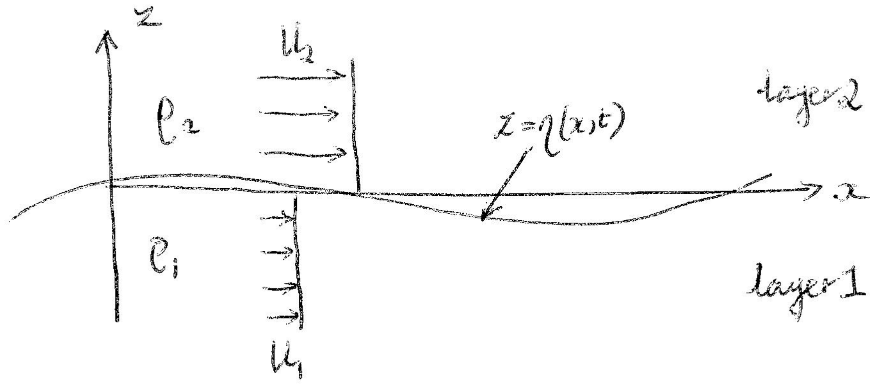


(a) Given incompressible, i.e. $\nabla \cdot \underset{\sim}{u} = 0$
For fluid to support given velocity profile it's inviscid, so $\underset{\sim}{u} = \nabla\phi$ for velocity potential given $\nabla \times \underset{\sim}{u} = 0$

so that $\nabla \cdot \underset{\sim}{u} = \nabla \cdot \nabla\phi = \underline{\nabla^2\phi = 0}$.

(b). Four boundary conditions
- 2 at interface (dynamic & kinematic)
- 1 for flow as $z \to \infty$
- 1 for flow as $z \to -\infty$

- In far field, i.e. away from region of disturbance (interface) we recover base flow i.e.
$$\nabla\phi_2 \to U_2 \quad \text{as } z \to -\infty$$
$$\nabla\phi_1 \to U_1 \quad \text{as } z \to \infty$$
- kinematic b.c. (particles on interface remain on interface), we define $F = z - \eta(x,t) = 0$
Thus $\frac{\partial F}{\partial t} + (\underset{\sim}{u}\cdot\nabla)F = \frac{DF}{Dt} = 0$ & with $u = \frac{\partial\phi}{\partial x}$, $w = \frac{\partial\phi}{\partial z}$
we have $\frac{\partial \eta}{\partial t} + \frac{\partial\phi}{\partial x}\frac{\partial\eta}{\partial x} = \frac{\partial\phi}{\partial z}$, $\phi = \{\phi_1, \phi_2\}$ on $z = \eta(x,t)$.
- dynamic b.c (pressure continuous across interface). For unsteady irrotational flow
$\frac{\partial\phi_1}{\partial t} + \frac{p}{\rho_1} + \frac{1}{2}u^2 + gz = G_1(t)$, so for continuous press. across interface

on integrating Euler equation

$$\rho_1\left(\frac{\partial\phi_1}{\partial t} + \frac{u_1^2}{2} + gz + G_1\right) = \rho_2\left(\frac{\partial\phi_2}{\partial t} + \frac{u_2^2}{2} + gz + G_2\right) \quad \text{on } z = \eta(x,t)$$

[In base state $z=0$ and $\partial/\partial t = 0$ $\Rightarrow$ $\rho_1(\tfrac{1}{2}U_1^2 + G_1) = \rho_2(\tfrac{1}{2}U_2^2 + G_2)$ on $z=0$.

(c) To assess stability of flow, introduce small amplitude perturbations via the perturbation quantities

$$\begin{cases} \underset{\sim}{u} = \underset{\sim}{U} + u'(x,z,t), \text{ so that } & \phi_2 = U_2 x + \phi_2' \\ & \phi_1 = U_1 x + \phi_1' \\ p = P + p'(x,z,t) & \\ \eta = 0 + \eta'(x,t) & \end{cases}$$

base state

Substituting into governing eq^n

$\nabla^2\phi_1 = 0 \Rightarrow \nabla^2(U_1 x + \phi_1') = 0 + \nabla^2\phi_1' = 0$. Thus $\nabla^2\phi_1' = 0$ for $z < \eta'(x,t)$
$\nabla^2\phi_2' = 0$ for $z > \eta'(x,t)$

& hence on linearising
$$\begin{cases} \nabla^2\phi_2' = 0 & z > 0 \\ \nabla^2\phi_1' = 0 & z < 0 \end{cases}$$
given perturbation behaviour

Q1 (c) cont^d.

Sub perturbation quantities into b.c's & linearising....

- in far field $\nabla\phi_1 \to U_1$ as $x \to -\infty$ reduces to $\nabla(U_1 x + \phi_1') \to U_1$, ie. $\nabla\phi_1' \to 0$ as $x \to -\infty$
  & similarly $\underline{\nabla\phi_2' \to 0 \text{ as } x \to \infty}$

  (ie. perturbation dies away to zero in far field)

- in kinematic b.c. $\frac{\partial \eta}{\partial t} + \frac{\partial \phi}{\partial x}\frac{\partial \eta}{\partial x} = \frac{\partial \phi}{\partial z}$ on $z = \eta(x,t)$ becomes $\frac{\partial}{\partial t}\eta' + \frac{\partial}{\partial x}(U_x + \phi')\frac{\partial \eta'}{\partial x} = \frac{\partial}{\partial z}(U_x + \phi')$ on $z = \eta'(x,t)$

  & neglecting products of small terms $\underline{\frac{\partial \eta'}{\partial t} + U_1 \frac{\partial \eta'}{\partial x} = \frac{\partial \phi_1'}{\partial z} \quad \& \quad \frac{\partial \eta'}{\partial t} + U_2 \frac{\partial \eta'}{\partial x} = \frac{\partial \phi_2'}{\partial z}}$ on $z=0$

- in dynamic b.c. $\rho_1\left(C_1 + \tfrac{1}{2}u_1^2 + g\eta + \frac{\partial \phi_1}{\partial t}\right) = \rho_2\left(C_2 + \tfrac{1}{2}u_2^2 + g\eta + \frac{\partial \phi_2}{\partial t}\right)$ on $z = \eta(x,t)$.

  we note $\frac{u_1^2}{2} = \frac{1}{2}(\nabla\phi_1)^2 = \frac{1}{2}[\nabla(U_1 x + \phi_1')]^2 = \frac{1}{2}(U_1 + \nabla\phi_1')^2 = \frac{1}{2}\left[U_1^2 + 2U_1\frac{\partial \phi_1'}{\partial x} + (\nabla\phi_1')^2\right] \equiv \frac{1}{2}\left(U_1^2 + 2U_1\frac{\partial \phi_1'}{\partial x}\right)$ on linearising

  thus $\rho_1\left[\underbrace{C_1 + \tfrac{1}{2}U_1^2}\ \overset{+U_1\partial\phi_1'/\partial x}{\wedge} + g\eta' + \frac{\partial}{\partial t}(\phi_1' + U_1 x)\right] = \rho_2\left[\underbrace{C_2 + \tfrac{1}{2}U_2^2}\ \overset{+U_2\partial\phi_2'/\partial x}{\wedge} + g\eta' + \frac{\partial}{\partial t}(\phi_2' + U_2 x)\right]$ on $z = \eta'(x,t)$

  reduces to $z=0$ on linearising

  & on $z=0$ we found earlier that $\rho_1(C_1 + \tfrac{1}{2}U_1^2) = \rho_2(C_2 + \tfrac{1}{2}U_2^2)$

  Hence linearised b.c. is

$$\underline{\rho_1\left(U_1\frac{\partial \phi_1'}{\partial x} + g\eta' + \frac{\partial \phi_1'}{\partial t}\right) = \rho_2\left(U_2\frac{\partial \phi_2'}{\partial x} + g\eta' + \frac{\partial \phi_2'}{\partial t}\right)} \text{ on } z=0$$

Now seek normal mode solutions of form:

$$\eta(x,t) = \hat{\eta}\, e^{ikx + st}$$
$$\phi_1'(x,z,t) = \hat{\phi}_1(z)\, e^{ikx + st} \quad z<0$$
$$\phi_2'(x,z,t) = \hat{\phi}_2(z)\, e^{ikx + st} \quad z>0$$

Substituting into $\nabla^2\phi' = 0$ gives $\frac{d^2\hat{\phi}_2}{dz^2} - k^2\hat{\phi}_2 = 0 \Rightarrow \hat{\phi}_2(z) = Ae^{kz} + Be^{-kz}$ & bounded solution requires $A=0$

& so $\hat{\phi}_2(z) = Ae^{-kz}$ & similarly $\hat{\phi}_1(z) = Ce^{+kz}$ for constants (A, B, C).

Thus $\phi_2' = Ae^{-kz}e^{ikx+st}$
$\phi_1' = Ce^{+kz}e^{ikx+st}$.

Using kinematic b.c.'s

$$s\hat{\eta}\cancel{e^{ikx+st}} + U_1 ik\hat{\eta}\cancel{e^{ikx+st}} = kCe^{kz}\cancel{e^{ikx+st}} \quad \text{on } z=0$$
$$\Rightarrow C = \frac{\hat{\eta}}{k}(s + U_1 ik).$$
$$s\hat{\eta}\cancel{e^{ikx+st}} + U_2 ik\hat{\eta}\cancel{e^{ikx+st}} = -Ake^{-kz}\cancel{e^{ikx+st}} \quad \text{on } z=0$$
$$\Rightarrow A = -\frac{\hat{\eta}}{k}(s + U_2 ik).$$

Finally, using dynamic b.c.

$$\rho_1\left(U_1 Ce^{kz}ike^{ikx+st} + g\hat{\eta}e^{ikx+st} + sCe^{kz}e^{ikx+st}\right) = \rho_2\left(U_2 Ae^{-kz}ike^{ikx+st} + g\hat{\eta}e^{ikx+st} + sAe^{-kz}e^{ikx+st}\right) \text{ on } z=0$$

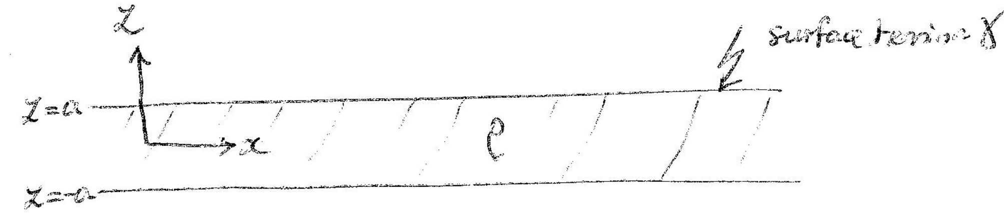
$$\Rightarrow \rho_1\left[\underbrace{ikU_1\cdot\frac{\hat{\eta}}{k}(s+U_1 ik)} + g\hat{\eta} + \underbrace{s\frac{\hat{\eta}}{k}(s+U_1 ik)}_{c}\right] = \rho_2\left(ikU_2\cdot -\frac{\hat{\eta}}{k}(s+U_2 ik) + g\hat{\eta} - s\frac{\hat{\eta}}{k}(s+U_2 ik)\right)$$

$$\text{LHS} = \left(\frac{\hat{\eta}}{k}\right)\rho_1\left(\underbrace{ikU_1(s+U_1 ik) + gk + s(s+U_1 ik)}_{= (s+U_1 ik)^2 + gk}\right) \Rightarrow \underline{\rho_1\left(kg + (s+ikU_1)^2\right) = \rho_2\left(kg - (s+ikU_2)^2\right)}$$

□

Q2. GRH

(a)



Growth rate of disturbance $s = f(k, a, \rho, \gamma)$

where each has following dimensions
$k \sim [1/L]$
$a \sim [L]$
$\rho \sim [M/L^3]$
$\gamma \sim \left[\frac{M.L}{T^2.L}\right] = [M/T^2]$

Thus, $s\left(\frac{\rho a^3}{\gamma}\right)^{1/2} = F(k,a)$

ie. $\underline{s\left(\frac{\rho a^3}{\gamma}\right)^{1/2} = F(ka)}$ ← dimensionless growth rate scales as indicated & is a function of dimensionless wavenumber ka.

(*) If Real $(s(k)) < 0$ for all k the system is stable.

(b). The essence of the approach is as follows:

Small amplitude perturbations (marked below with a prime) are introduced about the pressure, velocity, etc, of the steady base flow (say $\underline{u}_0, p_0$) so that

$$\underline{u} = \underline{u}_0 + \underline{u}'(x, y, z, t)$$
$$p = p_0 + p'(x, y, z, t)$$

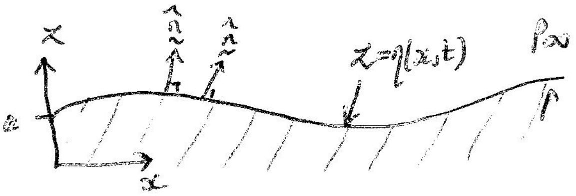
and substituted into the governing equations of motion & boundary conditions. This system is then linearised, ie. products of small terms are neglected (as diminishingly small). As a given disturbance to the base flow can be Fourier analysed spatially & expressed as an integral sum of normal modes over a range of wavenumbers k. Owing to there being an absence of terms in the governing equations involving products of perturbations, we can solve for the growth rate $s(k)$ by taking a single mode for which k is treated as a parameter, - subsequently sweeping through all values of k. Solution to linearised system sought in terms of normal mode solutions, eg $p' = \hat{p}(z)e^{ikx + st}$ (*)

2

(c) Liquid jet is inviscid $\frac{\partial \underline{u}}{\partial t} + (\underline{u}\cdot\nabla)\underline{u} = -\frac{1}{\rho}\nabla p$ i.e. Euler's eq^n & continuity govern motion.

& incompressible $\nabla\cdot\underline{u} = 0$

Boundary conditions for flow are kinematic (i.e. particles on interface remain on interface) and the Laplace result from which the pressure on interface (liquid-air) is obtained.



• Laplace's result gives $\underline{p - p_\infty = \gamma \nabla\cdot\hat{n}}$
where unit normal $\hat{\underline{n}} = \frac{\nabla F}{|\nabla F|}$ where
$F=0$ is equation of surface.

Note in undisturbed base state $\hat{\underline{n}} = \underline{k} \Rightarrow \nabla\cdot\hat{\underline{n}} = 0$ so that $\underline{p = p_\infty \text{ on } z = a, -a}$.
(we neglect effect of gravity for this slender jet)

• Kinematic b.c. in general: $z = \eta(x,t)$ and surface defined by $F = z - \eta(x,t) = 0 \Rightarrow \frac{DF}{Dt} = 0$

so that $-\frac{\partial\eta}{\partial t} + u\left(-\frac{\partial\eta}{\partial x}\right) + w(1) = 0 \Rightarrow \underline{w = \frac{\partial\eta}{\partial t} + u\frac{\partial\eta}{\partial x}}$ on $z=\eta(x,t)$

Moreover, with $F = z - \eta(x,t) = 0$, $\hat{\underline{n}} = \frac{\nabla F}{|\nabla F|} = \frac{-\frac{\partial\eta}{\partial x}\underline{i} + 0 + 1\underline{k}}{\sqrt{(\partial\eta/\partial x)^2 + 1}}$

Suppose the jet has steady velocity U, we make a Galilean transformation so that jet is stationary & moves in an environment of velocity $-U$. Thus, using the above, the base state may be expressed as $p = p_\infty$ on $z = \pm a$, $U = 0$, $\eta = \pm a$.

Now introduce perturbation quantities & write
$$\begin{cases} \underline{u} = 0 + \underline{u}'(x,z,t) \\ p = p_\infty + p'(x,z,t) \\ \eta = a + \eta'(x,t) \end{cases}$$
[focus on upper interface]

Substituting into governing equations gives
$$\begin{cases} \frac{\partial \underline{u}'}{\partial t} + \underbrace{(\underline{u}'\cdot\nabla)\underline{u}'}_{=0 \text{ on linearising as product of small terms}} = -\frac{1}{\rho}\nabla(p_\infty + p') = -\frac{1}{\rho}\nabla p' \Rightarrow \frac{\partial \underline{u}'}{\partial t} = -\frac{1}{\rho}\nabla p' \\ \nabla\cdot\underline{u}' = 0 \end{cases}$$

Combining as a single eq^n, take divergence of first to yield $\frac{\partial}{\partial t}(\nabla\cdot\underline{u}') = -\frac{1}{\rho}\nabla\cdot\nabla p' = -\frac{1}{\rho}\nabla^2 p'$
& using $\nabla\cdot\underline{u}' = 0$ gives $\underline{\nabla^2 p' = 0}$

Substituting perturbation quantities into b.c.'s gives:

(kinematic) $w' = \frac{\partial\eta'}{\partial t} + u'\frac{\partial\eta'}{\partial x}$ on $z = a + \eta'(x,t)$

i.e. $\underline{w' = \frac{\partial\eta'}{\partial t}}$ on $z = a$ on linearising.

(Laplace) $\hat{\underline{n}} = \frac{-\partial\eta'/\partial x\,\underline{i} + \underline{k}}{\sqrt{(\partial\eta'/\partial x)^2 + 1}} = \frac{-\frac{\partial\eta'}{\partial x}\underline{i} + \underline{k}}{\sqrt{1}}$ on linearising

PTO.

Q2 (c) cont^{d}.

Thus, $\nabla\cdot\hat{\underline{n}} = \frac{\partial}{\partial x}\left(-\frac{\partial\eta'}{\partial x}+\underline{k}\right)+\frac{\partial}{\partial y}\left(-\frac{\partial\eta'}{\partial y}+\underline{k}\right)+\frac{\partial}{\partial z}\left(-\frac{\partial\eta'}{\partial x}+\underline{k}\right) = -\frac{\partial^2\eta'}{\partial x^2}$.

So that $p - p_\infty = \gamma\,\nabla\cdot\hat{\underline{n}}$ reduces to

$$(p_\infty + p') - p_\infty = \gamma\cdot -\frac{\partial^2\eta'}{\partial x^2} \quad \text{on } z = a+\eta'$$

$$p' = -\gamma\frac{\partial^2\eta'}{\partial x^2} \quad \text{on } z = a \qquad \text{(on linearising)}$$

Combining b.c.'s, note $\frac{\partial^2 p'}{\partial t^2} = -\gamma\frac{\partial^2}{\partial x^2}\left(\frac{\partial^2\eta'}{\partial t^2}\right) = -\gamma\frac{\partial^2}{\partial x^2}\frac{\partial}{\partial t}\underbrace{\left(\frac{\partial\eta'}{\partial t}\right)}_{=w'}$ & $\frac{\partial\eta'}{\partial t} = w'$ from kinematic b.c.

& from $\frac{\partial\underline{u}'}{\partial t} = -\frac{1}{\rho}\nabla p'$ we have vertical component $\frac{\partial w'}{\partial t} = -\frac{1}{\rho}\frac{\partial p'}{\partial z}$

Thus $\underline{\rho\frac{\partial^2 p'}{\partial t^2} = +\gamma\frac{\partial^2}{\partial x^2}\left(\frac{\partial p'}{\partial z}\right)}$ on $z = a$ $\quad *$

Now seek normal mode sol^n to $\nabla^2 p' = 0$, introduce $p' = \hat{p}(z)e^{ikx+st}$ as a normal mode solution, thus $\frac{d^2\hat{p}}{dz^2} - k^2\hat{p} = 0$ giving $\hat{p}(z) = Ae^{kz} + Be^{-kz}$

As we have no preferred vertical direction (we neglected effects gravity), we require $A = B$

$$\Rightarrow \hat{p}(z) = A(e^{kz}+e^{-kz}) \Rightarrow p' = A(e^{kz}+e^{-kz})e^{ikx+st}.$$

Now use combined b.c. $*$, to give

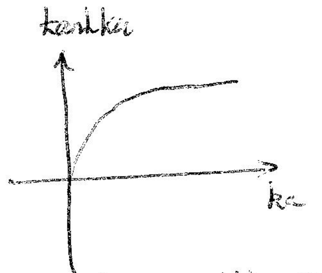
$$\rho\cdot s^2 A(e^{kz}+e^{-kz})e^{ikx+st} = \gamma\frac{\partial^2}{\partial x^2}\left[Ak(e^{kz}-e^{-kz})\cdot e^{ikx+st}\right] \quad \text{on } z = a$$

$$= \gamma Ak(e^{kz}-e^{-kz})\cdot(-1)k^2 e^{ikx+st} \quad \text{on } z = a.$$

$$\Rightarrow \quad s^2 = -\frac{\gamma}{\rho}k^3\,\frac{e^{kz}-e^{-kz}}{e^{kz}+e^{-kz}}\Bigg|_{z=a}$$

$$= -\frac{\gamma k^3}{\rho}\tanh(ka)$$

so finally $s^2 = -\frac{\gamma(ka)^3}{\rho a^3}\tanh(ka)$



Growth rate always negative for $ka > 0$

$\Rightarrow$ det stable.

$\tanh(ka) > 0 \ \forall\, ka > 0$

3 a)

i) Vortex shedding The lamp post is a bluff body. Vortices will be shed from either side, causing an oscillating side-to-side force on the lamp post at a frequency determined by the wind speed and the lamppost diameter. If the vortex shedding frequency is sufficiently close to the natural frequency of side-to-side vibrations of the lamppost then the vortex shedding will lock on to the natural frequency and thereby force the lamppost at exactly its resonant frequency. This will cause large amplitude oscillations.

ii) Changes in drag coefficient with Reynolds number The drag force on the lamp post is $F = \frac{1}{2}\rho V^2 C_D D$ per unit length, where V is the relative velocity between the air and the moving lamppost. Consider motion around an equilibrium position at some wind speed. If dF/dV is always positive then, if the post moves towards the wind, F increases as V increases and there is an extra drag force, which acts against the motion, meaning that oscillations are damped. If dF/dV is negative then, if the post moves towards the wind, F decreases as V increases and there is a reduction in drag force, meaning that oscillations are negatively damped - i.e. encouraged. Looking at the chart of $C_D(Re)$, there is a sharp drop in C_D around $Re = 2\times10^5$, which is sufficient to cause dF/dV to be negative.

iii) Galloping There is ice on the road, showing that the temperature is below freezing and therefore that there could be ice on the lampposts. The symmetry of the lamp-post cross-section could be broken and this can cause a fluid mechanical force in the direction of motion of the lamppost when the lamppost oscillates perpendicular to the wind direction. If this force is sufficient to overcome mechanical damping then oscillations will start.

b) Estimates: wind speed, $U = 10\,ms^{-1}$
lamppost diameter, $D = 0.1\,m$
air temperature $= T = -5°C = 268K$
air viscosity $= 1.8\times10^{-5}$ kg/(ms)

$Re = 7\times10^4$
$\rho = 1.3\,kg\,m^{-3}$

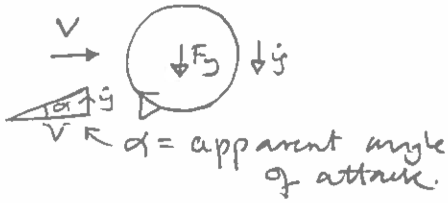
Now consider the three mechanisms in turn

i) <u>vortex shedding</u> At this Reynolds number, $St = 0.2 \Rightarrow f = \dfrac{0.2 \times 10}{0.1} = 20\,\text{Hz}$

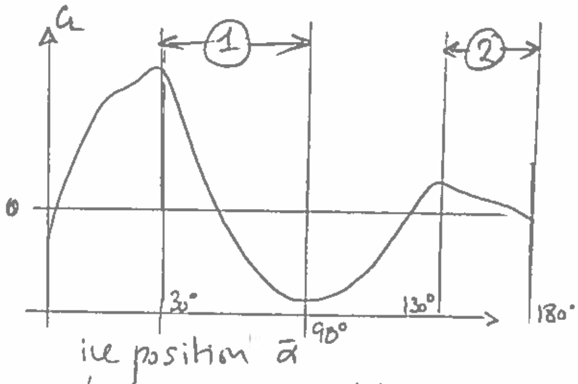
The natural frequency of vortex shedding is too far from the observed frequency of oscillation (1.2 Hz) for the oscillations to be caused by lock in of vortex shedding and mechanical oscillation. For lock-in at 1.2 Hz a wind speed of $0.6\,\text{ms}^{-1}$ would be required. We know from experience that wind speeds of $0.6\,\text{ms}^{-1}$ do not cause lampposts to oscillate with amplitudes of $\pm 1\,\text{m}$.

ii) <u>Changes in the drag coefficient with Reynolds number</u> The Reynolds number is significantly less than 2×10^5, which is the value at which dC_D/dRe is strongly negative. (Note that $F \sim Re^2 C_D$ so, for dF/dv to be negative, $d(Re^2 C_D)/dRe$ must be negative. This could only occur at the steep drop of $C_D(Re)$ around $Re = 2 \times 10^5$.) For a lamppost with diameter 0.1 m, a wind speed of $40\,\text{ms}^{-1}$ (140 km/hr) would be required for this mechanism to be active. This is above hurricane speed. For a lamppost with diameter 0.2m, a 70 km/hr wind would be required, which is unlikely given the cars on the road, but not impossible.

ii) <u>Galloping</u> There is ice on the road, so ice may well have formed on the lampposts. This will cause galloping if the fluid causes a force in the direction of motion of the lamppost. This can only occur if $dF_y/d\alpha > 0$, when $F_y = \frac{1}{2}\rho \bar{V}^2 D C_y$



and $C_y \simeq -C_L$. Therefore it can occur if $dC_L/d\alpha < 0$. From the chart of $C_L(\bar{\alpha})$ provided, this can occur in regions ① or ②. Given the seemingly moderate wind speed, the ice on the road, the relatively high chance that any ice formed would lie in a position that causes gallop,



and the fact that lampposts never seem to oscillate like that on non-freezing days, gallop is the most likely explanation.

Matthew Juniper.

4 a) U represents convection of perturbations by the mean flow, where U is the convection velocity.

ν represents diffusion of perturbations, where ν is the diffusion coefficient

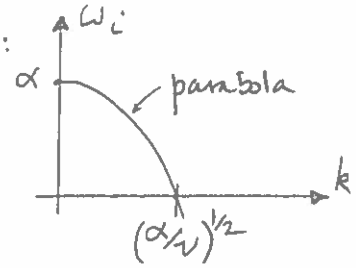
α bears no relation to p/ρ in the Navier-Stokes equations. It is a parameter that drives growth or decay of perturbations. E.g. if $\alpha > 0$ then this term is destabilizing, as can easily be seen by setting $U = 0$ and $\alpha = 0$.

b) Substitute $\phi = \phi_0 e^{i(kx - \omega t)}$ into $\dfrac{\partial \phi}{\partial t} + U \dfrac{\partial \phi}{\partial x} - \nu \dfrac{\partial^2 \phi}{\partial x^2} = \alpha \phi$

$\Rightarrow \phi_0 = 0$ or $\omega - Uk + i(\nu k^2 - \alpha) = 0$

$\Rightarrow \omega = Uk + i(\alpha - \nu k^2)$, which is the most convenient form.

c) The temporal growth rate, ω_i, equals $(\alpha - \nu k^2)$:

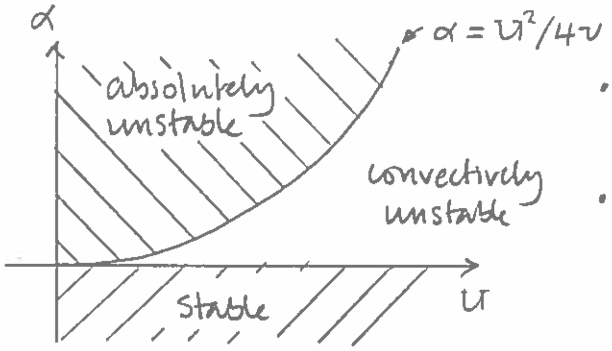


d) The flow is unstable if $\omega_i > 0$ for any k.
The flow is therefore unstable if $\alpha > 0$, for any values of U and ν.

e) The group velocity, $\partial\omega/\partial k$, is equal to zero when $\partial\omega/\partial k = U - 2ik\nu = 0$

$\Rightarrow k_0 = \dfrac{U}{2i\nu} = -\dfrac{U}{2\nu} i$ is the absolute complex wavenumber.

$\Rightarrow \omega_0 = U k_0 + i(\alpha - \nu k_0^2) = i\alpha - \dfrac{U^2}{2\nu} i + i\dfrac{U^2}{4\nu} = i\left(\alpha - \dfrac{U^2}{4\nu}\right) =$ abs. complex frequency



- The flow is absolutely unstable if $\omega_{0i} > 0$, which occurs if $\alpha > U^2/4\nu$.
- The flow is stable if $\alpha < 0$ for any U

<u>Physical Insight</u> The parameter α is the only parameter that determines whether or not a flow is unstable, in this model. (This is not particularly useful.) The main physical insight from the behaviour of the Ginzburg-Landau equation comes from considering the influences of U and ν. For small U^2/ν (ie. small convection velocity / large diffusion) the flow tends to be absolutely unstable. For large U^2/ν it tends to be convectively unstable. There is a competition between convection of perturbations away from their source (driven by U^2) and diffusion of perturbations both upstream and downstream (driven by ν). Put simply, the flow will be absolutely unstable if $\alpha > 0$ and if perturbations are not quickly swept away by the mean flow.

Matthew Juniper

Q1
Not a very popular question. This question focused on vortex-induced vibration of a structure. Surprisingly, students performed less well on the descriptive, than on the analytical, aspects of the question.

Q2
A popular question on the use of energy arguments in the study of the stability of a capillary jet that was tackled well by most. Students performed, in general, less well where required to provide physical explanations to clarify workings – on the whole, no explanations were offered.

Q3
The first part of this question asked the candidates to explain three physical mechanisms for the oscillations of lamposts on a frozen motorway. This was reasonably well answered by most candidates. Many candidates wrote around the subject, rather than answering the exact question, which did not gain them any marks.

The second part of this question asked the candidates to deduce which mechanism is responsible for the oscillation. Around one quarter of candidates answered this reasonably well, but most answered very briefly without much content. A good approach is to estimate the Reynolds number of the flow and hence the Strouhal number of vortex shedding, which is much larger than the observed vortex shedding frequency.

Q4
The first half of this question required knowledge of the course and some elementary algebra. This half was well-answered by almost every candidate. The second half required the candidates to derive the absolute complex wavenumber and frequency, sketch a graph of the convectively and absolutely unstable regions and then to comment on the physical insight that this gives. Most candidates derived the absolute complex wavenumber and frequency. Only around half the candidates sketched the graph well, however. Only three commented meaningfully on the physical insight.