Engineering Tripos Part IIA

3F4 - Data Transmission

Solutions 2017

(a) To permit self-synchronisation, i.e., there should be sufficient information in the transitions and zero crossings to enable symbol timing clock regeneration. 2

Create a transmitted power spectrum to suit the channel, ie, the PSD of the tx signal should be compatible with the channel frequency response $H_c(\omega)$, e.g.

- Many channels cannot pass d.c. (zero frequency) or have poor response (high attenuation) at low frequencies owing to the use of a.c. coupling.

- Channels usually exhibit some kind of lowpass response that limits the transmission of high frequency signal components. 2

[4]

(b) AMI or bipolar line coding inherently is zero mean and ~~so~~ has low PSD near to zero frequency making it suitable for a.c. coupled channels. However, long runs of binary zero will give problems for symbol timing recovery owing to lack of transitions. 1

HDB3 overcomes this problem by replacing runs of 4 zeros with a special 4-bit symbol that limits runs of zero value tx symbols to 3.

Special code is B00D

D is sent as $\pm A$ such that successive Ds have alternating polarity (1st ~~is~~ with arbitrary polarity) 1

B is sent as 0 or $\pm A$ such that following D violates the AMI rule. All other symbols follow AMI rule. Code maintains zero mean, but sends more marks than AMI and so will consume more Tx energy. 1

Violations enable special symbols to be detected at the receiver and replaced with 4 zero symbols.

1

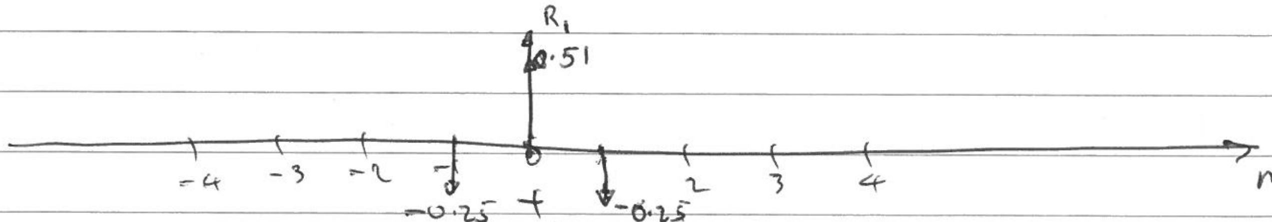
[4]

(c)

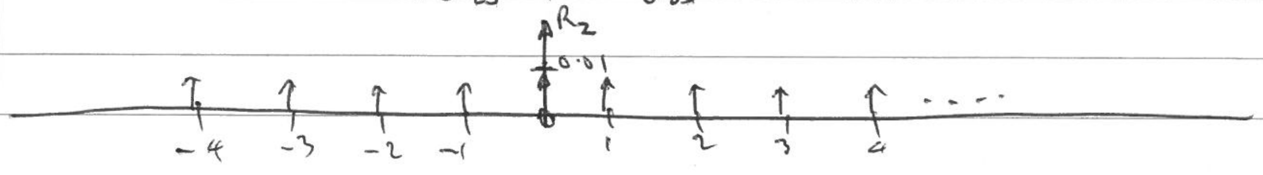
m	b_k	b_{k+m}	a_k	a_{k+m}	$R_i = a_k a_{k+m}$	p_i	$R(m)$
0	0	0	0	0	0	0.5	
	1	1	1.2	1.2	1.44	0.25	0.52
			−0.8	−0.8	0.64	0.25	
1	0	0	0	0	0	$0.5^2 = 0.25$	
	0	1	0 0	1.2 −0.8	0 0	$0.5^2/2 = 0.125$ $= 0.125$	
	1	0	1.2 −0.8	0 0	0 0	0.125 0.125	−0.24
	1	1	−0.8 1.2	1.2 −0.8	−0.96 −0.96	0.125 0.125	
2	0 (0)	0	0	0	0	$0.5^3 = 0.125$	
	0 (1)	0	0	0	0	$0.5^3 = 0.125$	
	0 (0)	1	0	1.2	0	0.125	
	0 (1)	1	0	−0.8	0	0.125	
	1 (0)	0	1.2	0	0	0.125	
	1 (1)	0	−0.8	0	0	0.125	
	1 (0)	1	1.2 −0.8	−0.8 1.2	−0.96 −0.96	0.125/2 0.125/2	
	1 (1)	1	1.2 −0.8	1.2 −0.8	1.44 0.64	0.125/2 0.125/2	0.01
3	0(**)	0	0	0	0	$0.5^4 \times 4$	
	0(**)	1	0 0	1.2 −0.8	0 0	$(0.5^4/2)\times 4$ $(0.5^4/2)\times 4$	
	1(**)	0	1.2 −0.8	0 0	0 0	$(0.5)^4/2 \times 4$ $(0.5)^4/2 \times 4$	
	1(00)	1	1.2 −0.8	−0.8 1.2	−0.96 −0.96	$0.5^4/2 = 0.03125$ 0.03125	
	1(01)	1	1.2 −0.8	1.2 −0.8	1.44 0.64	0.03125 0.03125	
	1(10)	1	1.2 −0.8	1.2 −0.8	1.44 0.64	0.03125 0.03125	
	1(11)	1	1.2 −0.8	−0.8 1.2	−0.96 −0.96	0.03125 0.03125	0.01

3

So for $m \geq 2$, $R(m) = 0.01$.
See $R(m)$ has a finite value over $m \to \infty$, so to permit a more meaningful physical representation, $R(m)$ will be expressed as a sum of 2 parts.



+



See $R_1 + R_2 = R$ and that $R_2(m) = 0.01$ for $m \pm \infty$.
Thus $R_2(m)$ can be represented as a series of impulses in the frequency domain

$$S_{x2}(w) = \frac{R}{T_s} \sum_{m=-\infty}^{\infty} e^{jmwT_s}$$

$$= \frac{2\pi R}{T_s^2} \sum_{m=-\infty}^{\infty} \delta\left(w - m\frac{2\pi}{T_s}\right)$$

where $R = 0.01$, so

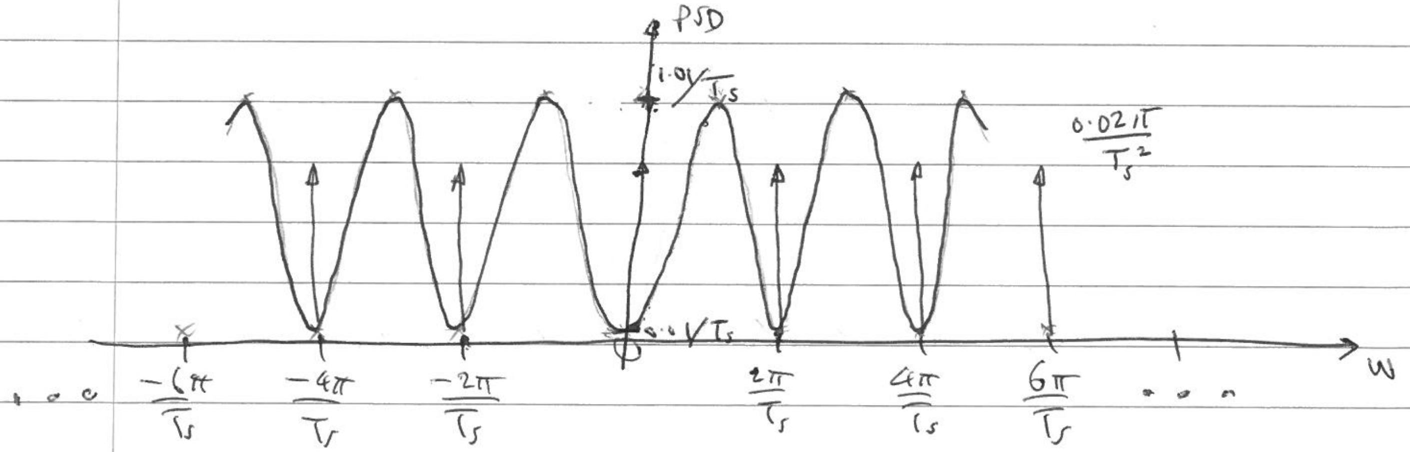
$$S_{x2}(w) = \frac{0.02\pi}{T_s^2} \sum_{m=-\infty}^{\infty} \delta\left(w - \frac{m2\pi}{T_s}\right) .$$ 2

For ~~the~~ $R_1(m)$ that is non-zero over a finite range of m we have,

$$S_{x1}(w) = \frac{1}{T_s}\left[R(0) + 2\sum_{m=1}^{\infty} R(m)\cos wmT_s\right]$$

$$= \frac{1}{T_s}\left[0.51 + 2(-0.25)\cos w\,T_s\right]$$

$$= \frac{1}{T_s}\left[0.51 - 0.5\cos w\,T_s\right]$$ 2



1

[8]

(d) From tables,

$$H(\omega) = T_s \quad \mathrm{sinc}\left(\frac{\omega T_s}{2}\right)$$ 1

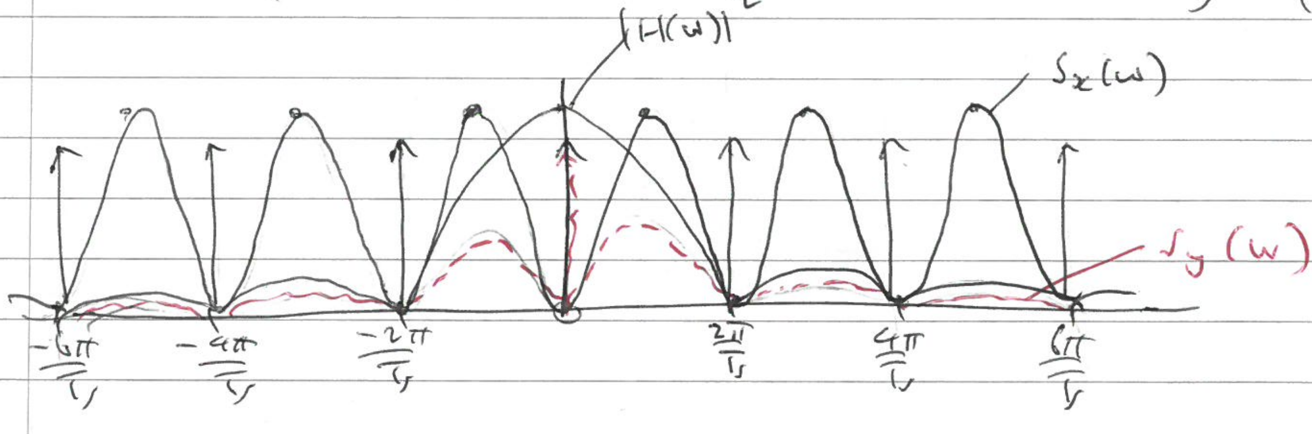
The power spectrum at the filter output is

$$S_y(\omega) = S_x(\omega)\, |H(\omega)|^2$$

$$= \left[\frac{1}{T_s}\left[0\cdot51 - 0\cdot5\cos\omega T_s\right) + \frac{0\cdot02\pi}{T_s^2}\sum_{m=-\infty}^{\infty}\delta\left(\omega - m\frac{2\pi}{T_s}\right)\right]$$

$$\times\ T_s^2\ \mathrm{sinc}^2\left(\frac{\omega T_s}{2}\right)$$

$$\left(T_s\left[0\cdot51 - 0\cdot5\cos\omega T_s\right] + 0\cdot02\pi\sum_{m=-\infty}^{\infty}\delta\left(\omega - m\frac{2\pi}{T_s}\right)\right)\mathrm{sinc}^2\left(\frac{\omega T_s}{2}\right)$$ 1



1

Advantage over polar line coding is low power density near zero frequency which is advantageous for a.c. coupled channels.

1

[4]

2. (a) Codewords $\underline{c}$ are generated from G and a data row-vector $\underline{x}$ using

$$\underline{c} = \underline{x}\, G$$

Hence when G is a 4×8 matrix, the dimension k of $\underline{x}$ is 4, and the 1 blocklength n of $\underline{c}$ is 8. 1 [2] [10%]

(b) G represents a systematic code because it is of the form $G = [\, I_k \mid P \,]$ where I_k is the $k \times k$ identity matrix.

Hence
$$P = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

When G is systematic, the parity check matrix H is given by

$$H = [\, P^T \mid I_{n-k} \,] \qquad \text{so that } G H^T = 0$$

$$\cancel{\therefore H = \begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}}$$

2 (b) (cont)

$$\therefore \; H = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad 2$$

d_{min} is the smallest number of columns of H which sum to zero. In this case $d_{min} = 3$, since column 4 can be combined with columns 1 & 6, or 2 & 7, or 3 & 8 to give a zero column vector for the syndrome. 2

[4] [20%]

(c) The max number of correctable errors is $\left\lfloor \frac{d_{min} - 1}{2} \right\rfloor = 1$. 1

Double errors could be detected if error correction is <u>not</u> enabled, but they ~~could~~ would be regarded as an incorrect single-error pattern if error correction is enabled. 2

[3] [15%]

[Note that this is not the best (8, 4) code.

If $P = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}$, then $d_{min} = 4$ & double errors could always be detected.]

2(d) $\quad \underline{x} = \begin{array}{c} \\ \left[\begin{array}{ccccc} \ldots\, 0, & \overset{m=1}{\downarrow} \\ & 1, & 0, & 0, & \ldots \end{array} \right] \end{array}$

For $m < 1$, x_m to x_{m-3} are all zero and so $c_{1,m} = c_{2,m} = 0$. Similarly for $m > 4$.

For $m = 1, 2, 3, 4$, the row vectors $\underline{c}_1$ & $\underline{c}_2$ are given by:

$$\begin{array}{cc} & \begin{array}{cccc} m=1 & & & m=4 \\ \downarrow & & & \downarrow \end{array} \\ \underline{c}_1 = & \left[\begin{array}{cccc} 1 & 1 & 1 & 1 \end{array} \right] \\ \underline{c}_2 = & \left[\begin{array}{cccc} 1 & 1 & 0 & 1 \end{array} \right] \end{array}$$ 2

Combining these cases for the given $\underline{x}$:

$$\begin{array}{cc} & \begin{array}{cccccccccc} & & & m=1 & & & m=4 & & & \\ & & & \downarrow & & & \downarrow & & & \end{array} \\ \underline{c}_1 = & \left[\begin{array}{cccccccccc} \ldots, & 0, & 0, & 1, & 1, & 1, & 1, & 0, & 0, & \ldots \end{array} \right] \\ \underline{c}_2 = & \left[\begin{array}{cccccccccc} \ldots, & 0, & 0, & 1, & 1, & 0, & 1, & 0, & 0, & \ldots \end{array} \right] \end{array}$$ 1

The combination of $\underline{c}_1$ & $\underline{c}_2$ has a Hamming weight of 7, but this is not d_{min}. 1

If $\underline{x} = [\ldots 0, 1, 1, 0 \ldots]$ then $\underline{c}_1$ & $\underline{c}_2$ are given by the sum of the above patterns with a 1-sample shifted version of the same patterns:

$$\underline{c}_1 = [\ldots\, 0, 0, 1, 0, 0, 0, 1, 0, \ldots]$$
$$\underline{c}_2 = [\ldots\, 0, 0, 1, 0, 1, 1, 1, 0, \ldots]$$ 1

This has a Hamming weight of 6. All other combinations of ones in $\underline{x}$ produce higher (or equal) weights.

$\therefore\ d_{min} = \underline{6}$ 1 [6] [30%]

2(e) For the block code, $d_{min} = 3$ which provides single error correction in an 8-bit block.

For the convolutional code, $d_{min} = 6$ which provides double error correction <u>and</u> also triple error detection. Each data bit influences around 8 bits of coded data in $\underline{c}_1$ and $\underline{c}_2$, so the error patterns must exist within a similar interval (~8 bits) to that for the block code in order to cause decoding failure.

Hence the convolutional code is significantly more powerful than the given (8,4) block code, for the same ~~co~~ rate 1/2.

The designer would therefore choose the convolutional code - except that it is more complicated to decode, as it needs the Viterbi algorithm instead of calculation of a simple 4-bit syndrome to determine the error pattern.

[5] [25%]

3. (a) By using more than 2 levels for symbols in a modulation scheme, it is possible to convey $m = \log_2 M$ bits of information per symbol. In most modulation schemes, this means that, for a given bit rate, the symbol rate can be reduced by the ~~a~~ factor $1/m$, and this often reduces the bandwidth of the modulated signal by a similar amount. 1

For maximum energy efficiency, the symbols s_k should have states that are symmetrical about zero, so that the mean value for s_k is zero and no power is "wasted" on a carrier component that conveys no information. 1
Hence for M equally-spaced values, centred on zero, we usually use the integers $(2i+1-M)$ which go from $-(M-1)$ to $+(M-1)$ as i goes from zero to $(M-1)$. These become -1 and $+1$ when $M=2$, the binary case. 2

[4] [20%]

3. (b) In each equation, $g(t)$ is the pulse which shapes the waveform during each symbol period. $g(t-kT_s)$ is a pulse which defines the waveform from $t = kT_s$ to $t = (k+1)T_s$, ~~a~~ provided that $g(t)$ is only non-zero over the interval $t = 0$ to T_s. 1
The spectrum of the modulated signal is usually closely related to the spectrum of $g(t)$ 1

(i) represents multi-level frequency-shift keying, because the s_k term is in the complex exponential and controls the rate of change of phase of the ~~expone~~ phasor. ~~vector~~
ω_D defines the frequency deviation in rad/s.
ϕ_k is a constant phase offset during the k^{th} symbol period, which usually is updated at $t = kT_s$ to ensure phase continuity from symbol $k-1$. 1
ϕ_0 is an additional constant phase offset that is caused by unknown path delays etc. 1

[30%]

3(b) (ii) represents multi-level QAM, because each symbol comprises two symbols in quadrature (s_{2k} & s_{2k+1}) which amplitude modulate the pulse $g(t-kT_s)$. 1

(iii) represents multi-level PSK, because $s_k \pi/M$ determines the phase of the complex exponential term for the duration of the symbol period when $g(t-kT_s)$ is non-zero. 1

(6) [30%]

(c) If $M=64$, then $m = \log_2 M = \underline{\underline{6}}$ 1

Hence the symbol period $T_s = 6T_b = 6/R_b$

MPSK & M-QAM are both schemes in which the modulating signal remains constant throughout each symbol period and in which symbols are uncorrelated with adjacent symbols. Hence the discrete autocorrelation function $R_{ss}(L) = 0$ except when $L=0$. 1

$$\therefore E\{|P(\omega)|^2\} \propto |G(\omega)|^2 \quad \text{where } G(\omega) \Leftrightarrow g(t)$$

$$\propto \operatorname{sinc}^2\left(\frac{\omega T_s}{2}\right)$$

First zeros of $\operatorname{sinc}^2\left(\frac{\omega T_s}{2}\right)$ are at $\omega T_s = \pm 2\pi$

$\omega = \pm 2\pi \frac{R_b}{6} = \pm \pi \frac{R_b}{3}$ or $f = \pm \frac{1}{T_s} = \pm \frac{R_b}{6}$ 1

or $2\pi \frac{R_b}{}$

3 (c) (cont)

Hence, for 64-PSK & 64-QAM,

$$\text{Bandwidth} = \frac{2}{T_S} = \frac{R_b}{3}\ \text{Hz}. \qquad 1$$

For 64-FSK, the centre frequency of the transmitted signal is shifted by $S_k \cdot (\omega_D/2\pi)$ Hz for each symbol. The value of $\omega_D/2\pi$ is chosen to be $\frac{1}{T_S}$ so that all the transmitted symbols are orthogonal to each other (as in DFT).

$\therefore$ Total bandwidth = bandwidth of 1 symbol + $(M-1)$(symbol frequency spacing)

$$= \frac{2}{T_S} + (M-1)\cdot\frac{1}{T_S} = \frac{M+1}{T_S} = \frac{65 \cdot R_b}{6}$$

$$= 10.83\ R_b. \qquad 2 \qquad [6] \quad [30\%]$$

(d) 64-PSK & 64-QAM have the same bandwidth, but 64-PSK is less resilient to noise because the 64 states are uniformly spaced around the edge of a unit circle, instead of being uniformly distributed inside an approximately unit ~~circle~~ square. 1
Hence 64-QAM is almost always preferred over 64-PSK. 1

64-FSK requires $\frac{65}{2} = 32.5$ times the bandwidth of 64-QAM but it is much more noise resilient because all the symbols are orthogonal to each other. 2 [4] [20%]

4. (a) Digital offers the following advantages over Analog transmission:

1) Less susceptible to cumulative degradations
2) Ultimate noise levels are determined by A-D converters & compression effects, rather than by signal/noise ratios on the channel.
3) Many multiplexing methods available (FDMA, TDMA, SSMA).
4) A wide range of source material can be used
5) More channels per unit of bandwidth if good digital compression is used.

This all requires <u>much</u> more complicated processing methods & these have only been cheaply available in the last 10-20 years. [3] [15%]

(b) Compressed HD video typically requires between 5 & 10 Mb/s of data capacity per channel. 1

Compressed audio typically requires between 100 kb/s & 250 kb/s for a stereo music signal. 1

Most wireless channels exhibit multiple paths (due to reflections) & high bit-rate serial transmissions are subject to severe ~~loss of~~ bit error rates if the spread of path delays exceeds the bit period (eg by 200 ns for a 5 Mb/s signal)! [15%]

[3]

4. (c) See diagrams in section 6.2 of the lecture notes, especially fig 6.1.

A COFDM modulator comprises an error-correction encoder, ~~and an~~ a demultiplexer into N parallel channels, and an inverse FFT modulator which modulates the N channels of data onto N adjacent subcarriers using the Fourier transform. The data rate on each channel is $\frac{1}{N}$ of the composite bit rate (after EC encoding), ~~so if~~ and so can tolerate N times as much multipath delay spread as a single serial data stream. 1

If N is large (~1000 to 2000) then large delay spreads can be tolerated. 1

The bandwidth efficiency of the OFDM scheme remains good, even with large N, because of the orthogonality properties of the FFT. 1

~~(d) Bit rate after error correction $= 30 \cdot \frac{3}{2} = 45$ Mb/s~~

~~Analysis period~~ of DFT

The demodulator for COFDM comprises an FFT block to recover the separate subcarrier amplitudes & phases, followed by a QPSK/QAM decoder & N channel multiplexer, and then an error-correction decoder.

4. (c) (cont)

Tolerance to multipath is improved by extending each transmitted block by a guard period, whose period equals the maximum delay spread to be tolerated. This reduces bandwidth efficiency but makes the system more resilient to noise and interference. The error correction coding is designed to overcome errors due to spectral fading (another multipath effect) which degrades a small percentage of the subcarriers at any given time. 1 1

[5] [25%]

(d) Composite bit rate after coding $= 30 . \frac{3}{2} = 45$ Mb/s.

Analysis period of FFT $= \frac{1}{\text{carrier spacing}} = \frac{1}{1{\cdot}25 . 10^{3}}$ s.

$= 800 \mu s$. 1

Guard period = delay spread = $10 \mu s$.

Symbol rate for each subcarrier $= \frac{1}{(800+10) . 10^{-6}}$ s^{-1}

Using 64-QAM, no of bits/s on each subcarrier $= \frac{6}{810 . 10^{-6}}$

$= 7407$ Hz 1

∴ No. of carriers to carry 45 Mb/s of coded data

$= \frac{45 . 10^{6}}{7407} = 6075$

No. of carriers, including 10% extra pilots $= 6075 \times 1{\cdot}1 = 6{,}683$ 1

4. (d) (cont.)

Estimated bandwidth for 6,684 carriers (using a null slot at the centre) with 1·25 kHz spacing and including spectrum to first zeros at ends

$$= (6{,}684 + 1) \cdot 1{\cdot}25\,\text{kHz} = \underline{\underline{8{\cdot}356}}\ \text{MHz}$$

~~Spectral efficiency = Bandwidth / user bit rate = 8.5~~

$$\text{Spectral efficiency} = \frac{\text{User bit rate}}{\text{Bandwidth}} = \frac{30 \cdot 10^6}{8{\cdot}356 \cdot 10^6} = \underline{\underline{3{\cdot}59}}\ \text{bit s}^{-1}/\text{Hz}$$

1

[5] [25%]

(e) For video, Fixed directional receiver antennas will produce a relatively stable channel (not time-varying) with relatively small path delay spread (due to narrow angle of arrival ~~range~~ spread). High bit rates & hence high spectral efficiency is needed. Therefore we choose 64-QAM and relatively short guard periods for good spectral efficiency. 1 1

For audio, moving omni-directional antennas will result in a time-varying channel with large path delay spread. Audio bit rates are much lower than for video (about $\frac{1}{10}$ of video rates), so robustness to noise is ~~needed~~ more important than high spectral efficiency. Hence we choose QPSK modulation & long guard periods. 1 1

[4] [20%]

NGK, 2017.

Engineering Tripos Part IIA 2017
Module 3F4: Data Transmission

Question 1 – Line Coding and transmit PSD
This question was in general well answered. Candidates had most problems with part (c) where they often managed to calculate the required discrete autocorrelation function values but could not use them to yield the required power spectral density.

Question 2 – Forward error correction coding
This question has the lowest mean mark. Part (c) gave most problems and only a few candidates could determine the correct minimum distance for the specified convolutional code. In part (d) a surprisingly large number of candidates could not describe the relative merits of convolutional versus block codes.

Question 3 – M-level complex baseband modulation, PSD and performance trade-off
This question was the least popular but was generally well answered. A surprising number of candidates failed to correctly identify the modulation types represented in part (b). In addition, a number of candidates could not calculate the bandwidth for FSK required in part (c).

Question 4 – Digital techniques and OFDM
This question received the highest scoring answers from the candidates. That said, some candidates gave wildly inaccurate estimates for the bit rates required to transmit compressed audio and video signals. Some of the answers describing the trade-offs required in part (e) were not particularly clear.

I. J. Wassell
(Principal Assessor)
8th May 2017