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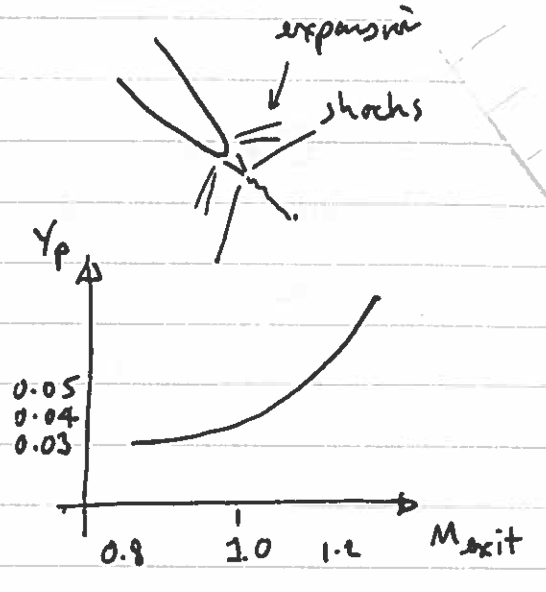
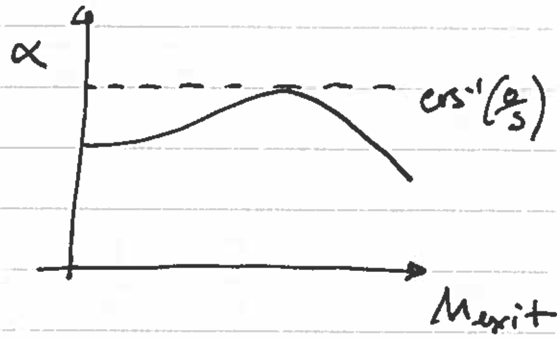
CRIB 4A3 2016

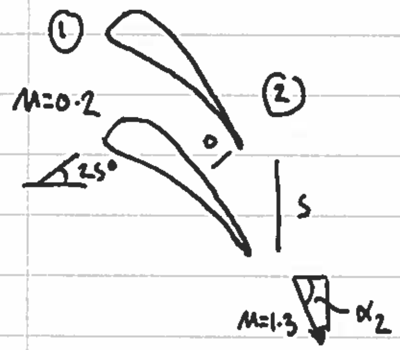
Q1) a) The loss coefficient of a turbine rises rapidly as M_{exit} approaches 1. The rise is closely related to the thickness of the trailing edge. The loss increase is partly caused by shock loss but mainly due to the complex TE flow pattern, which generates low pressure, causing drag.



The flow angle decreases as the exit Mach number increases above 1.0. The drop in downstream pressure and the choked throat mean the axial velocity must increase for continuity.



b)



$$\frac{P_{01}}{P_2} = 2.8$$

From the tables:

	M = 0.2	1.3
$\frac{\dot{m}\sqrt{c_pT_0}}{A\,P_0}$	0.4323	1.2014
$\frac{P}{P_0}$		0.3609

i) Stagnation pressure loss coefficient:

$$Y_p = \frac{P_{01} - P_{02}}{P_{02} - P_2} \Rightarrow Y_p = \frac{P_{01}/P_{02} - 1}{1 - P_2/P_{02}}$$

$$\frac{P_{01}}{P_{02}} = \frac{P_{01}}{P_2} \cdot \frac{P_2}{P_{02}} = 2.8 \times 0.3609 = 1.0105$$

$$\text{so} \quad Y_p = \frac{1.0105 - 1}{1 - 0.3609} = 0.0165$$

Exit angle:

using continuity from inlet to outlet $\quad T_{01} = T_{02}$

$$\frac{\dot{m}\sqrt{c_pT_{01}}}{hs\cos\alpha_1 P_{01}} = 0.4323 \qquad \frac{\dot{m}\sqrt{c_pT_{02}}}{hs\cos\alpha_2 P_{02}} = 1.2014$$

$$\text{so} \quad \cos\alpha_2 = \cos\alpha_1 \frac{P_{01}}{P_{02}} \frac{0.4323}{1.2014} = \cos(-25°) \times 1.0105 \times \frac{0.4323}{1.2014}$$

$$= 0.3295$$

$$\text{so} \quad \alpha_2 = 70.76°$$

ii) Find supersonic deviation:

from tables: $M = 1.0$

$\frac{\dot{m}\sqrt{c_p T_0}}{A P_0}$ 1.2810

$\frac{P^*}{P_0}$ 0.5283

i.e. $\delta_{ss} = \alpha_2\big|_{M=1} - \alpha_2\big|_{M=1.5}$

continuity: $\frac{\dot{m}\sqrt{c_p T_0}}{h s \cos\alpha_1 P_{01}} = f(M=0.2)$ and $\frac{\dot{m}\sqrt{c_p T_0}}{h s \cos\alpha_2 P_{02}} = f(M=1)$

$$\Rightarrow \quad \cos\alpha_2\big|_{M=1} = \cos\alpha_1 \frac{P_{01}}{P_{02}}\bigg|_{M=1} \frac{f(M=0.2)}{f(M=1)}$$

Need to find new $\frac{P_{01}}{P_{02}} = Y_{P_{new}}\left(1 - \frac{P_2}{P_{02}}\bigg|_{M=1}\right) + 1$

$$= \frac{0.0165}{2}\left(1 - 0.5283\right) + 1 = 1.003882$$

$$\Rightarrow \quad \cos\alpha_{2\,M=1} = \cos(25^\circ) \times 1.003882 \times \frac{0.4323}{1.2810}$$

$$= 0.30116$$

$$\alpha_{2\,M=1} = 72.19^\circ$$

$$\Rightarrow \quad \delta_{ss} = 72.19^\circ - 70.76^\circ$$

$$= 1.4^\circ$$

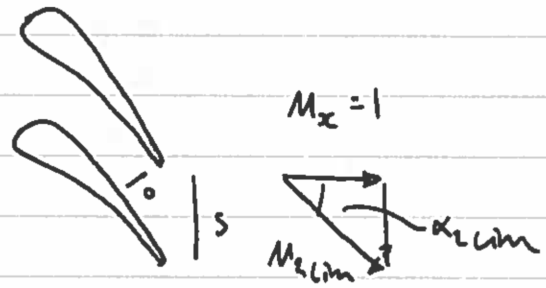
iii) $Y_p = 0.0165$ loss is <u>high</u> compared to $M=0.8$ at 2-5%

however one stage might do the work of several.

lower weight, cost and perhaps shaft speed.

c)

i)



At limit load $\quad M_x = 1$ so $M_{2lim} \cos\alpha_{2lim} = 1.$

$$\text{so} \quad \cos\alpha_{2lim} = \frac{1}{M_{2lim}} \quad // \quad (1)$$

continuity

$$\frac{\dot{m}\sqrt{c_pT_0}}{ho\,P_0^*} = f(M=1)$$

$$\frac{\dot{m}\sqrt{c_pT_0}}{hs\cos\alpha_{2lim}P_{2lim}} = f(M_{2lim})$$

neglect loss

$$\text{so} \quad \cos\alpha_{2lim} = \frac{o}{s} \cdot \cancel{\frac{P_0^*}{P_{2lim}}} \cdot \frac{f(M=1)}{f(M_{2lim})}$$

$$\cos\alpha_{2lim} = \frac{0.3 \, . \, 1.2810}{f(M_{2,lim})} \quad // \quad (2)$$

$$\text{so} \quad \alpha_{2lim} = \cos^{-1}\left\{\frac{1}{M_{2lim}}\right\} \quad \text{and} \quad \alpha_{2lim} = \cos^{-1}\left\{\frac{0.3843}{f(M_{2lim})}\right\}$$

ii) Iteration: not used

α_2	$M_{2\,lim} = 1/\cos\alpha_2$	$f(M_{2\,lim})$	$\alpha_2 = \cos^{-1}\left(\frac{0.3843}{f(M_{2\,lim})}\right)$
65	2.367	0.5499	45.7°
60	2.0	0.7591	59.6°

lucky guess! $\alpha_2 \approx 60 \pm 0.5 = 59.6°$ $M_{2\,lim} \approx 2.0$

(59.845   1.99   0.5023   59.845°)

Exact value for this iteration.

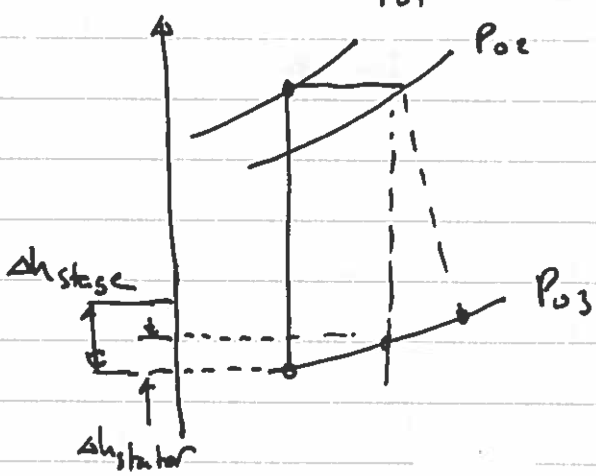
ii) new version: 59.85° is given so

$$M_{lim} = 1.99$$

d)

$$@\ M_2\ \text{lim} = 1.99.$$

$$\frac{T_{03}}{T_{01}} = 0.6 \qquad \left.\frac{P_2}{P_{02}}\right|_{\text{lim}} = 0.1298$$



$$\Delta s \approx -R\ \frac{P_{02} - P_{01}}{P_{01}}$$

$$\Delta h \approx T_{03}\ \Delta s$$

$$\text{so } \Delta h_{stator} = T_{03} \cdot -R\ \frac{P_{02} - P_{01}}{P_{01}}$$

$$P_{01} \approx P_{02}$$

$$P_{02} - P_{01} = Y_p\left(P_{02} - P_{2}\right)$$

$$\frac{P_{02} - P_{01}}{P_{02}} = Y_p\left(1 - \frac{P_2}{P_{02}}\right)$$

$$\text{so } Y_p\left(1 - \frac{P_2}{P_{02}}\right) = \frac{P_{02} - P_{01}}{P_{02}}$$

$$\text{so } \Delta h_{stator} = T_{03}\ R\ Y_p\left(1 - \frac{P_2}{P_{02}}\right)$$

$$\text{so } \frac{\Delta h_{stator}}{\Delta h_{stage}} = \frac{T_{03}\ R\ Y_p\left(1 - P_2/P_{02}\right)}{C_p\left(T_{01} - T_{03}\right)}$$

$$= \frac{\cancel{T_{03}}\ R\ Y_p\left(1 - \frac{P_2}{P_{02}}\right)}{\cancel{T_{03}}\left(C_p\left(\frac{T_{01}}{T_{03}} - 1\right)\right)}$$

$$RY_p\left(1-\frac{P_2}{P_{0c}}\right) = 287.1\times0.2\times(1-0.1298)$$

$$= 49.98$$

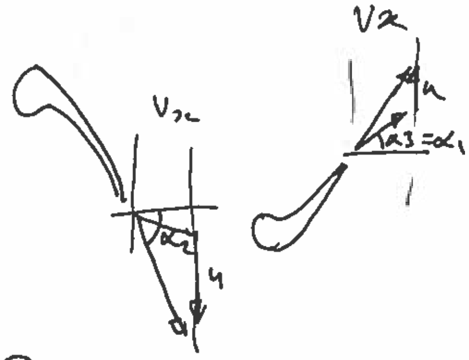
$$c_p\left(\frac{T_{01}}{T_{03}}-1\right) = 1005\left(\frac{1}{0.6}-1\right)$$

$$= 670$$

$$\Rightarrow \frac{\Delta h_{stator}}{\Delta h_{stage}} = \frac{49.98}{670} \approx 7.5\%$$

2) a)

$$\psi = \frac{\Delta V_\theta}{u} \quad \text{by definition.}$$



$$= \frac{V_x(\tan\alpha_2 - \tan\alpha_1)}{u}$$

$$\psi = \phi(\tan\alpha_2 - \tan\alpha_1) \quad \text{— (1)}$$

$$\Lambda = \frac{\Delta h_{\text{rotor}}}{\Delta h_{\text{stage}}} = 1 - \frac{\Delta h_{\text{stator}}}{\Delta h_{\text{stage}}}.$$

$$\Delta h_{\text{stator}} = \frac{1}{2}(V_2^2 - V_1^2) = \frac{1}{2} V_x^2(\tan^2\alpha_2 - \tan^2\alpha_1)$$

and $\Delta h_{\text{stage}} = u^2\psi$ (repeating stage so $V_1 = V_3$ so $\Delta h_0 = \Delta h$)

$$\Lambda = 1 - \frac{1}{2}\frac{\phi^2}{\psi}(\tan^2\alpha_2 - \tan^2\alpha_1) \quad \text{— (2)}$$

subs (2) into (1) $\quad \Lambda = 1 - \frac{1}{2}\phi(\tan\alpha_2 + \tan\alpha_1) \quad$ — (3)

$2 \times$ (3) + (1) gives $\quad \psi = 2(1 - \Lambda - \phi\tan\alpha_1)$

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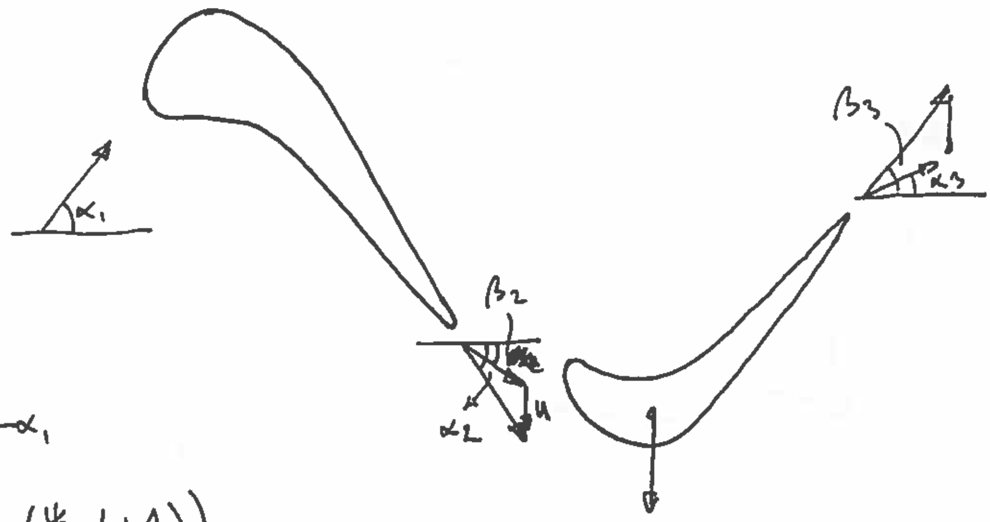
2) Continued

iii) $\psi = 1.7$, $\phi = 0.45$ and $\Lambda = 0.5$

V_x = const +ve is in direction of rotor rotation

$\psi = 2(1 - \Lambda - \phi \tan\alpha_1)$



First get α_1 $\frac{\psi}{2} = 1 - \Lambda - \phi\tan\alpha_1$

$$-\alpha_1 = \tan^{-1}\left(\frac{1}{\phi}\left(\frac{\psi}{2} - 1 + \Lambda\right)\right)$$

$$\alpha_1 = -37.9^\circ$$

use $\psi = \phi(\tan\alpha_2 - \tan\alpha_1)$

$$\alpha_2 = \tan^{-1}\left(\frac{\psi}{\phi} + \tan\alpha_1\right)$$

$$= 71.6^\circ$$

from vel. triangles: $\tan\beta_2 = \tan\alpha_2 - \frac{1}{\phi}$

$$\beta_2 = \tan^{-1}\left(\tan\alpha_2 - \frac{1}{\phi}\right)$$

$$= 37.9^\circ$$

$\tan(-\beta_3) = \frac{1}{\phi} + \tan(-\alpha_3)$

$$\beta_3 = -\tan^{-1}\left(\frac{1}{\phi} + \tan(-\alpha_3)\right)$$

$$= -71.6^\circ$$

As $\Lambda = 0.5$ full marks given for $\beta_3 = -\alpha_2$

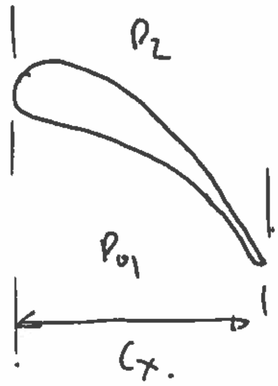
$\beta_2 = -\alpha_1$

2) Continued

b)

i)

$$Z = \frac{\dot{m}\,|V_{\theta_2} - V_{\theta_1}|}{c_x h\,(p_{01} - p_2)}$$

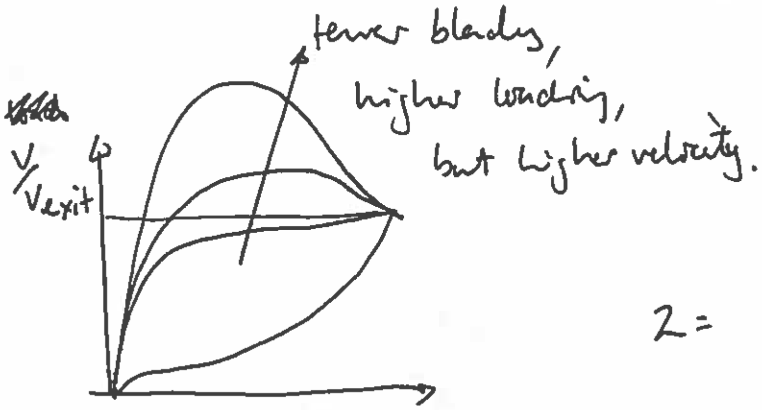


max possible loading:

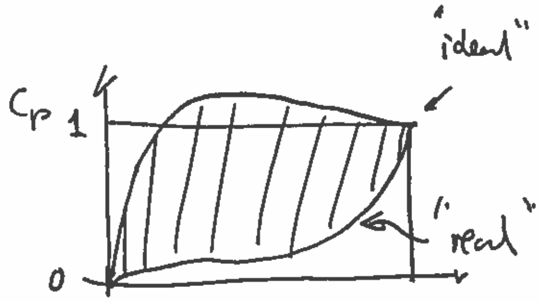
$F = h c_x (p_{01} - p_2)$

actual loading:

$F = \dot{m}(V_{\theta_2} - V_{\theta_1})$



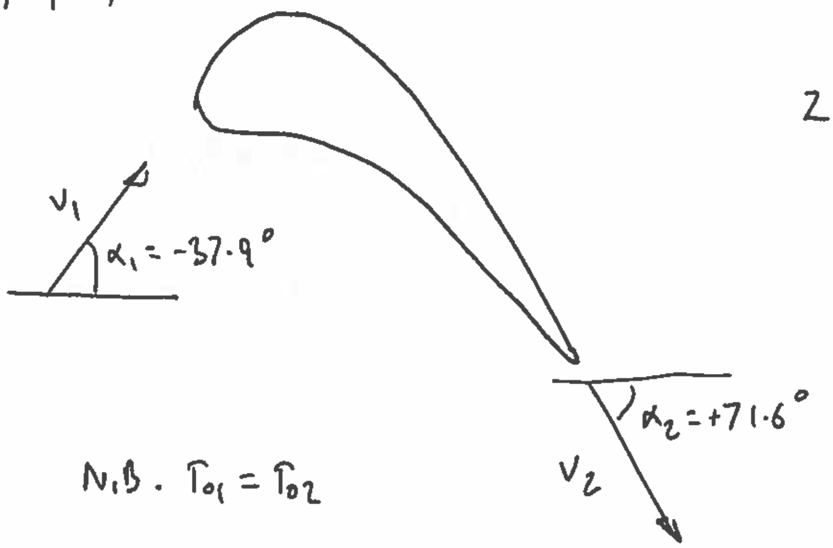
$$Z = \frac{\text{actual loading}}{\text{max possible}}.$$



$$C_p = \frac{p_{01} - p}{p_{01} - p_2}$$

ideal loading (impossible!) is a rectangle. So Zweifel indicates the area of overshoot necessary to achieve loading, so points to (optimum) balance of loss due to wetted area versus reduction in peak velocities. N.B. loss $\propto V^3$.

2) b) ii)



N.B. $T_{01} = T_{02}$

$$Z = \frac{\dot{m}\,|(V_{\theta_2} - V_{\theta_1})|}{h c_x (P_{01} - P_2)}$$

$$V_{\theta_2} = V_2 \sin\alpha_2 = f_2(M_2)\sqrt{c_p T_{02}}\,\sin\alpha_2$$

$$V_{\theta_1} = V_1 \sin\alpha_1 = f_2(M_1)\sqrt{c_p T_{01}}\,\sin\alpha_1$$

$$\dot{m} = \frac{f_1(M_1)\, h s \cos\alpha_1\, P_{01}}{\sqrt{c_p T_{01}}}$$

where $f_2(M) = \dfrac{V}{\sqrt{c_p T_0}}$ and $f_1(M) = \dfrac{\dot{m}\sqrt{c_p T_0}}{A P_0}$

$$\text{so} \quad Z = \frac{f_1(M_1)\cancel{h} s \cos\alpha_1\, \cancel{P_{01}} \left| \left\{ f_2(M_2)\cancel{\sqrt{c_p T_{02}}}\sin\alpha_2 - f_2(M_1)\cancel{\sqrt{c_p T_{02}}}\sin\alpha_1 \right\} \right|}{\cancel{\sqrt{c_p T_{01}}}\ \cancel{P_{01}}\ \cancel{h} c_x \left(1 - \frac{P_2}{P_{01}}\right)}$$

$$\text{so} \quad \frac{s}{c_x} = Z\left(1 - \frac{P_2}{P_{01}}\right) \Big/ f_1(M_1)\cos\alpha_1 \left| \left\{ f_2(M_2)\sin\alpha_2 - f_2(M_1)\sin\alpha_1 \right\} \right|$$

Data $\alpha_1 = -37.9°$, $\alpha_2 = 71.6°$ $M_1 = 0.2$, $Y_p = \dfrac{(P_{01} - P_{02})}{(P_{01} - P_2)} = 0.05$

$M_2 = 0.8$

$P_2/P_{02} = 0.6560$

$$\frac{P_{01}}{P_{02}} = Y_p\left(1 - \frac{P_2}{P_{02}}\right) + 1 = 1.0172$$

$$\frac{P_2}{P_{01}} = \frac{P_2}{P_{02}} \cdot \frac{P_{02}}{P_{01}} = 0.6449$$

$f_1(M_1) = 0.4323$

$f_1(M_2) = 1.2338$

so $f_2(M_1) = 0.1260$

$f_2(M_2) = 0.4764$

So $$\frac{\xi}{C_x} = \frac{0.8(1 - 0.6449)}{0.4332 \cos(-37.9)\left\{0.4764 \sin(71.6) - 0.1260 \sin(-37.9)\right\}}$$

$$= 1.57$$

(~~maybe change to isentropic blade.~~)

~~$\frac{p}{p_0}$ @ M=0.8~~

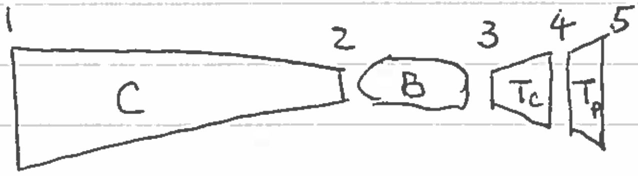
If ~~so~~ $\cancel{\frac{\xi}{C_x} = 0.8(1 - 0.6560)}$

~~(same)~~

$\cancel{= 0.53}$

$\cancel{= 1.52}$

Q3.



a). By continuity. $\dot{m}_1 = \dot{m}_3$, $\dot{m}_{fuel} << \dot{m}_1$ and can be neglected.

Section (3) choked

$$\frac{\dot{m}_1\sqrt{C_{p\,air}T_{01}}}{A_1 P_{01}} = \frac{\dot{m}_3\sqrt{C_{p\,gas}T_{03}}}{\sqrt{C_{p\,gas}T_{03}}}\cdot\frac{A_3 P_{03}}{A_3 P_{03}}\cdot\frac{\sqrt{C_{p\,air}T_{01}}}{A_1 P_{01}} = F_{gas}(1)\cdot\frac{A_3}{A_1}\cdot\frac{P_{03}}{P_{01}}\sqrt{\frac{C_{p\,air}T_{01}}{C_{p\,gas}T_{03}}}$$

$P_{02} \approx P_{03}$ for small loss in combustor

$$\therefore \frac{P_{03}}{P_{01}} = \frac{P_{02}}{P_{01}} = \pi = \frac{\dot{m}_1\sqrt{C_{p\,air}T_{01}}}{A_1 P_{01}}\cdot\frac{A_1}{A_3 F_{gas}(1)}\sqrt{\frac{C_{p\,gas}T_{03}}{C_{p\,air}T_{01}}}$$

assume both $C_{p\,air}$ & $C_{p\,gas}$ are constant. let $C = \frac{A_1}{A_3 F_{gas}(1)}$

$$\pi = C\cdot\sqrt{\frac{T_{03}C_{p\,gas}}{T_{01}C_{p\,air}}}\;\frac{\dot{m}\sqrt{C_{p\,air}T_{01}}}{P_{01}A_1} \qquad (1) \quad \text{q.e.d.}$$

(b). Both NGVs at (3) and (4) are choked assume the losses in NGVs are small upstream of the throat, neglect mechanical losses. system in equilibrium; $\dot{W}_c = \dot{W}_T \Rightarrow$

$$C_{p\,air}(T_{02}-T_{01}) = C_{p\,gas}(T_{03}-T_{04})$$

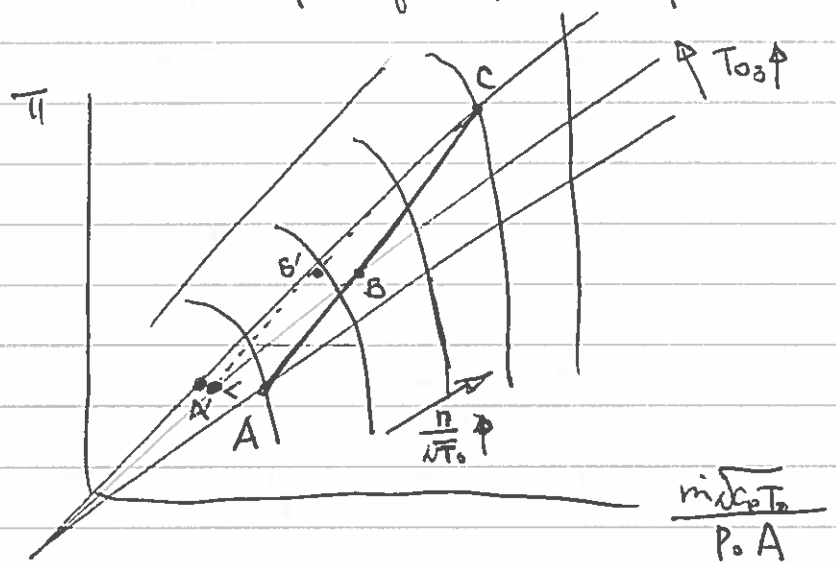
$$C_{p\,air}T_{01}\left(\frac{T_{02}}{T_{01}}-1\right) = C_{p\,gas}T_{03}\left(1-\frac{T_{04}}{T_{03}}\right)$$

by continuity $\sqrt{\frac{T_{04}}{T_{03}}} = \frac{P_{04}A_4}{P_{03}A_3} \Rightarrow \frac{A_3}{A_4} = \frac{P_{04}}{P_{03}}\sqrt{\frac{T_{03}}{T_{04}}} = \left(\frac{P_{04}}{P_{03}}\right)^{1-\frac{\eta_{pt}(\gamma-1)}{2\gamma}}$

So the turbine work is fixed by area ratio A_3/A_4

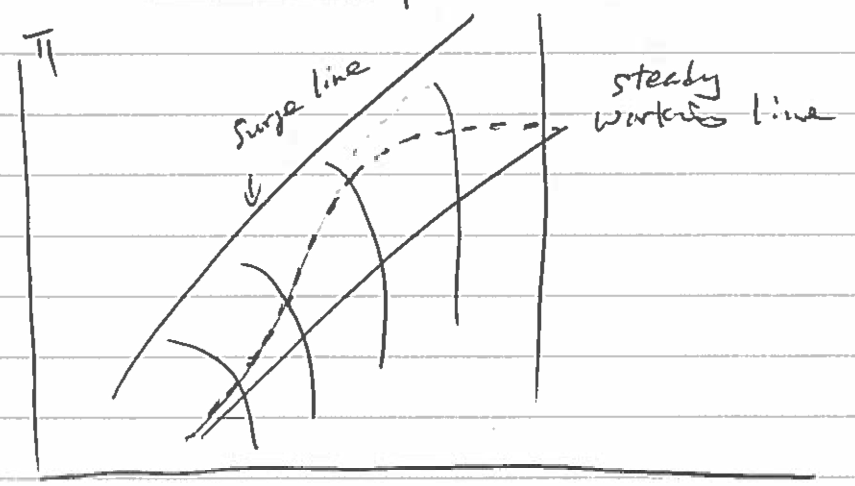
re exam equation (1) derived in (a).
for given T_{03}/T_{01}. the relationship between π and $\frac{\dot{m}\sqrt{C_p T_{01}}}{A_1 P_0}$ is a straight line on compressor map. with slop $C\cdot\sqrt{\frac{T_{03}}{T_{01}}}$, with lines of

higher T_{03} on the top-left of the map.



As engine speed increases, fuel input increases to boost turbine power output so the T_{03} increases. for a given η_{pc}, if the working line follows line A-B-C. With lower η_{pc} at lower speeds, it means the compressor requires higher turbine power thus higher T_{04} for the same pressure ratio π. So the working line would have to move to the left of A to start with, which drives the working line closer to the surge line. Similar situation will happen when engine acceleration requires more turbine power. results in a work line A'-B'-C with lower slope the line A-B-C.

(C)



During acceleration, turbine power is larger than the compressor power. more fuel is needed to increase T_{03}. this shall limit mass flow rate trough the turbine throats thus shifts the worki line to the left. ⇒ closer to the surge line ⇒ prone to instability.

(d).

(i). The new gas turbine shall have.

① lower compressor (and possibily lower turbine) efficiency. due to shorter blade to have higher proportion of endwall and tip clearance losses; lower mean blade speeds also likely leads to lower efficiency due ~~to~~ higher aerodynamic loadings.

② lower compressor throughflow capacity due to higher endwall blockage (higher clearance flows).

③ lower pressure rise capacity due to loss of η and surge margin, and requires extra turbine power.

These are reflected in higher exhaust gas temperature which is also indicative for higher T_{03}.

As discussed in part (b) & (c), this shall reduce surge margin.

(ii) high T_{03} reduces effective throat of the turbine NGVs so ~~to~~ open up NGVs throats shall move the working line back down ~~to~~ the lower right.

(iii) It is likely after restoring the stability the gas turbine can be operational but with lower cycle pressure ratio (for lower) and higher T_{03}, lower compressor efficiency and possibily lower turbine efficiency as well. The engine mass flow rate could be restored if designed correctly but specific work thus the total work output is likely to reduce. and the cycle efficiency lower than the original gas turbine.