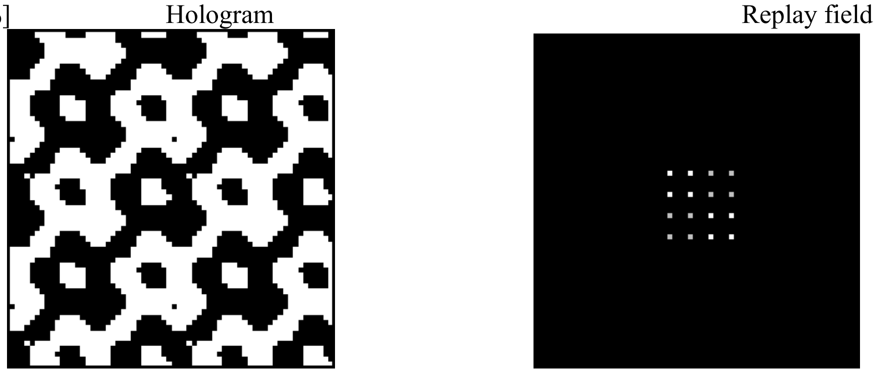
Cribs 4B11 2016 - Text here is longer than expected from candidates

Question 1: Computer generated holograms: 6 out of 8 candidates answered this question. The average raw mark was 12.5 out of 20.

a) Rather poor definitions of a CGH and Fourier transform, good assumptions made. b) Fairly standard book work question, well answered in general, but many answers followed a poorly laid out proof without much logic. Quite a few missed the relevance of N. c) Not so well answered with most getting the sinc envelope but very few able to numerically evaluate the peak heights

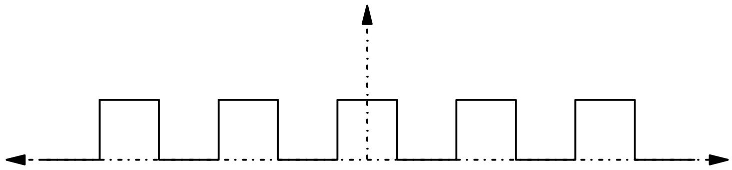
Q1 a) [20%] Hologram Replay field



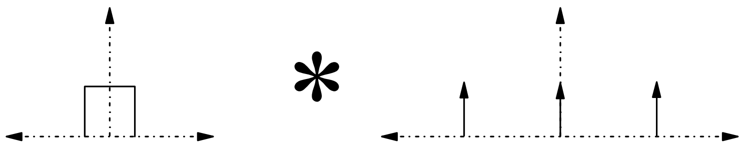
The hologram is an array of pixels in either phase or intensity which will generate a desired replay field. The relationship between the two is via the Fourier transform which converts one into the other and is reversible. The FT can be done by either a positive focal length lens or by going into the Fraunhopher region of free-space.

Assumptions: A perfect FT from the lens or free-space, perfect reproduction of the hologram on each pixel, no dead-space, no apodisation

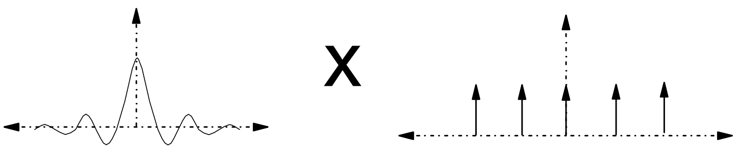
b) [25%] If a pixellated pattern such as a grating is viewed from the end it can be modelled as a as a repetitive 1-D function. The repetition rate is defined by the pixel pitch or period.



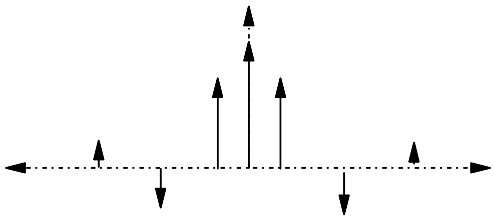
This can be expressed as a convolution of two functions and Δ is the width of a pixel and its periodicity.



Where the delta function train represents the sampling or pixellation function and * represents a convolution. After the Fourier transform we have the replay field by Fourier analysis and the .

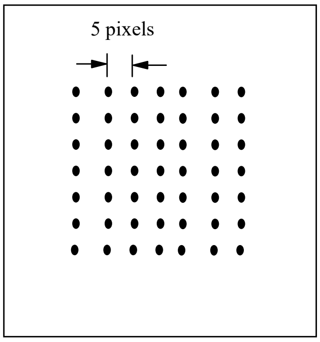


Gives the final result.

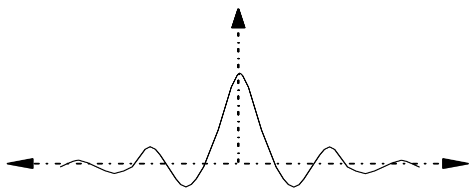


The train of delta functions has a sinc envelope and every second delta function is suppressed by the zeros of the sinc. Each symmetric pair of delta functions above represents a separate order and is repeated every odd harmonic.
The overall size of the replay field is dictated inversely by the wavelength λ. The number of pixels N does not have any effect on the replay field but it does effectively define the resolution of the replay field, but only assuming a one to one sampling in the Fourier domain.

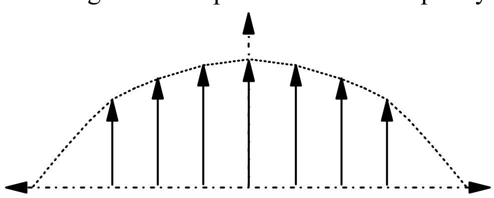
c) i) [15%]



The square shape of the pixels in the CGH means that there will be an overall sinc (sine(x)/x) shaped envelope in the replay field.



The original design of the CGH was to have all of the spots of equal height in the 7x7 array, however the outer spots will be reduced in height with respect to the central spot by the sinc envelope.



ii) [15%]
From the FT of a single pixel we have the envelope function for the CGH.
$F(u,v) = Aa^2 \mathrm{sinc}(\pi au)\mathrm{sinc}(\pi av)$ which is in (u,v), normalised spatial frequencies.

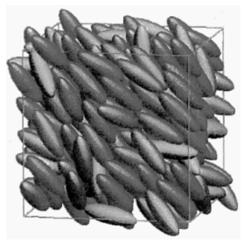
We assume that for an NxN CGH we will have NxN spatial frequency positions in the central lobe of the sinc envelope in the replay field. We also assume no deadspace, apodisation and a perfect FT.

The position of the central spot in the replay field will be at (0,0) and the top rightmost spot will be at (17,17) in the replay field. In order to find the first zero of the sinc envelope (where $\mathrm{sinc}(\pi) = 0$) we must have $(u,v) = 1/a$. Where a is the pixel pitch of the CGH. At the position $u = 1/a$, $v = 1/a$ we have the spatial frequency point of (128,128) hence the normalised co-ordinates will be at $(u,v) = (17/128a, 17/128a)$ which can be fed in the $F(u,v)$. This gives a ratio of 5.7% in height of the spots.

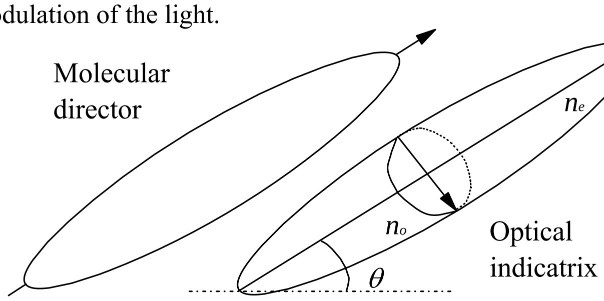
iii) [15%] The DBS algorithm is not a good choice for designing this CGH as there it does not use the correct optimisation cost function. D uses a pure power difference in the replay field before and after pixel flips to define the cost function. This gives no control over the height of the spots in the replay field of the CGH, hence they will be uneven. This is especially the case if the sinc envelope has to be corrected for in the CGH.

A better algorithm is simulated annealing, however even this does not generate uniform replay fields, so a modified cost function is used based a round the difference between the spots desired and the average of the spots in the replay field.

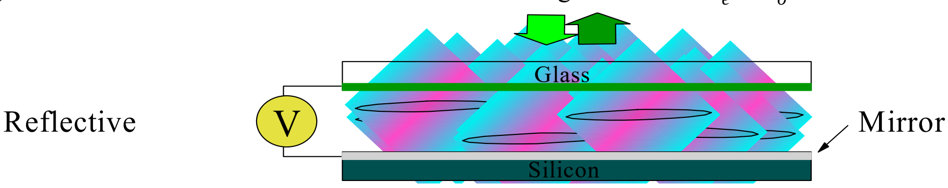
Question 2: Liquid crystal modulation: 8 out of 8 candidates answered this question. The average raw mark was 14.4 out of 20.
a) Overall a well answered section with most defining phase modulation quite well. b) Very well answered which was pleasing as this was not directly in the notes. Good observations of the failings of TN c) Standard book work maths, but quite a few got the wrong answer. d) quite a few got the relevance of part c but several tried unsuccessfully to prove it with Jones matrices.



Q2 a) [25%] The nematic phase is the least ordered mesophase before the isotropic. The molecules have only long range order and no longitudinal order. This means that the molecules retain a low viscosity, like a liquid, and are prone to flow. The anisotropy in the shape of the calimatic molecules leads to a dielectric anisotropy which is represented by a pair of dielectric constants, ε_{perp} and ε_{para}. The total dielectric anisotropy $\Delta\varepsilon$ is defined as the difference between the two. It is important to note that can be both positive as well as negative (vertically aligned nematic (VAN) devices). Molecules with a negative $\Delta\varepsilon$ behave in the opposite way electrically to those with a positive $\Delta\varepsilon$. The existence of this dielectric anisotropy means that we can move the molecules around by applying an electric field across them, combined with the flow properties means that a nematic molecule can be oriented in any direction with the use of an electric field which leads to their ability to perform greyscale modulation of the light.



The calimatic molecular shape also leads to an optical anisotropy in nematic LCs, with the two axes of the molecule appearing as the refractive index. The refractive index along the long axis of the molecules is often referred to as the extraordinary n_e (or fast n_s) and the short axis the ordinary n_o (or slow n_s) axis. The difference between the two is the birefringence. $\Delta n = n_e - n_o$

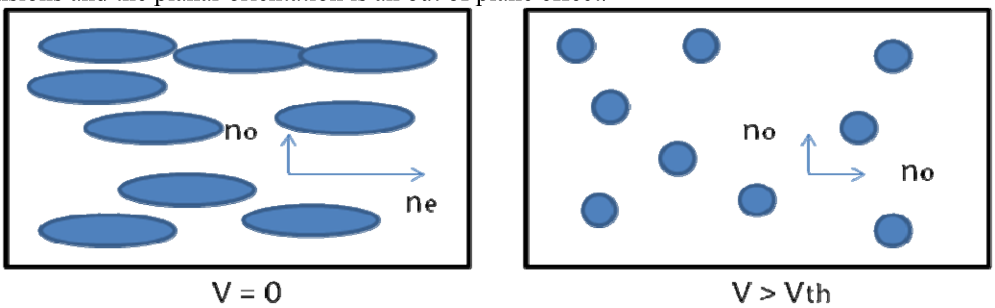


The optical anisotropy means that we can effectively rotate the axes of the indicatrix as the molecules move, creating a moveable wave plate or optical retarder. This along with polarising optics makes the basis of phase modulation. If we have an optical indicatrix oriented at an angle of θ to the plane of the cell (usually this corresponds to the plane with the glass walls and ITO electrodes), then we can calculate the refractive index seen by light passing perpendicular to the cell walls. We can then calculate the retardance Γ of the liquid crystal layer for a given cell thickness d and wavelength λ.

$$\Gamma = \frac{2\pi d\left(n(\theta)-n_o\right)}{\lambda}$$

We can now use this expression in a Jones matrix representation of the optical LC retarder to get the optical characteristics of the LC material. In the case of nematic LC materials, there is little restriction on the flow properties of the material, hence it is possible to continuously vary Γ through several rotations of π providing a thick enough cell. The main drawback of these materials is due to the fact that the flow of the material means that the speed of the modulation is often very slow (10's of msec to seconds). It is by definition an out of plane electro-optical effect.

b) [25%] One of the main limitations of nematic LCs is that they are inherently polarisation sensitive when in the planar geometry. This comes from the fact that the optical anisotropy is only in two dimensions and the planar orientation is an out of plane effect.



When seen from above in the 0V state, light polarised parallel to the long axis of the molecules sees ne and light perpendicular sees no. When the LC is switched into the homeotropic state by the applied electric field, the light polarised parallel to the long axis of the molecules now sees no but the light perpendicular still sees no. Hence one polarisation is phase modulated, while the perpendicular polarisation is not.

The effects discussed above is the planar effect, where there is just the simple angle θ with respect to the cell walls. Because the nematic LC molecules are free to rotate in any position, it is possible to make more complex geometries such as twisted structures. In these devices, the molecules follow a twisted or helical path often rotating through an angle of 90^{o} as is the case with twisted nematic or TN displays. This reduces the polarisation dependence as both polarisation states will see a change in reflective index when the LC material is switched, however it is not fully polarisation insensitive as each polarisation direction will see differing degrees of phase modulation. There is no twisting direction that will solve this inequality between the two polarisation states.

c) [25%] For a quarter waveplate, Γ = 90 degrees and ψ = 45 degrees.

$$W = \begin{pmatrix} e^{-j\Gamma/2}\cos^2\psi + e^{j\Gamma/2}\sin^2\psi & -j\sin\frac{\Gamma}{2}\sin(2\psi) \\ -j\sin\frac{\Gamma}{2}\sin(2\psi) & e^{j\Gamma/2}\cos^2\psi + e^{-j\Gamma/2}\sin^2\psi \end{pmatrix}$$

$$e^{-j\Gamma/2}\cos^2\psi + e^{j\Gamma/2}\sin^2\psi = \frac{1}{2\sqrt{2}}(1-j)\left(\frac{1}{\sqrt{2}}\right)^2 + \frac{1}{2\sqrt{2}}(1+j)\left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{\sqrt{2}}$$

$$e^{j\Gamma/2}\cos^2\psi + e^{-j\Gamma/2}\sin^2\psi = \frac{1}{2\sqrt{2}}(1+j)\left(\frac{1}{\sqrt{2}}\right)^2 + \frac{1}{2\sqrt{2}}(1-j)\left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{\sqrt{2}}$$

$$-j\sin\frac{\Gamma}{2}\sin(2\psi) = -j\frac{1}{\sqrt{2}}$$

$$QWP = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -j \\ -j & 1 \end{pmatrix}$$

d) [25%] Multiple passes through a cell with waveplate optics is required to make them insensitive. The NLC is aligned parallel to the vertical (y) axis.

The vertical component will pass through the NLC and be fully phase modulated before passing through the QWP and being reflected back again. The two passes through the QWP are effectively the same as passing through a HWP with the same orientation of 45 degrees to the vertical

$$QWP(two\ \ passes) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -j \\ -j & 1 \end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -j \\ -j & 1 \end{pmatrix} = \begin{pmatrix} 0 & -j \\ -j & 0 \end{pmatrix}$$

For vertical light in the result of the QWP will be:

$$\begin{pmatrix} 0 & -j \\ -j & 0 \end{pmatrix}\begin{pmatrix} 0 \\ V \end{pmatrix} = \begin{pmatrix} -jV \\ 0 \end{pmatrix}$$

Which is horizontally polarised and so will not see the phase modulation on the return pass back through the NLC. For horizontally polarised light, process is reversed, there is no modulation from the NLC on the pass in, then the QWP:

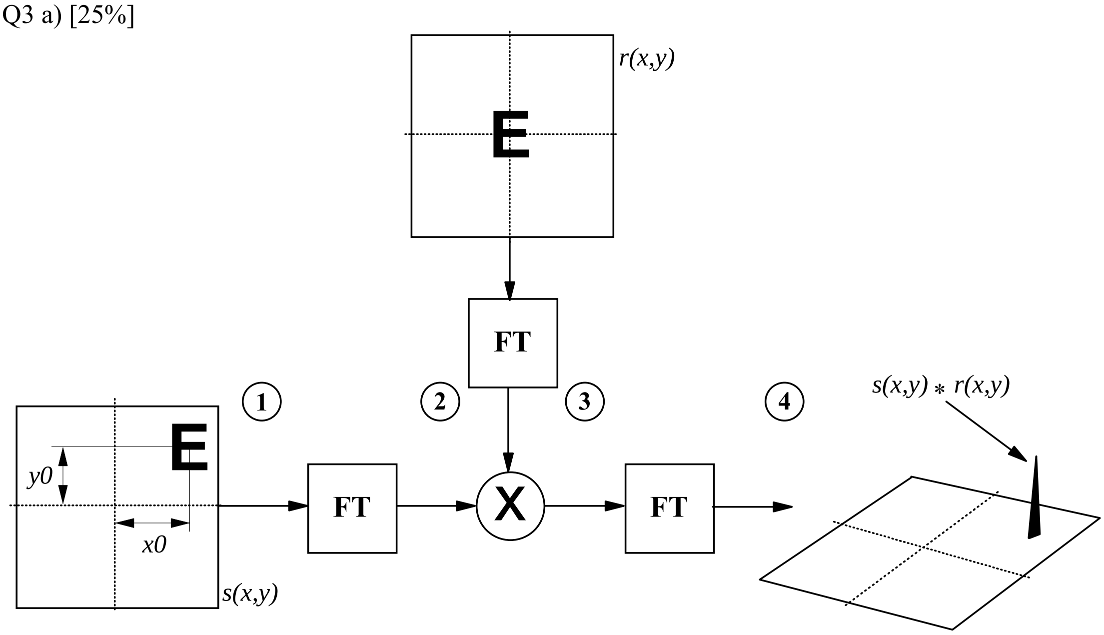
$$\begin{pmatrix} 0 & -j \\ -j & 0 \end{pmatrix}\begin{pmatrix} V \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -jV \end{pmatrix}$$

Hence the light is now vertically polarised and will see the NLC phase modulation on the way back out of the cell.

Thus both states are modulated in the same way by the NLC on each pass through the cell giving a polarisation insensitive phase modulation.

Question 3: Correlators: 7 out of 8 candidates answered this question. The average raw mark was 12 out of 20.
a) Well answered book work section. Only a few candidates pointed out the simpler mechanical properties and several drew unnecessary full optical diagrams. No one mentioned the speed issues. b) Some good answers but only a few pointed out the limits of the complex nature of the filter. A few mentioned the benefits of speed with the binary phase modulation. A few rather random OASLM ideas as well c) Varied answers here with few pointing out the extra information required to represent the SDF as a matched filter. d) quite well answered despite being speculative.

Q3 a) [25%]

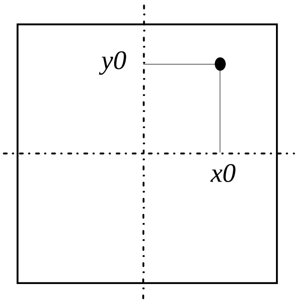


The input image *s(x,y)* is displayed in plane 1 before the FT into plane 2.

$$S(u,v)e^{-j2\pi(x_0u+y_0v)}$$

The FT of *s(x,y)* is then multiplied by the FT of the reference *r(x,y)*.

$$R(u,v)S(u,v)e^{-j2\pi(x_0u+y_0v)}$$



The FT of the reference is done off line on a computer and is defined as the <u>matched filter</u> *R(u,v)* for that particular reference *r(x,y)*. The product of the input FT and the filter then undergoes a further FT to give the correlation in plane 4. Advantages: the object in the reference *r(x,y)* is centred in the process of generating the filter *R(u,v)*, so that if a correlation peak occurs, its position is directly proportional to the object in the input image, with no need for any decoding. Unlike in the JTC, there is only one correlation peak and there are no DC terms to degrade the correlator output. Disadvantage: optomechanics and alignment of filter and FT are difficult.

b) [30%] The filter is matched to the reference via a Fourier transform such that.

$$F(u,v) = F_T[r(x,y)]$$

The best test for any matched filter is to perform an autocorrelation with the filter that has been generated. If the matched filter above is used for the reference image of a letter E, then the autocorrelation will have optimum SNR. The autocorrelation peak is very broad and has a huge SNR, as there is no appreciable noise in the outer regions of the correlation plane. Such a filter is not very useful for pattern recognition. Such a broad peak could lead to confusion when the position of the peak is to be determined. Also, similar shaped objects (such as the letter F) will correlate well with the filter leading to incorrect recognition. Another identical E which is placed in the input along with the original one will also cause problems as the correlation peak will take an extremely complex structure. Finally, the filter is a complex function and there is no technology available to display the filter in an optical system.

Great improvements can be made to the usefulness of the correlation peak, by using a phase only matched filter (POMF). The matched filter *F(u,v)* is stripped of its phase information (i.e. the phase angle of the complex data at each pixel) and this is used as the filter in the correlator.

$$F(u,v) = F_{amp}(u,v)e^{i\phi(u,v)}$$

Hence

$$F_{POMF}(u,v) = e^{i\phi(u,v)}$$

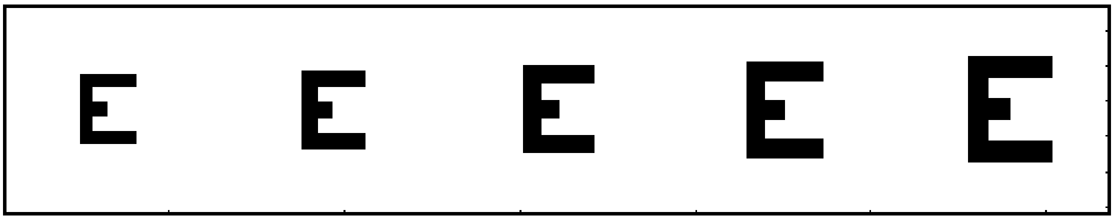
The autocorrelation for the POMF is much more desirable even though there is a reduction in the SNR due to the increase in the background noise. The correlation peak is much narrower which is due to the information which is stored in the phase of the matched filter. The POMF is the most desirable filter to use as it has good narrow peaks and still remains selective of similar structured objects (like Fs). The POMF is also a complex light modulation scheme, so the problems associated with binary phase (180° symmetry) will not occur. However, the continuous phase structure of $\phi(u,v)$ means that it cannot be easily displayed in an optical system.

The penalties associated with going to binary phase are greatly out weighed by the advantages gained by using FLC SLMs in the optical system. The binary phase is selected from the POMF by two thresholds δ_1 and δ_2.

$$F_{BPOMF} = \begin{cases} 0 & \delta_1 \le \phi(u,v) \le \delta_2 \\ \pi & \text{Otherwise} \end{cases}$$

1) The SNR is up to 6dB worse than the case of the POMF.
2) The filter cannot differentiate between an object and the same object rotated by 180° (due to the fact that the BPOMF is a real function).
3) The BPOMF is not as selective as the POMF due to the loss of information in the thresholding.

c) [30%] The SDF provides a way of achieving a limited form of invariance to the effects of image rotation and scaling. The idea is to take a series of reference images $r_1(x,y)$ to $r_n(x.y)$ and combine them by a linear summation. The final composite SDF image contains all of the references and so will correlate with the input image. Unlike mathematical transform techniques, the SDF does not destroy information and still retains the ability to be shift (position) invariant.



If we define a *1xn* vector **a**, whose elements are the weighting coefficients for each respective reference image, then the SDF is defined as.

$$h_{SDF}(x,y)=\sum_{n}^{i=1} a_i r_i(x,y)$$

We can easily calculate the weight coefficients from the cross correlations between all the reference images and the correlation peak value desired for each reference.

$$\mathbf{a}=\mathbf{R}^{-1}\mathbf{c}$$

Where **c** is a *nx1* vector whose elements are the desired correlation peak values for each image correlated with the SDF and **R** is the *nxn* correlation matrix. The elements of **R** are calculated from the cross correlation of the images in the reference set. Hence the element $R_{i,j}$ of the matrix **R** is the correlation between the reference images $r_i(x.y)$ and $r_j(x.y)$.

When the SDF is used in a matched filter and displayed as continuous phase only filter then the correlation peaks are reasonably close to the expected value of 1.0. If the phase only filter is then thresholded to form a binary phase only filter then the results are much worse. The correlator cannot distinguish between similar objects. This is the main drawback with the SDF, as the more references that are included, the more difficult it is to display the SDF on a modulator in a realistic optical correlator.

d) [15%] There are other possible applications which go beyond the optical comparator which has now given rise the optical processor or 'Fourier Engine'. An *n*th order derivative function can be solved in Fourier space by using the identity

$$\frac{d^n g(x)}{dx^n} = FT[(j2\pi u)^n G(u)]$$

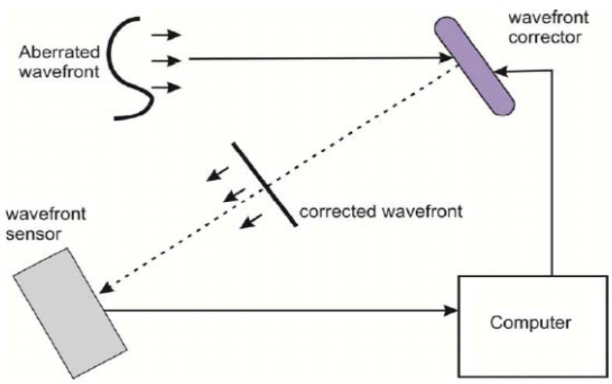
.

This is essentially a matched filter where the filter is the function in the () above. This could be implemented as a matched filter by displaying the () term on a SLM. If the order of the derivative *n*, is fixed, then the filter is purely a fixed value and does not need an SLM. When $n = 1$, the filter is purely a π phase step and a graduated amplitude mask in the direction of the derivative. There are still problems however with the representation of data in these optical systems, such as bipolar number systems, input and output data speed and dynamic range.

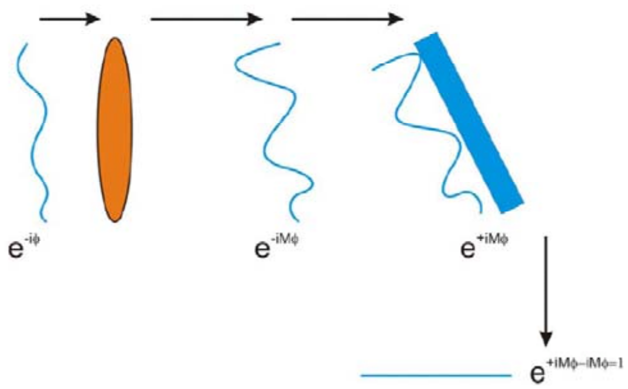
Question 4: Adaptive optics: 3 out of 12 candidates answered this question. The average raw mark was 13.33 out of 20.

a) Mostly answered well, but a few omitted the theory behind the concept of conjugation. b) Some quite good answers with several pointing out the key features of the Zernike polynomials over a circular aperture. c) This was rather poorly answered with a lot of candidates drawing a shuttered switch. d) Good answers in this section showing a good understanding of the principles of an adaptive CGH.

Q4 a) [30%]

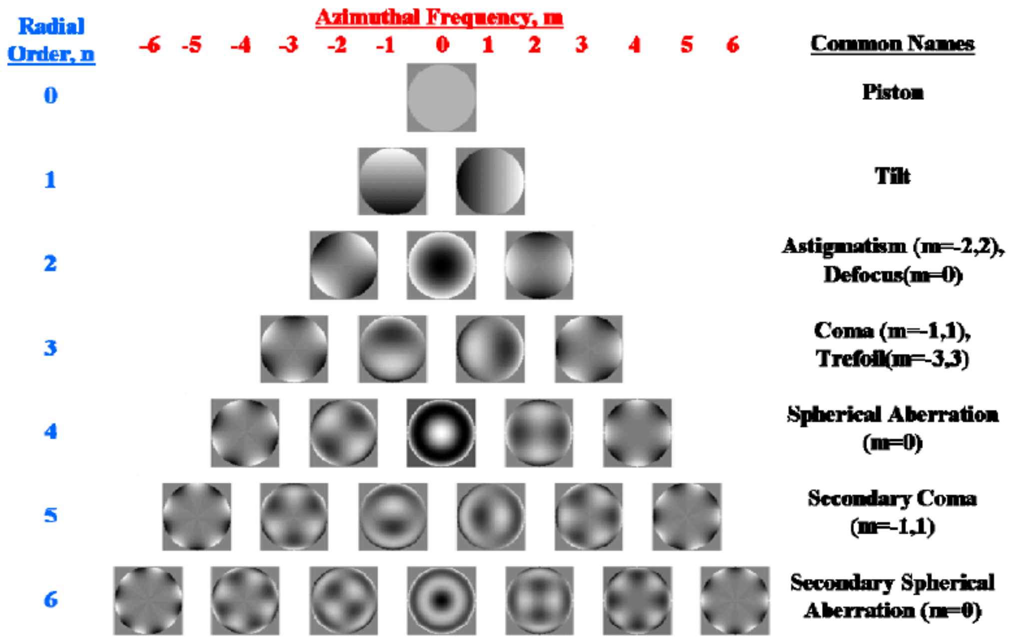


The basic AO principle is to determine in a rapid timescale how an image is being distorted by the medium or environment through which it passes and from the measurement of the distortions apply a correction to the wavefront and then use a "closed loop" feedback to generate a distortion-free image. Adaptive optics are heavily used in astronomy to correct for the aberrations introduced by the earth's atmosphere. Adaptive optic systems remove the phase aberrations that arise in propagating wavefronts to ensure that a high quality image is obtained. Generally, adaptive optic systems use the principle of phase conjugation to correct the distortions in the wavefronts. This is done by adding aberrations of equal amplitudes of the opposite sign.



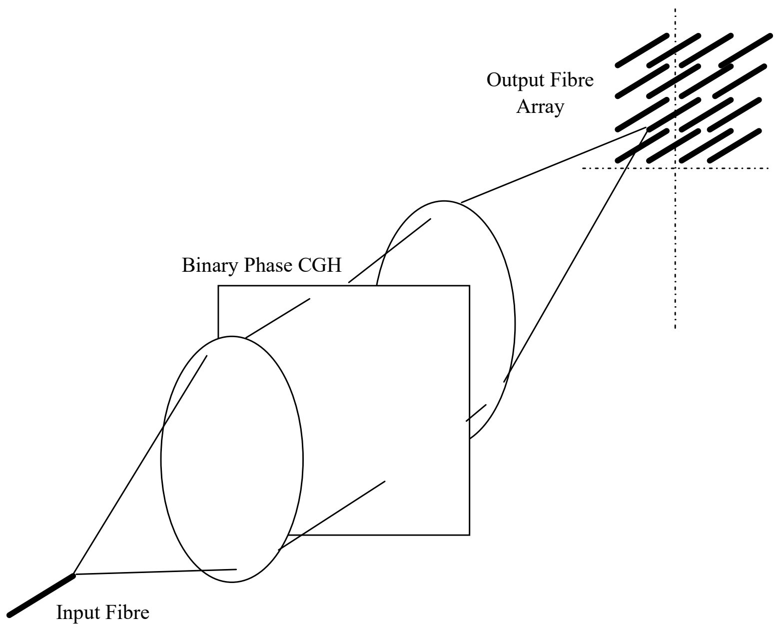
b) [30%] For optical systems aberrations can be defined mathematically by characterizing the aberrations in terms of power series expansions. As most optical systems have an axis based on circular symmetry, it is useful to expand the wave aberration in terms of a complete set of basis functions that are orthogonal over the interior of a circle. Zernike polynomials satisfy this condition as they form a complete set of functions or modes that are orthogonal over a circle of unit radius.

We use Zernike polynomials because they have a number of beneficial mathematical properties. They are orthogonal over the continuous unit circle and all their derivatives are continuous. They are efficient at representing errors that are common in optics such as coma, spherical aberration, and astigmatism. Finally, they form a complete set, meaning that they can represent arbitrarily complex continuous surfaces given enough terms. Examples of Zernike polynomials are shown in below. The lowest order is generally referred to as Piston and corresponds to a movement of the wavefront up and down.



In order to represent an aberration optically, we calculate a series of Zernike polynomials that best match the shape of the aberration. In most applications, the dominant effects of the aberration can be corrected for using only the lowest few Zernike orders, greatly simplifying the correction task. Optical modulators such as deformable mirrors are well suited to the use of Zernikes as they match the modal shapes formed when the membrane in the mirror is deformed.

c) [25%]



There are many different sources of optical aberration in this switch architecture.
1) There may be wavefront distortion from the optical souces and the apodisation of the optical fibre plus distorion to the intensity profile (non-Gaussian)
2) Aberrations from the collimation lens. Errors from the edges of the lens due to fabrication. Plus mis-alignmnet ot the input beam (shift/tip/tilt)
3) There may be imperfections in the SLM, especially due to non flatness or non uniformity of the FLC cell gap
4) Aberrations from the field lens. Errors from the edges of the lens due to fabrication. Plus mis-alignmnet ot the input beam (shift/tip/tilt). There are also higher order aberrations as the lens has to focus the outer fibre channels off-axis which creates a high degree of coma and astigmatism.
5) Outer channels are off-axis and also suffer from fan-in where the mode is not on axis with the fibre core.

d) [15%] There are two inherent features of the FLC SLM which can be harnessed as part of an AO system. The FLC will be binary phase, hence the lower half of the replay field will have a symmetric copy of the output fibre spots. This half plane could be used to detect the shape and structure of the spots to identify possible aberrations. Special holograms or even a closed iterative loop could be used to characterise the nett effect of all the above aberration sources and generate an overall estimate of the aberration in the system. The is effectively a wavefront detector.

Once the aberration is known, it can be either conjugated or even better, added to the routing hologram generation algorithm to create holograms with internally conjugated correction terms within it. This is using the diffractive approach to correct for the defects within the overall optical system if the switch.