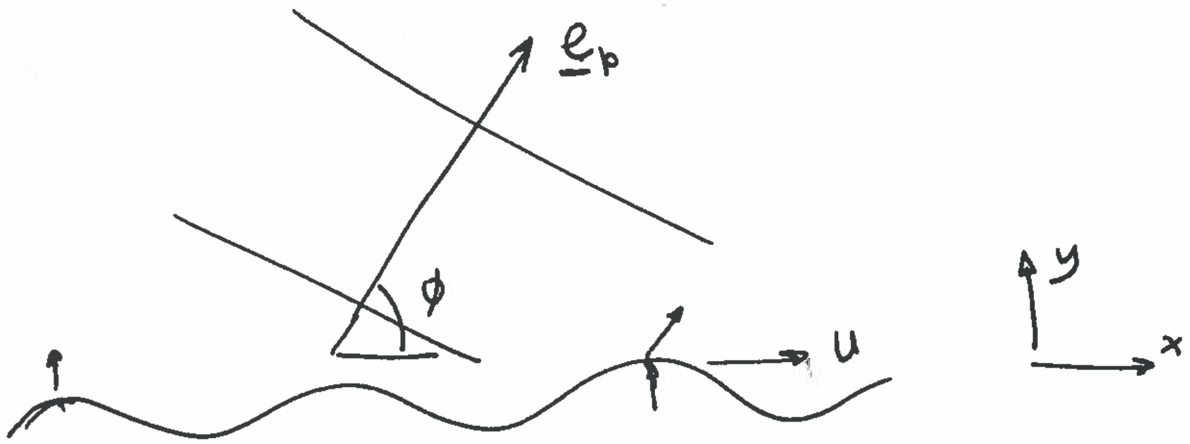
1.



(a) The acoustic potential field satisfies the wave eqn:

$$\left(\frac{1}{c_0^2} \frac{\partial^2}{\partial t^2} - \nabla^2 \right) \phi' = 0 \qquad -(1)$$

Let $\phi'(\underline{x}, t) = f(y)\, e^{i\omega\left(t - \frac{x}{U}\right)}$ $\quad -(2)$

where U = speed of the surface wave.

The boundary condition at $y=0$ is:

$$\left| \frac{\partial \phi'}{\partial y} \right|_{y=0} = V_0 = 1 \times 10^{-2} \text{ m/s}$$

~~Plugging (2) into~~

~~The direction of propagation~~ is

~~Take~~ (

Plugging (2) into (1) gives

$$\left(-\frac{\omega^2}{c_0^2}+\frac{\omega^2}{u^2}\right)f - f'' = 0$$

$$\Rightarrow \quad f'' + \left(\frac{\omega^2}{c_0^2}-\frac{\omega^2}{u^2}\right)f = 0$$

(Assuming that the incoming wave is zero).

$$f = Ae^{-i\omega\sqrt{\frac{1}{c_0^2}-\frac{1}{u^2}}\cdot y}$$

~~The p~~

$$f = Ae^{-i\frac{\omega}{c_0}\sqrt{1-\frac{1}{M^2}}\cdot y}.$$

$$M = \frac{u}{c_0} = 8$$

$$= A\,e^{-i\frac{\omega}{c_0}\sqrt{1-\frac{1}{64}}\cdot y}.$$

~~The direction of propa~~

$$\therefore \quad \phi'(\underline{x},t) = e^{i\omega\left(t-\frac{x}{u}-\frac{1}{c_0}\sqrt{1-\frac{1}{M^2}}\cdot y\right)}$$

$\therefore$ The direction of propagation is

$$\tan^{-1}\left(\frac{\frac{1}{c_0}\sqrt{1-\frac{1}{M^2}}}{\frac{1}{u}}\right) = \tan^{-1}\left(M\sqrt{1-\frac{1}{M^2}}\right)$$

$$= \tan^{-1}\left(\sqrt{M^2-1}\right.$$

(15)

$$\therefore \quad \phi = \cancel{82}\ 82.82^\circ$$

(b) $\underline{I} = \cancel{p v^*}\ \frac{1}{2} Re\{ p \underline{v}^* \}$

$$\underline{V}' = \frac{p'}{\rho_0 c_0} \underline{e}_\phi$$

$$\therefore \quad p' = -\rho_0 \frac{\partial \phi}{\partial t}$$

$$= -\rho_0 i \omega A \underbrace{\cancel{\;}\; E}$$

Exponential terms.

$$\therefore \quad \frac{1}{2} Re\{ p' \underline{v}'^* \} = \frac{1}{2} \frac{\rho_0^{\cancel{2}} \omega^2 A^2}{\cancel{\rho_0} c_0} \underline{e}_\phi$$

$$\Rightarrow \quad I = \frac{1}{2} \times 1.2 \times . \frac{(2\pi \times f)^2}{c_0} A^2 \quad \underline{e}_\phi .$$

to get A: $\quad A \sqrt{1 - \frac{1}{M^2}} \frac{\omega}{c_0} = V_0 .$

$$\therefore \quad A = \frac{V_0 c_0}{\omega \sqrt{1 - \frac{1}{M^2}}}$$

$$\therefore \quad I = \frac{1}{2} \frac{\rho_0 \cancel{\omega^2}}{\cancel{c_0}} \frac{V_0^2 c_0^{\cancel{2}}}{\cancel{\omega^2} (1 - \frac{1}{M^2})}$$

$$= \frac{1}{2} \rho_0 c_0 \frac{V_0^2}{(1 - \frac{1}{M^2})} = \frac{1}{2} \times 1.2 \times 340 \times \frac{(0.01)^2}{(1 - \frac{1}{M^2})}$$

$$I = 2.1 \times 10^{-3} \; N m^{-1} s^{-1} \cdot e_p$$

(c) $\rho_0 |V'|^2$ = constant analogy the plane wave.

$$\therefore \; \cancel{\rho_0} \frac{V_0^2}{\left(1 - \frac{1}{M^2}\right)} = 10^{-8} \cancel{\rho_0} V_s^2$$

$$\therefore \; V_s = \frac{V_0}{\sqrt{1 - \frac{1}{M^2}}} \cdot 10^4$$

$$= 100.8 \; m/s$$

2.

(a) $\frac{\partial}{\partial t}\left(\frac{\partial \rho'}{\partial t} + \rho_0 \nabla \cdot \underline{u}' = \mu(\underline{x}, t) \right)$

$- \nabla \cdot \left(\rho_0 \frac{\partial \underline{u}'}{\partial t} - \nabla p = 0 \right)$

$\Rightarrow \quad \frac{\partial^2 \rho'}{\partial t^2} - \nabla^2 p' = \dot{\mu}(\underline{x}, t)$

$p' = c_0^2 \rho'$

or $\quad \underline{\frac{\partial^2 \rho'}{\partial t^2} - c_0^2 \nabla^2 \rho' = \dot{\mu}(\underline{x}, t)}$

(b) Using the Green's f^n for the wave eq^n:

$\rho' = g(\underline{x}, t | \underline{y}, \tau) * \dot{\mu}(\underline{x}, t)$

$\therefore \rho' = \iint \frac{\delta\{|\underline{x} - \underline{y}| - c_0(t - \tau)\}}{4\pi c_0 |\underline{x} - \underline{y}|} \cdot \dot{\mu}(\underline{y}, \tau) \, d\underline{y} \, d\tau$

Far field $\Rightarrow \frac{1}{|\underline{x} - \underline{y}|} \sim \frac{1}{|\underline{x}|}$ (Assuming compact)

$\therefore \quad \rho' = \frac{1}{4\pi c_0 |\underline{x}|} \int \frac{1}{c_0} \, \dot{\mu}\left(\underline{y}, t - \frac{|\underline{x} - \cancel{\underline{y}}|}{c_0} \right) d\underline{y}$

$$\therefore \rho' = \frac{1}{4\pi c_0^2 |\underline{x}|} \int \dot{\mu}\left(\underline{y}, t - \frac{x}{c_0}\right) d\underline{y}$$

Using $\dot{m} = \int_V \dot{\mu}(\underline{y}, t)\, d\underline{y}$

$$\dot{m}\left(t - \frac{|x|}{c_0}\right) = \int_V \dot{\mu}\left(\underline{y}, t - \frac{|x|}{c_0}\right)$$

$$\therefore \rho' = \frac{1}{4\pi c_0^2 r} \ddot{m}\left(t - \frac{r}{c_0}\right) \qquad (r = |\underline{x}|)$$

$$p' = c_0^2 \rho'$$

$$\Rightarrow p' = \frac{1}{4\pi r} \ddot{m}\left(t - \frac{r}{c_0}\right)$$

(c) $\dot{m}(t) = \rho_0 Q\,(1 + \cos(2\pi f t))$

$$\ddot{m} = -\rho_0 Q\, 2\pi f \sin(2\pi f t)$$

$$p'_{rms} = \frac{1}{\cancel{4}\cancel{\pi} r} \rho_0 Q \frac{\cancel{2}\cancel{\pi} f}{\sqrt{2}}$$

l/min → m^3/s

$$= \frac{1.2 \times}{10} \; 12 \times \frac{10^{-3}}{60} \times \frac{261.6}{2\sqrt{2}}$$

$$= 2.22 \times 10^{-3} \; N/m^2$$

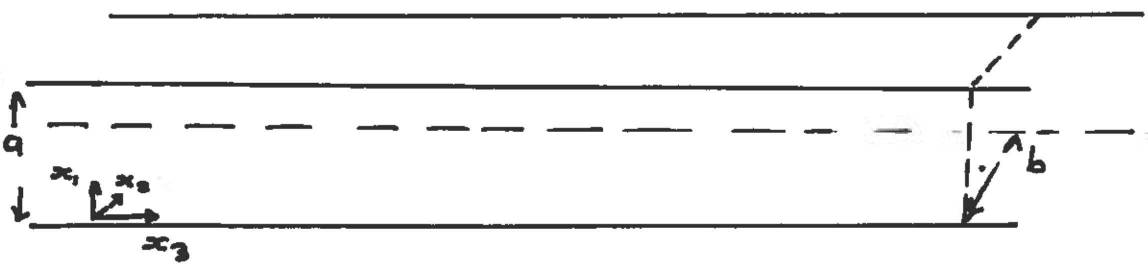
$$\therefore \quad SPL = 20\log\left(\frac{p_{rms}}{2\times10^{-5}}\right) \; dB$$

$$= 20\log\left(\frac{2.22\times10^{-3}}{2\times10^{-5}}\right)$$

$$= 40.1 \; dB$$

(c) We get only the dipole sound because we don't get the monopole sound from mass pulasations. ~~the~~ Diapole pressure field is $O(M)$ smaller that the monopole field. Here $M \ll 1$. So we would expect to get very little sound.

S (1)



(i) Seeking a separable solution of frequency ω of the form

$$p'(\underline{x},t) = e^{i\omega t} g(x_3)\, f(x_1)\, h(x_2)$$

leads to $\frac{f''}{f} = \text{constant} = -k_1^2$ say, where $'$ denotes a derivative with respect to the argument.

This has solutions of the form $f(x_1) = A\cos(k_1 x_1) + B\sin(k_1 x_1)$

but the rigid wall boundary condition of zero normal velocity and therefore $\frac{\partial p_1}{\partial x_1} = 0$ on $x_1 = 0$ and a

leads to $B=0$ and $k_1 = \frac{m\pi}{a}$ for integer m.

Similar reasoning gives $h(x_2) = A_2 \cos\left(\frac{n\pi x_2}{b}\right)$.

and hence the modes have the form

$$p'(\underline{x},t) = e^{i\omega t} g(x_3) \cos\left(\frac{m\pi x_1}{a}\right) \cos\left(\frac{n\pi x_2}{b}\right) \qquad (1) \qquad [10\%]$$

(ii) $g(x_3)$ is found by substituting from equation (1) into the convective wave equation

$$-\left(\frac{m^2\pi^2}{a^2} + \frac{n^2\pi^2}{b^2}\right) g + \frac{d^2g}{dx_3^2} - \left(-k_0^2 g + 2Mik_0 \frac{dg}{dx_3} + M^2\frac{d^2g}{dx_3^2}\right) = 0$$

$$(1-M^2)\frac{d^2g}{dx_3^2} - 2Mik_0\frac{dg}{dx_3} + \left(k_0^2 - \frac{m^2\pi^2}{a^2} - \frac{n^2\pi^2}{b^2}\right) g = 0.$$

Try a solution of the form $g(x_3) = A e^{ikx_3}$. This gives an equation for k. A can be any constant.

$$-k^2(1-M^2) + 2Mk_0k + \left(k_0^2 - \frac{m^2\pi^2}{a^2} - \frac{n^2\pi^2}{b^2}\right) = 0$$

i.e. $k^2(1-M^2) - 2Mk_0k - \left(k_0^2 - \frac{m^2\pi^2}{a^2} - \frac{n^2\pi^2}{b^2}\right) = 0$

$$k_\mp = \frac{k_0 M \mp \sqrt{M^2k_0^2 + (1-M^2)\left(k_0^2 - \frac{m^2\pi^2}{a^2} - \frac{n^2\pi^2}{b^2}\right)}}{1-M^2}$$

$$k_\mp = \frac{k_0 M \mp \sqrt{k_0^2 - (1-M^2)\left(\frac{m^2\pi^2}{a^2} + \frac{n^2\pi^2}{b^2}\right)}}{1-M^2} \qquad (2)$$

The solution for the mode of frequency ω is therefore

$$p'(\underline{x},t) = e^{i\omega t}\left(Ae^{ik_+x} + Be^{ik_-x}\right)\cos\left(\frac{m\pi x_1}{a}\right)\cos\left(\frac{n\pi x_2}{b}\right) \quad [60\%]$$

where A and B are arbitrary constants and k_+ and k_- are defined in equation (2)

(iii) When $m=n=0$, the wave is plane (i.e. independent of x_1 & x_2)

$$k_{\mp} = \frac{k_0 M \mp k_0}{1-M^2}$$

$$k_+ = \frac{k_0(M+1)}{1-M^2} = \frac{k_0}{1-M} = \frac{\omega}{c_0-\bar{U}}$$ ← describes a wave travelling upstream with speed $c_0-\bar{U}$

$$k_- = \frac{k_0(M-1)}{1-M^2} = -\frac{k_0}{1+M} = -\frac{\omega}{c_0+\bar{U}}$$ ← describes a wave travelling downstream with speed $c_0+\bar{U}$

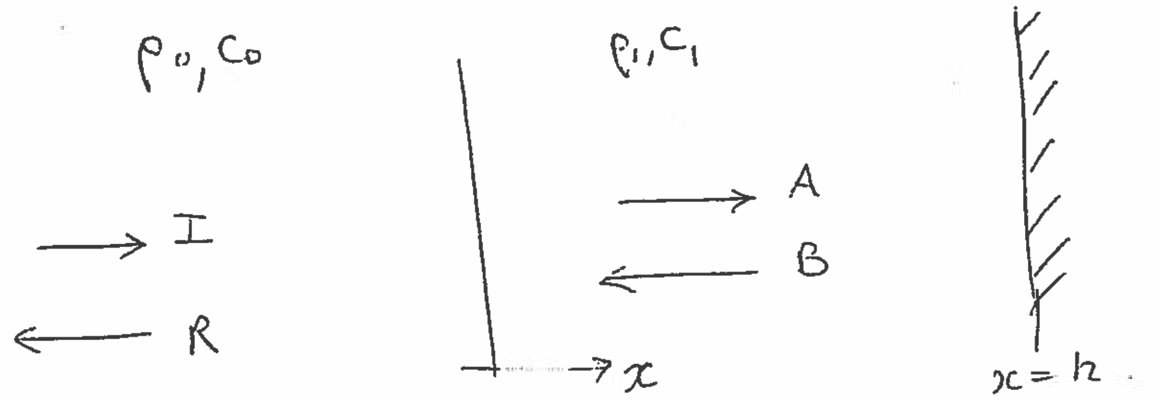
where $\bar{U} = c_0 M$ [15%]

iv) Modes are cut-off if

$$k_0 < (1-M^2)^{1/2}\left(\frac{m^2\pi^2}{a^2} + \frac{n^2\pi^2}{b^2}\right)^{1/2}.$$

i.e. $$\omega < c_0(1-M^2)^{1/2}\pi\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)^{1/2}$$ [15%]

(i)



(a)

In $x < 0$ $\quad p = I e^{-ik_0 x + i\omega t} + R e^{ik_0 x + i\omega t}$ $\quad k_0 = \omega/c_0$

$0 < x < h$ $\quad p = A e^{-ik_1 x + i\omega t} + B e^{ik_1 x + i\omega t}$ $\quad k_1 = \omega/c_1$

pressure → normal velocity? $\quad p = -\rho \frac{\partial \phi}{\partial t} = -i\rho\omega\phi$

$$v = \frac{\partial \phi}{\partial x} = \frac{\partial}{\partial x}\left(\frac{ip}{\rho\omega}\right)$$

∴ in $x < 0$,

$$v = \frac{i}{\rho_0 \omega}\left\{-ik_0 I e^{-ik_0 x + i\omega t} + ik_0 R e^{ik_0 x + i\omega t}\right\}$$

$$v = \frac{1}{\rho_0 c_0}\left\{I e^{-ik_0 x + i\omega t} - R e^{ik_0 x + i\omega t}\right\}$$

in $x > 0$

$$v = \frac{1}{\rho_1 c_1}\left\{A e^{-ik_1 x + i\omega t} - B e^{ik_1 x + i\omega t}\right\}$$

continuity of pressure across $x = 0$

$$I + R = A + B \quad \text{.......... (1)}$$

continuity of normal velocity across $x = 0$

$$\frac{I - R}{\rho_0 c_0} = \frac{A - B}{\rho_1 c_1} \rightarrow \rho^*(I - R) = A - B \quad \text{\_\_(2)}$$

$$\rho^* = \frac{\rho_1 c_1}{\rho_0 c_0}$$

Impedence on $x = h$

$$Ae^{-ik_1h} + Be^{ik_1h} = \frac{Z}{\rho c_1}\left\{Ae^{-ik_1h} - Be^{ik_1h}\right\}$$

$$Be^{ik_1h}(1+z^*) = Ae^{-ik_1h}(z^*-1) \qquad z^* = \frac{Z}{\rho_1 c_1}$$

$$\therefore A = \frac{Be^{2ik_1h}}{\frac{(1+z^*)}{z^*-1}}$$

$$(1) \rightarrow I+R = B\left\{1 + e^{2ik_1h}\frac{(z^*+1)}{z^*-1}\right\}$$

$$(2) \rightarrow (I-R)\rho^* = B\left\{-1 + e^{2ik_1h}\frac{(z^*+1)}{z^*-1}\right\}$$

dividing

$$\frac{I+R}{(I-R)\rho^*} = \frac{z^*-1+e^{2ik_1h}(z^*+1)}{-z^*+1+e^{2ik_1h}(z^*+1)}$$

$$I\left\{-z^*+1+e^{2ik_1h}(z^*+1) - \rho^*(z^*-1) - \rho^* e^{2ik_1h}(z^*+1)\right\}$$

$$= R\left[+(z^*-1) + e^{2ik_1h}(z^*+1) - \rho^*(z^*-1) - \rho^* e^{2ik_1h}(z^*+1)\right]$$

$$\frac{R}{I} = \frac{(1-z^*)(1+\rho^*) + (1+z^*)(1-\rho^*)e^{2ikh}}{(1-z^*)(\rho^*-1) - (1+z^*)(1+\rho^*)e^{2ikh}}$$

(b) $z^* = 1$ ⑧

$$\frac{R}{I} = \frac{2(1-\rho^*)e^{2ik_1h}}{-2(1+\rho^*)e^{2ik_1h}}$$

$$\therefore \frac{R}{I} = \frac{\rho^*-1}{\rho^*+1}.$$

$$|R/I| = \left|\frac{\rho^*-1}{\rho^*+1}\right|$$

$$\text{phase}(R/I) = 0 \quad \text{if } \rho_1 c_1 > \rho_0 c_0$$
$$= \pi \quad \text{if } \rho_1 c_1 < \rho_0 c_0$$

$$F = |I|^2\left\{1 - \frac{(\rho^*-1)^2}{(\rho^*+1)^2}\right\} = \frac{4|I|^2\rho^*}{(\rho^*+1)^2} \quad [2]$$

not all energy is reflected. impedance of wall matches impedance of fluid in $x>0$, so same result as if wall were absent.

Q1 Sound propagation from earthquakes: the wavy-wall problem

Although the problem was very similar to a worked example in class, this question was unpopular. There were only 4 attempts. This could have been because the new application of the problem and the descriptive nature of the questions, particularly part (c).

Q2 Sound radiation from a free reed: monopole vs dipole sources

In theory this was the most difficult question requiring complex mathematics. But this was a popular question with the highest average mark. The students had some the background maths in multiple example questions and coped well with the new practical application. There was some confusion with the physical interpretation in the final part of the question. The demonstration in one of the lectures should have made it easy. Those who attended the lecture did well.

Q3 Duct acoustics

This was a popular question. Most candidates knew how to approach all parts of the question and most gave very good answers, as demonstrated by the high average mark. In spite of that, only a few where able to fully interpret the plane wave result obtained in part c) as describing plane waves travelling with speed c 0 (±1+M). A few thought that cut-off modes required Real(k)=0, rather thanjust Imag(k)≠0.

Q4 Sound transmission and reflection

This question was on the whole very well done, with candidates able to identify the correct waves in each region and state the appropriate boundary conditions at each interface. The algebraic manipulations required to derive the reflection coefficient were also well handled. Many candidates were able to make sensible comments about the physical interpretation of the limit considered.