1. Data from the question
   $p_{01} = 1 \times 10^5\,\mathrm{Pa}$
   $T_{01} = 300\,\mathrm{K}$
   $\mathrm{M}_{1rel} = 0.65$
   $Y_p = 0.035$

   (a) **Mass-flow per unit frontal-area**

   $$\frac{\dot{m}}{A_x} = \frac{\dot{m}\cos(\alpha_1)}{A_1}$$

   using the compressible flow tables

   $$\frac{\dot{m}\sqrt{c_p T_{01}}}{A_1 p_{01}} = f(\mathrm{M} = 0.65)$$

   so

   $$\frac{\dot{m}}{A_x} = f(\mathrm{M} = 0.65)\frac{p_{01}}{\sqrt{c_p T_{01}}}\cos\alpha_1$$

   giving

   $$\frac{\dot{m}}{A_x} = 1.128\frac{1 \times 10^5}{\sqrt{1005 \times 300}}\cos 47^\circ = 140.1\mathrm{kg\,s^{-1}\,m^{-2}}$$

   Total pressure ratio

   $$\frac{p_{02}}{p_{01}} = 1 - Y_p\left(1 - \frac{p_1}{p_{01}}\right)$$

   from the tables

   $$\frac{p_1}{p_{01}} = 0.7528 \quad \text{so} \quad \frac{p_{02}}{p_{01}} = 1 - 0.035\,(1 - 0.7528) = 0.9913$$

   Exit Mach number. The endwall boundary layers reduce the area such that

   $$1.1A_{2x} = A_{1x}$$

   and we can say that $T_{01} = T_{02}$ so

   $$\frac{\dot{m}\sqrt{c_p T_{02}}}{A_2 p_{02}} = \frac{\dot{m}\sqrt{c_p T_{01}}}{A_1 p_{01}}\frac{p_{01}}{p_{02}}\frac{A_1}{A_2} = 1.128\frac{1}{0.9913}\frac{1.1A_{1x}\cos 47^\circ}{A_{1x}\cos 18^\circ} = 0.8975$$

   which gives $\mathrm{M}_2 = 0.46$ using the tables.

   Inlet Mach number $\mathrm{M}_{1rel} = 0.65$, exit Mach number $\mathrm{M}_{2rel} = 0.459$. This gives a ratio of 0.46/0.65 = 0.71. The usual target from De Haller is a velocity ratio of 0.72. As such this is a fairly high diffusion blade, and might be prone to separation and stall in practice.

(b)

$$V_1 = \mathrm{M}_1\sqrt{\gamma R T_1}, \quad \gamma R = (\gamma-1)c_p, \quad c_p T_1 = c_p T_{01} - \frac{1}{2}V_1^2$$

$$V_1^2 = \mathrm{M}_{1rel}^2 \cos^2 47^\circ \left((\gamma-1)\, c_p T_{01} - \frac{(\gamma-1)}{2} V_1^2 \right)$$

so

$$V_1^2 = \frac{0.65^2 \cos^2 47^\circ \times 0.4 \times 1005 \times 300}{1 + 0.65^2 \cos^2 47^\circ \frac{0.4}{2}} \quad \text{and} \quad V_1 = 151\,\mathrm{m\,s^{-1}}$$

As such

$$U_1 = V_1 \tan 47^\circ = 161.9\,\mathrm{m\,s^{-1}}$$

[3]

Using the SFEE

$$T_{0,rel} = T_{01} + \frac{1}{2}\frac{U_1^2}{c_p} = 300 + \frac{161.9^2}{2 \times 1005} = 313\,\mathrm{K}$$

[1]

(c) The rotor can be treated just like the stator as there's no change in radius, and the AVDR is the same.

$$\mathrm{M}_{2,rel} = 0.46 \quad \text{and} \quad T_{01,rel} = T_{02,rel}$$

so, using the tables

$$\frac{T_2}{T_{02,rel}} = 0.9594 \quad \mathit{so} \quad T_2 = 300.3\mathrm{K}$$

$$V_{2,rel} = \sqrt{2c_p\left(T_{02,rel} - T_2\right)} = \sqrt{2 \times 1005 \times 313\,(1 - 0.9594) =} \,159.8\ \mathrm{m\ s^{-1}}$$

and the angle

$$\alpha_2 = \tan^{-1}\left\{\frac{U_2 - V_{2,rel}\sin 18^\circ}{V_{2,rel}\cos 18^\circ}\right\} = \tan^{-1}\left\{\frac{161.9 - 159.8\sin 18^\circ}{159.8\cos 18^\circ}\right\} = 36.5^\circ$$

(d) Stage loading

$$\psi = \frac{\Delta h_0}{U^2} = \frac{\Delta V_\theta U}{U^2} = \frac{V_{\theta 2}}{U} = \frac{V_{x2}\tan 36.5^\circ}{U}$$

$$V_{x2} = V_{2,rel}\cos 18^\circ = f\left(\mathrm{M}_{2,rel}\right)\sqrt{c_p T_{02,rel}}\cos 18^\circ = 0.2850\sqrt{1005 \times 313.0}\cos 18^\circ = 152\,\mathrm{m\,s^{-1}}$$

so

$$\psi = \frac{152\tan 36.5^\circ}{161.9} = 0.695$$

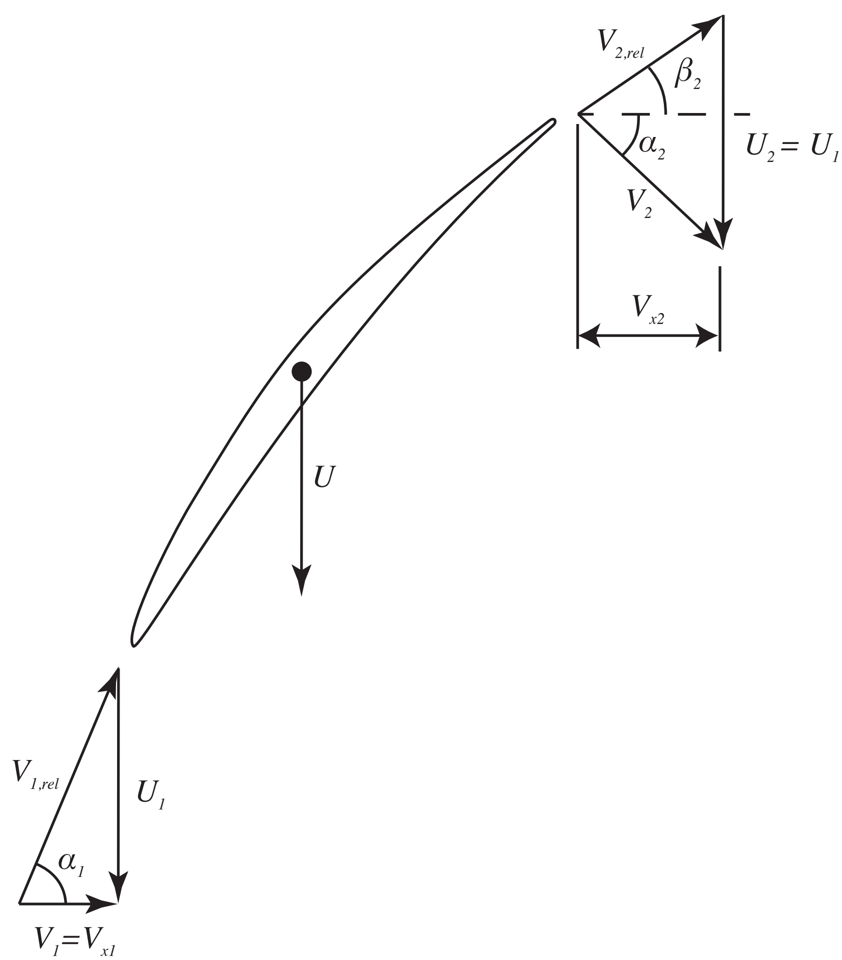


Figure 1: Velocity Triangles

2. (a) Measurements required to derive a mass averaged loss coefficient

$$Y_p = \frac{\int_0^s \frac{p_{01}-p_{02}}{p_{01}-p_1}\rho V_x}{\int_0^s \rho V_x}\mathrm{d}y$$

Measurements required for loss coefficient:

- Inlet total pressure survey, although this should be uniform for a well designed wind tunnel. This would need a Pitot tube.
- Static pressure measured with wall tappings.
- Temperature measurement to get the density, using a thermocouple or RTD.

Measurements of exit total pressure, p_{02}, static pressure, p_2 and flow angle, α_2 needs an aerodynamic probe. Options include a cobra probe + a pitot tube, or a three-hole probe. Both of these need a *calibration* at the relevant free stream Reynolds and Mach number. The yaw angle, α_2 can be measured using the calibration, or by nulling at each measurement location.

In order to measure the variation of loss and deviation with Mach number at constant Reynolds number (for a fixed geometry), both the inlet *and* exit pressures need to be adjusted. To reduce the Reynolds number at constant Mach number, the inlet temperature can be increased.

(b) Data: $p_{01} = 1 \times 10^5$ Pa, $\mathrm{M}_{exit} = \mathrm{M}_2 = 1.3$, $Y_p = 0.1$.
From the tables, at M = 1.3,

$$\frac{p_{02}}{p_2} = \frac{1}{0.3609}$$

Rearrange the Y_p expression to get (with $T_{01} = T_{02}$)

$$\frac{p_{01}}{p_{02}} = Y_p\left(1 - \frac{p_2}{p_{02}}\right) + 1 = 0.1(1 - 0.3609) + 1 = 1.06391$$

Using continuity

$$f(M_1) = \frac{\dot{m}\sqrt{c_p T_{01}}}{A_1 p_{01}} = \left(\frac{\dot{m}\sqrt{c_p T_{02}}}{A_2 p_{02}}\right)\frac{p_{02}}{p_{01}}\frac{A_2}{A_1} = \left(\frac{\dot{m}\sqrt{c_p T_{02}}}{A_2 p_{02}}\right)\frac{p_{02}}{p_{01}}\frac{hs\cos 70°}{hs\cos 40°}$$

so

$$f(M_1) = 1.2014\frac{1}{1.06391}\frac{\cos 70°}{\cos 40°} = 0.5042$$

so

$$M_1 = 0.235$$

(c) Two thirds of the total pressure loss occurs downstream of the throat plane (*).

$$\begin{aligned} p_0^* &= p_{01} - \frac{1}{3}\left[p_{01} - p_{02}\right] \\ &= p_{01}\left[1 - \frac{1}{3}\left(1 - \frac{p_{02}}{p_{01}}\right)\right] \end{aligned}$$

so

$$p_0^* = p_{01}\left[1 - \frac{1}{3}\left(1 - \frac{1}{1.06391}\right)\right] = p_{01}0.980$$

so

$$\frac{p_0^*}{p_{01}} = 0.980$$

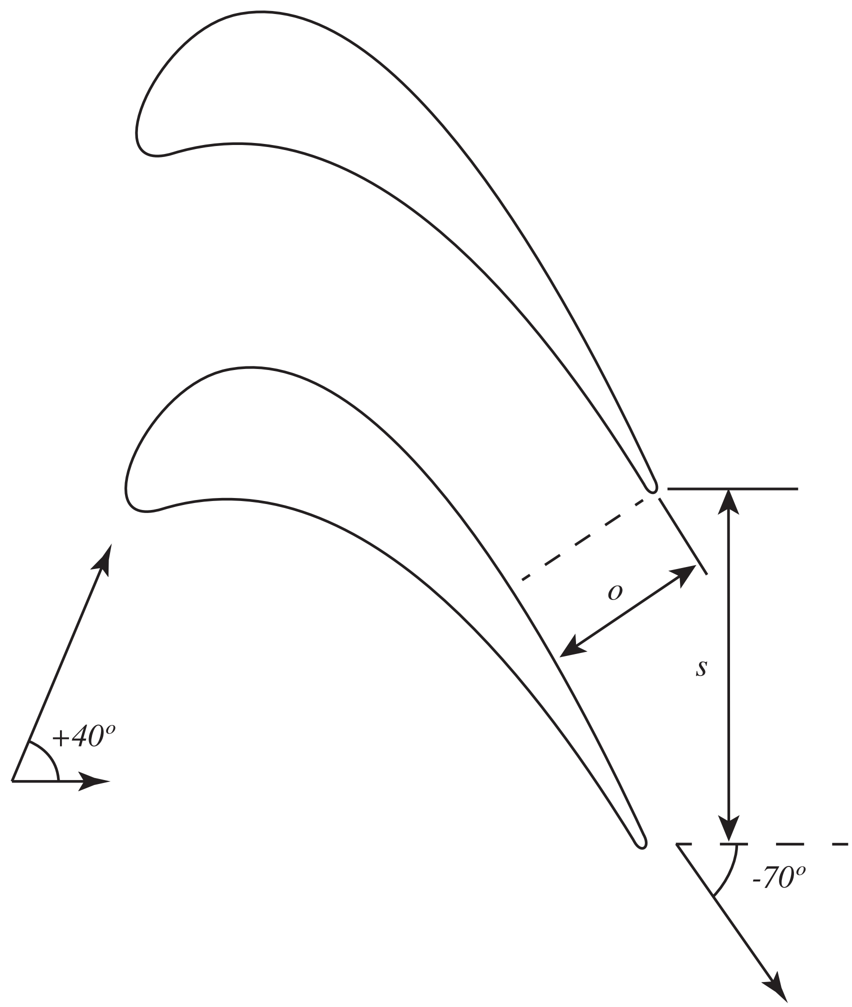


Figure 2: Velocity Triangles

from the tables

$$f(\mathrm{M}=1) = \frac{\dot{m}\sqrt{c_p T_0^*}}{hop_0*}$$

and

$$f(\mathrm{M}=1.3) = \frac{\dot{m}\sqrt{c_p T_{02}}}{hs\cos\alpha_2 p_{02}}$$

so

$$\frac{f(\mathrm{M}=1)}{f(\mathrm{M}=1.3)} = \frac{s\cos\alpha_2 p_{02}}{op_0*} = \frac{s\cos\alpha_2}{o}\left(\frac{p_{02}}{p_{01}}\frac{p_{01}}{p_0^*}\right)$$

so

$$\frac{o}{s} = \cos\alpha_2 \frac{f(\mathrm{M}=1.3)}{f(\mathrm{M}=1)}\left(\frac{p_{02}}{p_{01}}\frac{p_{01}}{p_0^*}\right) = \cos 70^\circ \left(\frac{1.2014}{1.281}\right)\left(\frac{1}{1.0426}\right) = 0.308$$

the first term is geometric, the second term is supersonic deviation, and the third term is the deviation due to loss.

the geometric term alone would only give

$$\frac{o}{s} = \cos\alpha_2 = 0.342$$

(d) Consider a control volume around the blade. The boundary between the blades is periodic, so the pressure and net momentum is balanced. As such

$$F_y = \dot{m}\left(+V_2 \sin 70^\circ + V_1 \sin 40^\circ\right)$$

so

$$\frac{F_y}{h} = \frac{\dot{m}\sqrt{c_p T_{02}}}{hs\cos\alpha_2 p_{02}}\left[+\frac{V_2 \sin 70^\circ}{\sqrt{c_p T_{02}}} + \frac{V_1 \sin 40^\circ}{\sqrt{c_p T_{01}}}\right] p_{02} s \cos\alpha_2$$

and so

$$\frac{F_y}{h} = f_1(\mathrm{M}=1.3)\left[f_2(\mathrm{M}=1.3)\sin 70^\circ + f_2(\mathrm{M}=0.235)\sin 40^\circ\right]\frac{1\times 10^5}{1.0639}0.05\cos 70^\circ$$

so

$$\frac{F_y}{h} = 1473\ \mathrm{N\,m^{-1}}\ \text{upwards on blade asdrawn.}$$

$$\frac{F_x}{h} = \frac{\dot{m}}{h}\Delta V_x + (p_1 - p_2)s$$

so

$$\frac{F_x}{h} = \frac{\dot{m}\sqrt{c_p T_{02}}}{hs\cos\alpha_2 p_{02}}\left[-\frac{V_2 \cos 70^\circ}{\sqrt{c_p T_{02}}} + \frac{V_1 \cos 40^\circ}{\sqrt{c_p T_{01}}}\right] p_{02} s \cos\alpha_2 + (p_1 - p_2)s$$

As before, the various terms can be found from the tables

$$\frac{F_x}{h} = -2864\ \mathrm{N\,m^{-1}}$$

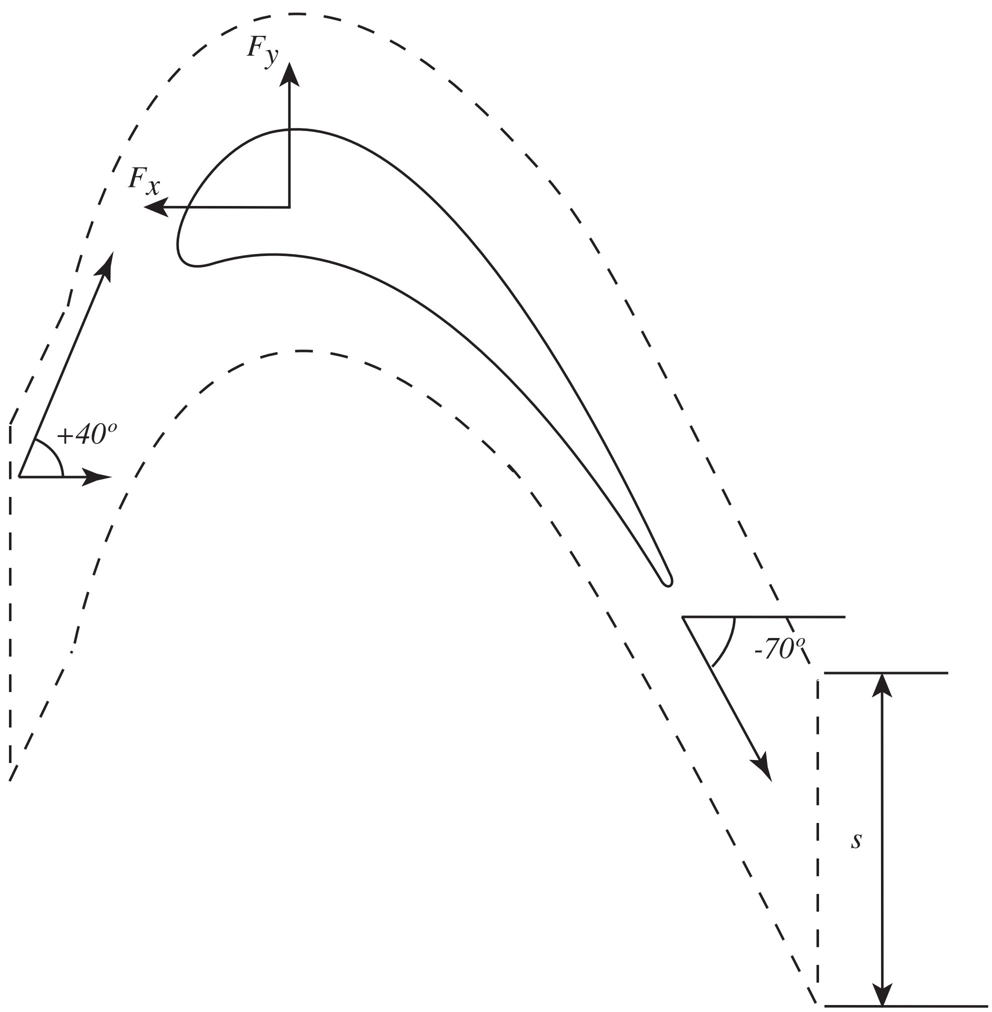


Figure 3: Control volume

4A3 Exam Paper Solutions

Q3 (a) The compressor isentropic efficiency at design is 86%.

$$\frac{T_{02}}{T_{01}} = 1 + \frac{1}{\eta_{is}}\left[\left(\frac{p_{02}}{p_{01}}\right)^{(\gamma-1)/\gamma} - 1\right] = 1 + \frac{1}{0.86}\left[8.3^{0.4/1.4} - 1\right] = 1.9658.$$

The polytropic efficiency is then given by:

$$\eta_p = \frac{\gamma-1}{\gamma}\frac{\ln\left(p_{02}/p_{01}\right)}{\ln\left(T_{02}/T_{01}\right)} = \frac{0.4}{1.4}\frac{\ln 8.3}{\ln 1.9658} = \underline{0.8946}. \quad (89.5\%)$$

[15%]

(b) Applying continuity between inlet ant exit,

$$\frac{\dot{m}\sqrt{c_p T_{01}}}{D^2 p_{01}} = \frac{\dot{m}\sqrt{c_p T_{02}}}{A_N p_{02}}\frac{A_N}{D^2}\frac{p_{02}}{p_{01}}\sqrt{\frac{T_{01}}{T_{02}}}$$

Applying the definition of polytropic efficieny

$$\frac{\dot{m}\sqrt{c_p T_{01}}}{D^2 p_{01}} = \frac{\dot{m}\sqrt{c_p T_{02}}}{A_N p_{02}}\frac{A_N}{D^2}\left(\frac{p_{02}}{p_{01}}\right)^{1-\frac{\gamma-1}{2\gamma\eta_p}}$$

Using the fact that the exit nozzle is choked, $\dfrac{\dot{m}\sqrt{c_p T_{02}}}{A_N p_{02}} = 1.281$, rearranging gives

$$\frac{p_{02}}{p_{01}} = \left(\frac{\dot{m}\sqrt{c_p T_{01}}}{D^2 p_{01}}\frac{D^2}{1.281 A_N}\right)^{\frac{2\gamma\eta_p}{(2\gamma\eta_p-\gamma+1)}}$$

Replacing constant terms with a single constant, *C*,

$$\Rightarrow \underline{\frac{p_{02}}{p_{01}} = C\left(\frac{\dot{m}\sqrt{T_{01}}}{p_{01}}\right)^{\frac{2\gamma\eta_p}{(2\gamma\eta_p-\gamma+1)}}}$$

[20%]

(c) Using conditions at the design point, the constant *C* = 8.3.

The equation of the working line on the characteristic is thus given by:

$$\frac{p_{02}}{p_{01}} = 8.3\left(\frac{\dot{m}\sqrt{T_{01}}}{p_{01}}\right)^{\frac{2\times1.4\times.895}{(2\times1.4\times.895-1.4+1)}} = 8.3\left(\frac{\dot{m}\sqrt{T_{01}}}{p_{01}}\right)^{1.19}$$

This can be plotted on the characteristic for various values of normalised $\dot{m}\sqrt{T_{01}}/p_{01}$

The working line is found to intersect the 80% speed line where

$$\underline{\frac{p_{02}}{p_{01}} = 4.933}, \quad \frac{\dot{m}\sqrt{T_{01}}}{p_{01}} = 0.644$$

For the first stage,

$$\phi = \frac{c_x}{U} \propto \frac{\dot{m}\sqrt{T_{01}}}{p_{01}} \bigg/ \frac{\Omega}{\sqrt{T_{01}}}$$

$$\therefore \frac{\phi_{80\%}}{\phi_{100\%}} = \left(\frac{\dot{m}\sqrt{T_{01}}}{p_{01}} \bigg/ \frac{\Omega}{\sqrt{T_{01}}}\right)_{80\%} \Bigg/ \left(\frac{\dot{m}\sqrt{T_{01}}}{p_{01}} \bigg/ \frac{\Omega}{\sqrt{T_{01}}}\right)_{100\%} = \frac{0.644}{80} \bigg/ \frac{1}{100} = 0.805$$

$$\Rightarrow \phi_{80\%} = 0.805 \times 0.5 = \underline{0.4}$$

The inlet relative flow angles at design (100%) and 80% speed are:

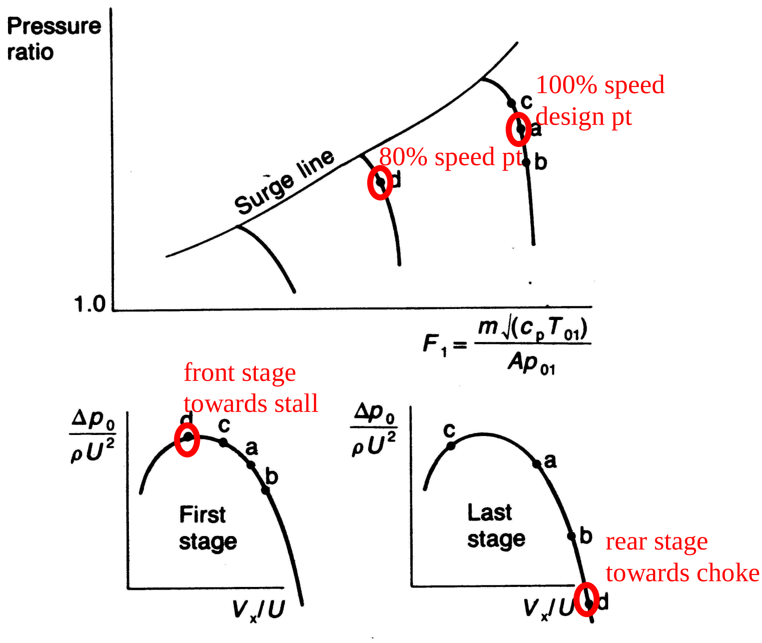
$$\beta_{100\%} = \tan^{-1}\left(\frac{1}{0.5}\right) = 63.43^0, \qquad \beta_{80\%} = \tan^{-1}\left(\frac{1}{0.4}\right) = 68.20^0$$

Thus, the incidence onto the front rotor at 80% speed = 68.2 - 63.43 = <u>4.8 deg</u>

As expected, the first rotor is at +ve incidence (towards stall) .

[30%]

(d)



At the design point (100% speed) all stages are at close to design incidence.

At 80% speed on the design working line, the front and rear stages will operate at different conditions simultaneously.

- Front stages of the compressor tend to operate towards stall.

At part speed, because the mass flow rate is greatly reduced, the incidence of the flow onto the front rotor blades increases (as shown in part (c)).

- Rear stages tend to operate towards choke.

At part speed, because the pressure ratio is low, the density in the rear stages is very low relative to design. This leads to high axial velocity, negative incidence and possible choking.

[20%]

(e) As the nozzle area is reduced all stages will operate at higher pressure ratios. This progressively increases the density above the design value through the machine. Hence, the last stages will have a much higher density than at design, a much lower axial velocity, and thus higher incidence onto the rotors. This will lead to stall initiating in the rear stages. As the blade height is small in the rear stages, the stall is likely to be full-span. If there is a significant plenum downstream, the pressure energy stored will also lead to surge.

[15%]