}^{*}=h \frac{E O Q_{i}}{2}+K_{i} \frac{\lambda_{i}}{E O Q_{i}}=\sqrt{2 K_{i} \lambda_{i} h_{i}}
\end{aligned}
$$

(ii) $[20 \%]$ Due to the economy of scale in setup costs, the company will incur a setup cost of $\alpha\left(K_{A}+K_{B}\right)$ for setup at the same time. Since the cycle length is $T$, the setup cost per unit time will be $\alpha\left(K_{A}+K_{B}\right) / T$. During each order, the company will produce $T * \lambda_{A}$ units of A and $T * \lambda_{B}$ units of B . The average inventory holding cost for these units will be $\frac{T}{2}\left(h_{A} \lambda_{A}+h_{B} \lambda_{B}\right)$. Therefore, the total cost (annual setup cost + annual holding cost) in terms of the cycle $T$, i.e., time elapsed between subsequent runs, can be expressed as:

$$
T C(T)=\frac{\alpha\left(K_{A}+K_{B}\right)}{T}+\frac{T}{2}\left(h_{A} \lambda_{A}+h_{B} \lambda_{B}\right)
$$

(iii)[20\%] Using the first-order optimality conditions, we can calculate the optimal $T^{*}$. Take the derivative $T C(T)$ with respect to $T$, which should be equal to 0 at optimality:

$$
-\frac{\alpha\left(K_{A}+K_{B}\right)}{T^{2}}+\frac{1}{2}\left(h_{A} \lambda_{A}+h_{B} \lambda_{B}\right)=0 .
$$

From the equation above, we can calculate the optimal $T^{*}$ as:

$$
T^{*}=\sqrt{2 \alpha\left(K_{A}+K_{B}\right) /\left(h_{A} \lambda_{A}+h_{B} \lambda_{B}\right)} .
$$

Insert optimal $T^{*}$ into the cost function to find the expression for the optimal total annual cost in terms of the given parameters $\left(\lambda_{i}, K_{i}, h_{i}, \alpha\right)$ :

$$
T C^{*}=\sqrt{2 \alpha\left(K_{A}+K_{B}\right)\left(h_{A} \lambda_{A}+h_{B} \lambda_{B}\right)} .
$$

(iv) $[20 \%]$ We can now express this condition for $\alpha$ in terms of the given parameters ( $\lambda_{i}, K_{i}, h_{i}$ ) using total cost expressions from parts (i) and (iii):

$$
\begin{aligned}
T C^{*} & \leq T C_{A}^{*}+T C_{B}^{*} \\
& \rightarrow \sqrt{2 \alpha\left(K_{A}+K_{B}\right)\left(h_{A} \lambda_{A}+h_{B} \lambda_{B}\right)} \leq \sqrt{2 K_{A} \lambda_{A} h_{A}}+\sqrt{2 K_{B} \lambda_{B} h_{B}} \\
& \rightarrow \alpha \leq\left(\sqrt{K_{A} \lambda_{A} h_{A}}+\sqrt{K_{B} \lambda_{B} h_{B}}\right)^{2} /\left(\left(K_{A}+K_{B}\right)\left(h_{A} \lambda_{A}+h_{B} \lambda_{B}\right)\right)
\end{aligned}
$$

