4A3: Turbomachinery I 2015
Exam Solutions

Q1)

$\frac{p_{02}}{p_{01}}$ = 4 [-], $\eta_{Comp} = 0.8$ [-], $T_{01} = 300$ K, $\alpha_2 = 70°$, $\chi_2 = -45°$, $\sigma = 0.85$

a) i) Stagnation temperature ratio from the isentropic relation

$$\frac{T_{02is}}{T_{01}} = \left(\frac{p_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}}$$

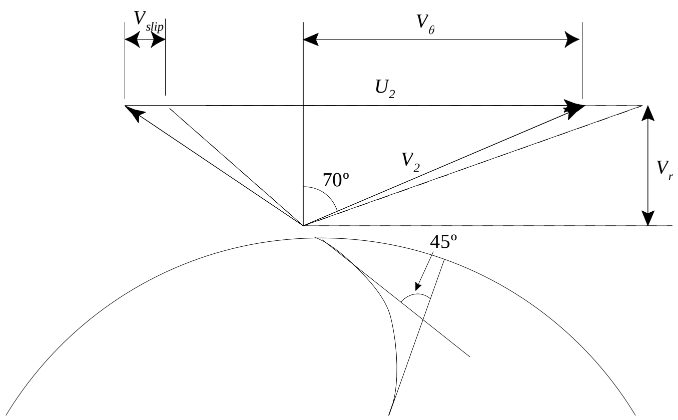
Isentropic temperature rise across impeller

$$\Delta T_{0is} = T_{01}\left(\frac{T_{02is}}{T_{01}} - 1\right)$$

Actual temperature at exit from the whole machine

$$\Delta T_0 = \frac{\Delta T_{0is}}{\eta_{Comp}} = 182.25 \text{ K and } T_{0exit} = 300 + 182.25 = \underline{\mathbf{482.3\ K}}$$

ii)



From the compressible flow tables, at:

$$M = 1.0,\ V = 0.5774\sqrt{c_p\, T_0}$$

There's no work in the diffuser, so the total temperature rise in the impeller is the same as for the whole machine (compressor + diffuser). So absolute speed of the blade tip is calculated from

$$V_2 = 0.5774\sqrt{1005 \times (T_{01} + \Delta T_0)}$$

Calculate blade speed from the definition of slip,

$$\sigma = \frac{V_{\theta 2}}{U_2 + V_{r2}\tan\chi_2}$$

(Remembering that backswept blades have a negative angle)

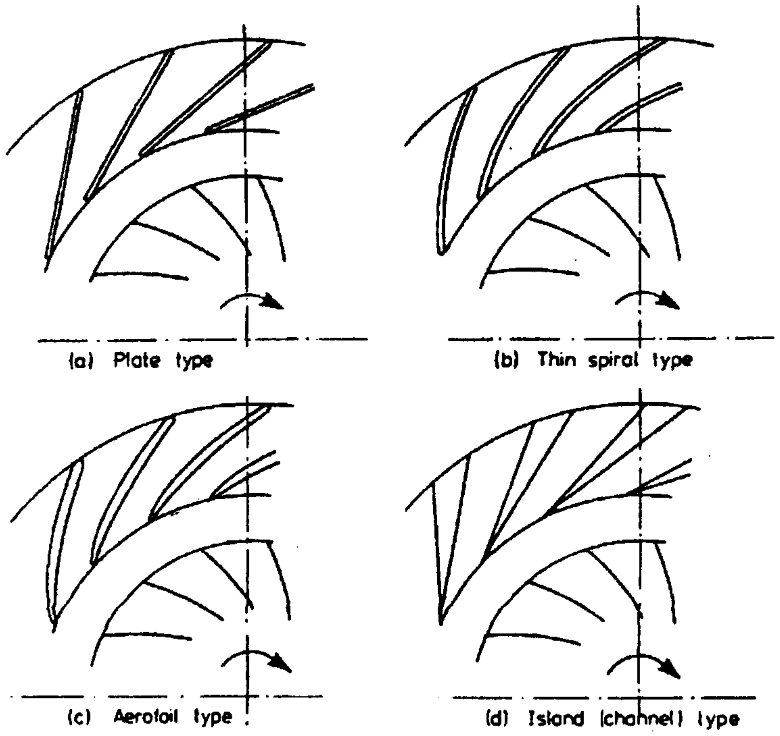
$$U_2 = V_2\left(\frac{\sin\alpha_2}{\sigma} - \cos\alpha_2 \tan\chi_2\right) = \underline{\mathbf{581.87\ ms^{-1}}}$$

and then the blade radius is

$$r_2 = \frac{U_2}{(25000/60)2\pi} = \underline{\mathbf{0.222\ m}}$$

iii) Slip can be reduced by increasing the number of blades, reducing the cross passage pressure gradient. However, increased blade numbers increase the wetted area and hence could actually lower the overall efficiency. Increased blade count may also lead to choking at inlet, although this can be reduced by using splitter vanes to lower the blockage in the inducer.

a) i)



a) Cheapest, but comparatively poor recovery, and very poor incidence range.
b) Better recovery than (a) at slightly higher expense, but still poor incidence range.
c) Most expensive, good recovery, and best incidence range.
d) Cheaper than c), good recovery, but very poor incidence range.

ii) Data from the question:

γ=1.4, c_p= 0.7, M_{inlet} = 0.8, $OPR = 4$, p_{01} = 1 bar.

Total to static isentropic relationship for Mach number, M

$$\frac{p_{inlet}}{p_{0inlet}} = \left(1 + \frac{\gamma-1}{2} M_{inlet}{}^2\right)^{-\frac{\gamma}{\gamma-1}} = 0.656$$

Maximum pressure rise

$$\Delta p_{max} = p_{0inlet} - p_{inlet} = p_{oinlet}\left(1 - \left(\frac{p_{inlet}}{p_{0inlet}}\right)\right) = p_{0inlet}(1 - 0.656)$$

Actual pressure rise is 0.7 of the maximum pressure rise

$$\Delta p_{actual} = 0.7(p_{0inlet} - p_{inlet}) = 0.2408 p_{0inlet}$$
$$\Delta p_{actual} = \frac{0.2408}{0.656} p_{inlet} = 0.376 p_{inlet}$$
$$p_{exit} - p_{inlet} = 0.376 p_{inlet}$$

so

$$p_{inlet} = \frac{OPR \times p_{01}}{(1+0.367)} = \frac{4\times1\times10^5}{(1+0.367)} = \underline{\mathbf{2.93\ bar}}$$

iii) Radius increase means that the flow area increases, so by continuity the radial velocity drops. Also, as there are no blades, the angular momentum must be conserved, so the tangential velocity drops as well. Both of which give rise to diffusion.

b) iii)
Definition of polytropic efficiency

$$\frac{p_{01}}{p_{02}} = \left(\frac{T_{01}}{T_{02}}\right)^{-\frac{\eta_p\gamma}{\gamma-1}} \quad \text{(same for static ratios)}$$

From the compressible flow tables,
Diffuser LE, $M_{Diff\,LE}$ = 0.8 so $\frac{T_{LE}}{T_{0LE}} = 0.8865$
Impeller TE (exit), $M_{Impeller\,TE}$ = 1.0 so $\frac{T_{TE}}{T_{0TE}}$ =0.8333
So the static temperature ratio is

$$p_{TE} = p_{LE}\left(\frac{0.8865}{0.8333}\right)^{-\frac{\eta_p\gamma}{\gamma-1}} = \underline{\mathbf{2.51\ bar}}$$

c) At impeller exit, M = 1.0so, $\frac{p}{p_0} = 0.582$ from the tables

$\frac{P_{TE}}{P_{0TE}} = 0.582$so impeller exit total pressure $P_{0TE} = \frac{P_{TE}}{0.582} = 4.32$ bar

Impeller isentropic exit total temperature

$$T_{0TEis} = T_{01}\left(\frac{P_{0TE}}{P_{01}}\right)^{\frac{\gamma-1}{\gamma}}$$

Impeller alone efficiency

$$\eta_{impeller} = \frac{T_{0TE}-T_{01}}{T_{0TEis}-T_{01}} = \underline{\mathbf{0.925}}$$

Comments:

This was the least popular question on the paper and was found to be the most difficult. Part (a) (i) was a straightforward use of isentropic efficiency and was well answered. In part (a) (ii) about half the candidates applied the slip factor incorrectly or didn't find the correct flow velocity using compressible flow. Design changes to change slip in (a) (iii) were described well. In part (b) (i) many candidates gave several drawings of possible diffusers and detailed explanations of their relative advantages and disadvantages. However, very few correctly answered part (b) (ii) due to not knowing how to apply a pressure recovery coefficient or not realising that at the diffuser exit the stagnation and static pressures would be equal. In part (b) (iii) many candidates struggled with how to apply a polytropic efficiency to static conditions and didn't manage to find the static temperature ratio across the vaneless space from the Mach numbers. Part (c) to derive the impeller alone efficiency was generally done well by the candidates who had managed to complete part (b).

Q2)

a) Stage loading coefficient, $\psi = 0.4$
Mean blade velocity, $U_{mean} = 245$ ms^{-1}
$OPR = 18$, $T_{01} = 285$ K, $\eta_C = 0.87$, $c_p = 1005$ J kg^{-1} K^{-1}, $\gamma = 1.4$

Total enthalpy and total temperature change across a stage

$$\Delta h_{0_\psi} = \psi U^2$$

$$\Delta T_{0_\psi} = \frac{\Delta h_{0_\psi}}{c_p}$$

Isentropic temperature ratio

$$T_{0is} = T_{01}(OPR)^{\frac{\gamma-1}{\gamma}}$$

Isentropic temperature rise

$$\Delta T_{0is} = T_{01}\left(\frac{T_{0is}}{T_{01}} - 1\right)$$

Actual temperature rise

$$\Delta T_0 = \frac{\Delta T_{0is}}{\eta_C}$$

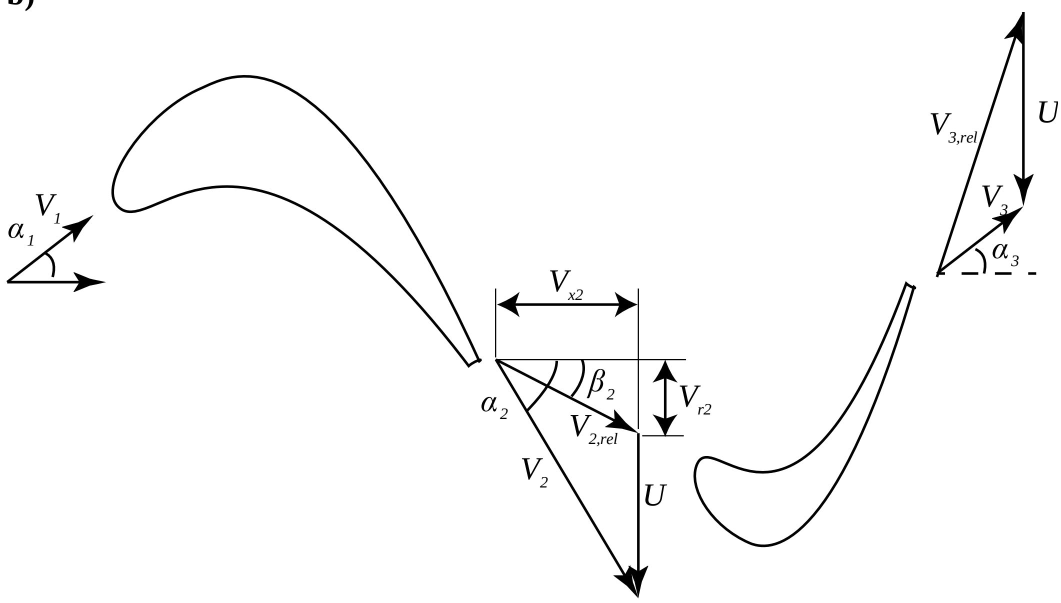
Minimum stages

$$stages = \frac{\Delta T_0}{\Delta T_{0\psi}} = 17.6 \text{ so } \underline{\textbf{18 stages}}$$

Now the inlet temperature is increased

$$\eta_{Cmin} = T_{01}\frac{\left(OPR^{\frac{\gamma-1}{\gamma}} - 1\right)}{\mathrm{N}\times\Delta T_{0\psi}} = \underline{\textbf{0.925}}$$

b)



Rotor relative inlet angle (NB signs integrated in equations below)

$$\tan\beta_2 = \tan\alpha_2 - \tan\beta_3 - \tan\alpha_3$$

$$\text{so}\beta_2 = \underline{\textbf{47.4°}}$$

c) T_{01} = 1600 K, and T_{02} = T_{01} as we're ignoring the cooling!

From tables, at M=0.75

$$V_2 = 0.4497\sqrt{1005\times1600} = \underline{\textbf{570.25 ms}^{-1}}$$

$$V_x = V_2\cos\alpha_2 = \quad = \underline{\textbf{176.2 ms}^{-1}}$$

$$U = V_x(\tan\beta_3 - \tan\alpha_3) = \underline{\textbf{351.0 ms}^{-1}}$$

Euler work equation

$$\Delta h_0 = \Delta(UV_\theta)\text{per stage}$$

$$\Delta(V_\theta) = V_x(\tan\alpha_2 + \tan\alpha_3)$$

$$\Delta T_0 = \frac{U\Delta(V_\theta)}{c_p}$$ = <u>**211.8 K per stage**</u>

Total work is 3x compressor work

$$W_{comp} = c_p\Delta T_{0comp} \text{ and so } W_{turb} = 3\times c_p\Delta T_{0comp}$$

$$stages = \frac{3\times c_p\Delta T_{0comp}}{\Delta T_{0turbine}}$$ = 5.96<u>**so 6 stages**</u>

d)The velocity triangles are the same, but the stagnation temperature is reducing. The axial velocity and blade speed are constant, so the Mach numbers must increase.

As the Mach numbers approach 1.0 at exit, then the losses will increase due to a rapid increase in **base pressure drag,** and extra losses due to the shock and the shock boundary layer interactions that arise. As the Mach number increases even more, the exit flow angle will change due to supersonic deviation, and eventually it would reach limit load. The annulus line would be modified to drop the Mach numbers in real machine.

Comments:

This was the most popular question, and overall completed very well. Part (a), determining the number of compressor stages and an efficiency, was completed accurately in almost all cases. Some of the velocity triangles produced for part (b) were of particularly poor quality or even the wrong way around. Part (c) involves the turbine design and some compressible flow. Those candidates who used non-dimensional velocity as a function of Mach number had no problems finding the blade speed. There were some mistakes applying the Euler work equation accurately to find the stagnation pressure drop and number of stages. Part (d) was descriptive and probably the worst answered part. Many candidates stated the velocity increased through the stages even though this was specified as constant. Few gave precise reasons of how higher Mach numbers lead to increased losses and deviation. Too many candidates discussed stall and choke in the context of multi-stage compressors.

Q3 (a) Work balance between the compressor and turbine:

$$c_p\left(T_{03}-T_{02}\right)=c_{pe}\left(T_{04}-T_{05}\right)\quad \Rightarrow \frac{c_pT_{02}}{c_{pe}T_{04}}\left(\frac{T_{03}}{T_{02}}-1\right)=\left(1-\frac{T_{05}}{T_{04}}\right)$$

$$\therefore k=\frac{c_pT_{02}}{c_{pe}T_{04}}\left(\left(\frac{p_{03}}{p_{02}}\right)^{\frac{\gamma-1}{\gamma\eta_{pc}}}-1\right)=\frac{1005\times 288}{1244\times 1200}\left((6)^{\frac{0.4}{1.4\times 0.85}}-1\right)=\underline{0.160}\qquad \text{<15\%>}$$

Given the turbine and nozzle are choked, $\frac{\dot{m}\sqrt{c_{pe}T_{09}}}{p_{09}A_9}=\frac{\dot{m}\sqrt{c_{pe}T_{04}}}{p_{04}A_4}=F(1)$

The nozzle can be assumed to be adiabatic with no losses, so this can be written as:

$$\frac{\sqrt{T_{05}}}{p_{05}A_9}=\frac{\sqrt{T_{04}}}{p_{04}A_4}$$

$$\therefore \left(\frac{A_9}{A_4}\right)^2=\frac{T_{05}}{T_{04}}\left(\frac{p_{04}}{p_{05}}\right)^2=\left(\frac{T_{05}}{T_{04}}\right)^{1-2\gamma_e/(\gamma_e-1)\eta_{pt}}$$

Hence, the temperature ratio T_{05}/T_{04} and k are fixed by the ratio of propulsive nozzle area to tuirbine vane throat area A_9/A_4. <10%>

[25%]

(b) From the shaft work balance (above),

$$k=\frac{c_pT_{02}}{c_{pe}T_{04}}\left(\left(\frac{p_{03}}{p_{02}}\right)^{\frac{\gamma-1}{\gamma\eta_{pc}}}-1\right)\qquad \Rightarrow \underline{\frac{p_{03}}{p_{02}}=\left(1+k\frac{c_{pe}T_{04}}{c_pT_{02}}\right)^{\frac{\gamma\eta_{pc}}{\gamma-1}}}$$

Applying continuity from compressor inlet to turbine inlet (with $p_{03}=p_{04}$),

$$\frac{\dot{m}\sqrt{c_pT_{02}}}{A_2p_{02}}=\frac{\dot{m}\sqrt{c_pT_{04}}}{A_4p_{04}}\frac{p_{03}}{p_{02}}\frac{A_4}{A_2}\sqrt{\frac{T_{02}}{T_{04}}}$$

Given the turbine is choked and areas fixed,

$$\underline{\frac{\dot{m}\sqrt{c_pT_{02}}}{A_2p_{02}}=C\frac{p_{03}}{p_{02}}\sqrt{\frac{T_{02}}{T_{04}}}}$$

[15%]

(c) At the new operating point,

$$T_{04}-T_{05}=T_{04}\left(1-\frac{T_{05}}{T_{04}}\right)=kT_{04}=0.160\times 950=\underline{152K}$$

$$\frac{p_{03}}{p_{02}}=\left(1+k\frac{c_{pe}T_{04}}{c_pT_{02}}\right)^{\frac{\gamma\eta_{pc}}{\gamma-1}}=\left(1+0.16\times\frac{1244\times 950}{1005\times 288}\right)^{\frac{1.4\times 0.85}{0.4}}=\underline{4.463}\qquad \text{<10\%>}$$

For the propulsive nozzle,

$$V_j^2 = 2c_{pe}T_{05}\left(1-\frac{T_j}{T_{05}}\right) = 2c_{pe}T_{05}\left(1-\left(\frac{p_a}{p_{05}}\right)^{\frac{\gamma_e-1}{\gamma_e}}\right)$$

$$T_{05} = T_{04} - 152 = 798K \qquad \frac{p_a}{p_{05}} = \frac{p_{02}}{p_{05}} = \frac{p_{02}}{p_{03}}\times\frac{p_{04}}{p_{05}}$$

$$\frac{p_{04}}{p_{05}} = \left(\frac{T_{04}}{T_{05}}\right)^{\gamma_e/\eta_{pt}(\gamma_e-1)} = (1-k)^{-\gamma_e/\eta_{pt}(\gamma_e-1)} = (1-0.16)^{-1.3/0.9(0.3)} = 2.315$$

$$V_j = \sqrt{2c_{pe}T_{05}\left(1-\left(\frac{p_{02}}{p_{03}}\times\frac{p_{04}}{p_{05}}\right)^{\frac{\gamma_e-1}{\gamma_e}}\right)} = \sqrt{2\times1244\times798\left(1-(2.315/4.463)^{0.3/1.3}\right)} = \underline{528\text{ m/s}}$$

<20%>

At the design point,

$$V_j^* = \sqrt{2c_{pe}T_{05}\left(1-\left(\frac{p_{02}}{p_{03}}\times\frac{p_{04}}{p_{05}}\right)^{\frac{\gamma_e-1}{\gamma_e}}\right)} = \sqrt{2\times1244\times1200\times0.84\left(1-(2.315/6)^{0.3/1.3}\right)} = 703\text{m/s}$$

Ratio of gross thrusts

$$\frac{\dot{m}V_j}{\dot{m}^*V_j^*} = \frac{p_{03}/p_{02}}{(p_{03}/p_{02})^*}\times\sqrt{\frac{T_{04}^*}{T_{04}}}\times\frac{V_j}{V_j^*} = \frac{4.463}{6}\times\sqrt{\frac{1200}{950}}\times\frac{528}{703} = 0.628$$

Percentage reduction in gross thrust

$$(1-0.628)\times100 = \underline{37.2\%}$$

<15%> [45%]

(d) For a multistage compressor as speed is reduced along the working line (equivalent to reducing fuel flow to the turbojet) incidence increases on the front stages and the rear stages move towards negative incidence and choking. Hence, the instability is likely to be stall in the front stages of the compressor.

Possible solutions:

(i) Restagger the compressor blades to reduce incidence on front stages and increase on rear stages (has a design point efficiency penalty)

(ii) Use Variable Guide Vanes (VGVs) on the front stages (to correct incidence at part speed).

(iii) Use bleed midway through compressor (to prevent rear stages choking).

[15%]

Comments:

This was the second most popular question and it was generally very well answered. Part (a) requires a work balance to find a constant that applies at all operating points on the working line. This was found correctly in almost all cases, but the explanations of why it remained constant required continuity to be applied from turbine stator to propelling nozzle throat and this was not always done. Part (b) involves two some fairly standard proofs that were completed accurately in almost all cases. Part (c) is the most challenging part of the question. Firstly, at a new operating point, the new compressor pressure ratio and turbine stagnation pressure drop are found from the previous parts. The tricky bit is determining the engine jet velocity, which requires the jet static-to-stagnation pressure ratio. Errors were often made in calculating this and some candidates incorrectly assumed the jet was just sonic. To find the ratio of gross thrusts at the two operating points the ratio of mass flow rate times jet velocity is required. The ratio of mass flow rate can be determined from the second result in part (b). Finally, part (d) is a discussion of compressor instability and possible solutions. The answers given were variable and some were lengthy but too vague. Overall though, it is clear the students knew the their stuff about stall and off-design operation, which is encouraging, as this has historically been an unpopular part of the course.