4B11 2019 cribs – more verbose than expected

Q1 Overall a popular question that was well answered as it followed the bookwork required from the notes. Quite a few candidates derived the FT in part a) rather than using it to prove the required formula. d) had quite a few good answered which showed that there was a good understanding of the principles of pixel shape.

a) [20%] Far field diffraction pattern = Fourier transform of the aperture.

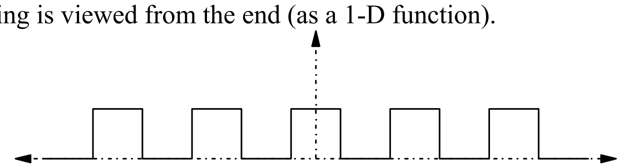
$$F(u,v) = \iint_{\pm\infty} f(x,y) e^{2\pi j(ux+vy)} dxdy = A\int_{-a/2}^{a/2} e^{2\pi j(ux)} dx \int_{-a/2}^{a/2} e^{2\pi j(vy)} dy$$

$$= A\left[\frac{e^{2\pi ju}}{2\pi ju}\right]_{-a/2}^{a/2}\left[\frac{e^{2\pi jv}}{2\pi jv}\right]_{-a/2}^{a/2} = \frac{A}{2\pi ju}\left(e^{a\pi ju} - e^{-a\pi ju}\right)\frac{1}{2\pi jv}\left(e^{a\pi jv} - e^{-a\pi jv}\right)$$

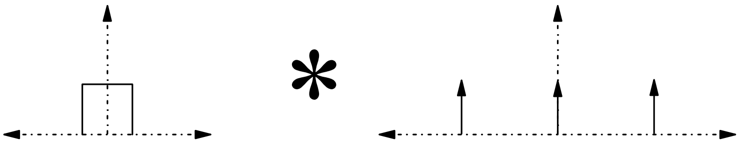
$$= \frac{A}{\pi u}\sin \pi u \frac{1}{\pi v}\sin \pi v = Aa^2 \text{sinc}(\pi au)\text{sinc}(\pi av)$$

Assumptions: far field is coaxial from the propagating axis. The far field is far enough from the aperture to be in the Faunhopher region. Aperture is illuminated by a coherent, monochromatic plane wave source. Aperture is constant and much smaller than the distance to the far field.

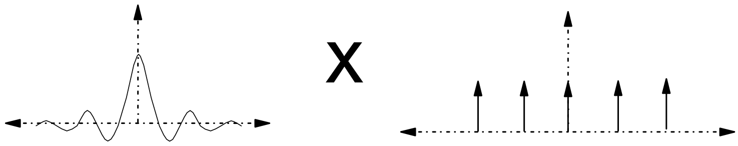
b) [30%] If the 1D amplitude grating is viewed from the end (as a 1-D function).



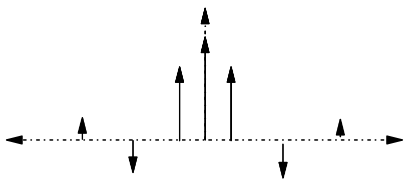
Which can be expressed as a convolution of two functions.



After the Fourier transform.

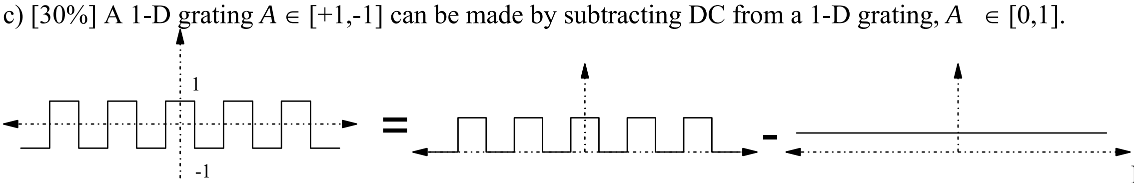


Gives the final result.

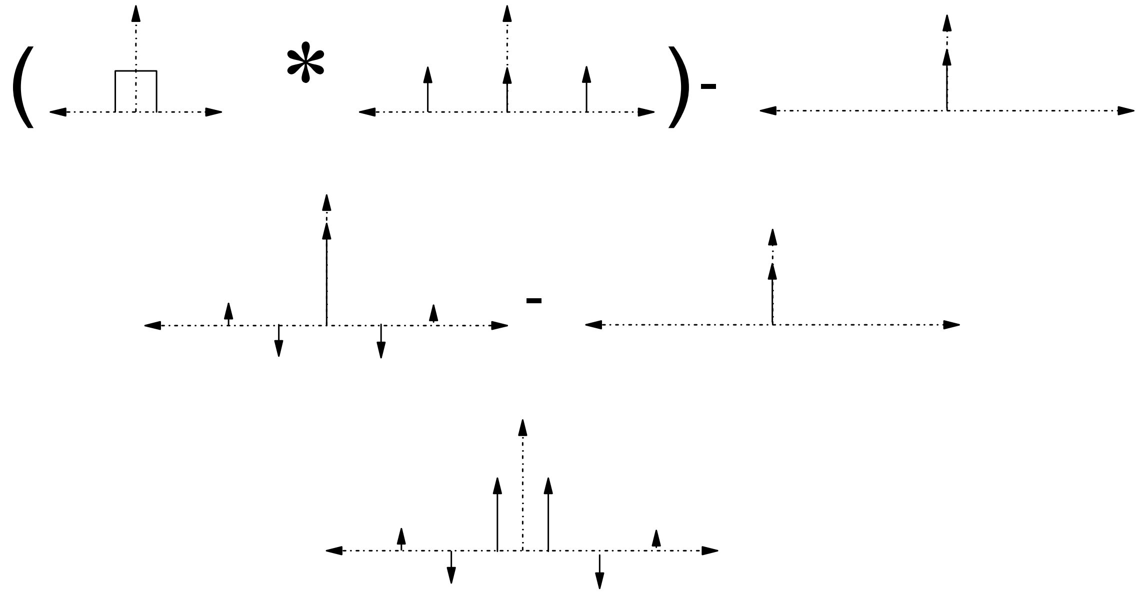


The train of delta functions has a sinc envelope and every second delta function is suppressed by the zeros of the sinc. The 2-D far field region of the 2-D grating shown above will be the same structure as the 1-D case (a sinc enveloped train of delta functions) superimposed onto the y axis of the 2-D far field plane.
Assumptions: No deadspace, 50:50 mark space ratio. Uniform coherent, monochromatic illumination.
Estimated efficiency = 50% (amplitude)*0.41 = 20%

c) [30%] A 1-D grating $A \in [+1,-1]$ can be made by subtracting DC from a 1-D grating, $A \in [0,1]$.



Hence in the Fourier domain we have:

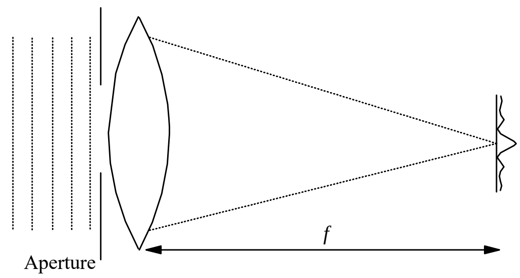


Hence the far field diffraction pattern is the same as for part b, but with no zero order and the efficiency is now 41%.

d) [20%] The 1D analogy shows that the envelope of the orders in the replay field are dominated by a sinc function that is proportional to the size of the overall aperture (a). It we extend this to the two dimensional plane, then the plane is now enveloped by a 2D sinc function, with the first order contained within the central sinc lobe. The geometry of the replay field still has a square symmetry all be it via a sinc envelope, which arises from the hard boundaries of the square aperture. If we move top a hexagonal pixel, the hard boundaries are now hexagonally symmetry is, hence we would expect the replay filed to have the envelope of a sinc function, but still retain the hexagonal geometry of the first order or central lobe. The outer orders would also retain the hexagonal packing.

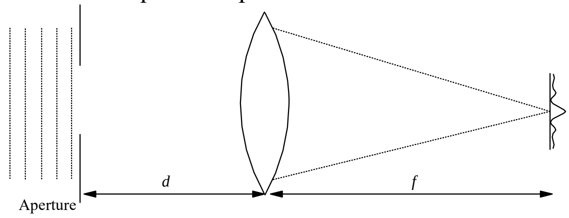
Q2 This was the least popular question in the exam, which was unusual as it contained a very similar theme to a question a couple of years back. First part was book work and fairly well answered. b) Was not so well answered which was a little surprising as it was basic theory. A few calculated the pixel pitch rather than the size of the RPF. c) was ok, but few got d) to scale the hologram targets.

a) [35%] If a positive focal length lens is included directly after the aperture, the far field pattern appears in the focal plane of the lens. A positive lens performs a Fourier transform of the aperture placed behind it. If we consider the aperture *A(x,y)* placed just before a positive lens of focal length *f*, then we can calculate the field just after the lens as described by Goodman, by the paraxial approximation. The application of Snell's law at the spherical lens/air boundaries of the lens shows that the lens converts plane waves incident upon it into spherical waves convergent on the focal plane. For this reason, the diffraction to the far field pattern now occurs at in the focal plane of the lens. The effect of this is to shift the far field or Fraunhofer region away from the distance *R* to the single position specified by the focal length *f*. The final result for the diffracted aperture *A(x,y)* through the lens is.



$$E(\alpha,\beta) = e^{\frac{jk}{f}\left(\alpha^2+\beta^2\right)} \iint_A A(x,y)\, e^{\frac{jk}{f}(\alpha x+\beta y)}\, dxdy$$

Far field region = Focal plane of a positive lens = FT{ Aperture function }. It looks like we are getting something for nothing, but this is not the case, as the lens introduces the quadratic phase distortion term in front of the transform.



If the aperture is placed a distance d behind the lens, then there will be a corresponding change in the phase distortion term of the Fourier transform.

$$E(\alpha,\beta)=e^{\frac{jk}{2f}\left(1-\frac{d}{f}\right)\left(\alpha^2+\beta^2\right)}\iint_A A(x,y)e^{\frac{jk}{f}(\alpha x+\beta y)}dxdy$$

From this equation we can see another way of removing the phase distortion. If the distance is set so that $d = f$, then the phase distortion is unity and we have the full Fourier transform scaled by the factor of the focal length, *f.* This is a very important feature used in the design of optical systems and is the principle behind the *4f* system. In a *4f* system, there are two identical lenses separated by a distance *2f*. This forms the basis of a low distortion optical system.

In spatial coordinates $u = k\alpha/2\pi f$ and $v = k\beta/2\pi f$, hence the RPF scale proportional to focal length and inversely with wavelength ($k = 2\pi/\lambda$).

b) [25%] A two dimensional grating or hologram comprises of *NxN* square apertures (pixels) with a pixel pitch Δ having an amplitude *A*. The two dimensional envelope due to the fundamental pixel which covers the far field diffraction pattern (or FT) of the hologram is just a 2-D sinc function (where $a = \Delta$).

$$A\Delta^2\mathrm{sinc}\left(\pi\Delta u\right)\mathrm{sinc}\left(\pi\Delta v\right)$$

The useful information of the replay field is contained in the central first order lobe of the sinc function, so we can calculate the width of the replay field as where the first zero of the sinc function occurs ($\pi\Delta u = \pi$, $\pi\Delta v = \pi$). We want the coordinates in terms of [α,β], so we use the above transformation to get.

$$\alpha_M=\frac{f\lambda}{\Delta}\quad\beta_M=\frac{f\lambda}{\Delta}$$

Hence f =150mm, $\Delta = 13\text{x}1^{-6}$m, λ=412, 532 and $635\text{x}10^{-9}$m and the size of the first order = $2\alpha_m$ = 9.5, 12.3, 14.7 mm

When used as a holographic projector, the problem will be that the size of the RPF of the three primary colours will be different and will not overlap to form a single colour image. The other problem is that the lens may not be an achromat and it may have different focal and aberration properties for different wavelengths.

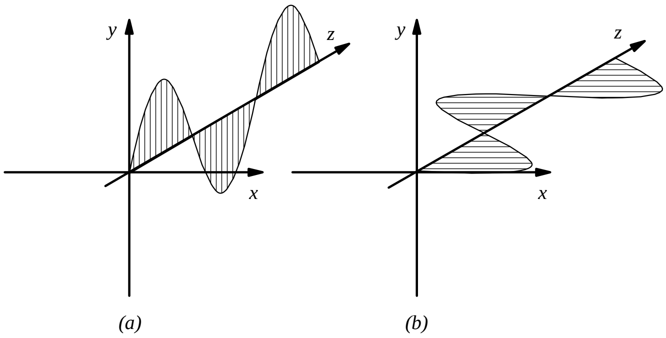
c) [25%] Solution 1. Use 3 SLMs, one for each wavelength and then scale the RPFs with different optical paths (focal lengths), or possibly different pixel pitch SLMs, to create a single frame form 3 colour all the same scale. Disadvantages – three times as expensive as there are now 3 SLMs and 3 sets of optics required. Optically difficult to alight the 3 sets of optics, may have mechanical issues. Advantage – all 3 SLMs and lasers can be run simultaneously if required which will increase the available frame rates per SLM.

Solution 2. All 3 wavelengths go through a single SLM and are switched in sequence to give an RG and B frame per full frame cycle. The holograms have to scaled to fit the common optics, hence the scales in part (b) can be used to scale the desired RPF images before the calculation of the hologram for each colour. Disadvantages – takes 3 times as long to display one frame or have to run the SLM and lasers at 3 times video frame rate. Optics have to be designed to work identically for all 3 colours. The scaling process will change the sampled resolution of the 3 frames to compensate for size given the constant pixel pitch, hence the resolution of the generated image will be lower. Hologram calculation is more difficult and has less time. Advantages – cheaper and simpler to build. Time averaging can be implemented as part of a colour algorithm over many frames in a sequence.

d) [15%] If the replay field is defined as being at 3f/4 rather than f, then the result will not be a Fourier transform of the original hologram but some other transformation. It can be calculated by taking the FT of the hologram and then diffracting it backwards by f/4 to form the Fresnel transform or fractional Fourier transform. The FT is calculated using extra terms that represent the break down of the original assumptions made when forming the far field diffraction pattern.

Q3 Popular question, well answered, with a good understanding of Jones matrices. Almost all got the derivation of the polarisor fully correct but there were a few errors on the FLC derivation as many used the nematic formula instead of the FLC

a) [30%] Monochromatic, coherent light sources such as lasers can be represented in terms of an orthogonal set of propagating eigenwaves which are usually aligned to the *x* and *y* axes in a coordinate system with the direction of propagation along the *z* axis.



Vertically $V = \begin{pmatrix} 0 \\ V_y \end{pmatrix}$ (a) and Horizontally $V = \begin{pmatrix} V_x \\ 0 \end{pmatrix}$ (b) polarised light

We can now combine these two eigenwaves to make any linear state of polarisation we require. We could represent this as V_x and V_y and use the two states combined into a single Jones matrix. $V = \begin{pmatrix} V_x \\ V_y \end{pmatrix}$ We can also represent more complex states of polarisation such as circular states. So far we have assumed that the eigenwaves are phase (i.e. they start at the same point). We can also introduce a phase difference ϕ between the two eigenwaves which leads to circularly polarised light. In these examples, the phase difference ϕ is positive in the direction of the z axis and is always measured with reference to the vertically polarised eigenwave (parallel to the y axis), hence we can write the Jones matrix. $V = \begin{pmatrix} V_x \\ V_y e^{j\phi} \end{pmatrix}$ There are two states, which express circularly polarised light. If ϕ is positive, then the horizontal component leads the vertical and the resultant director appears to rotate to the right around the z axis in a clockwise manner and is right circularly polarised light.

Right Circular $\quad V = V_x \begin{pmatrix} 1 \\ j \end{pmatrix} \qquad$ Left Circular $\quad V = V_x \begin{pmatrix} 1 \\ -j \end{pmatrix}$

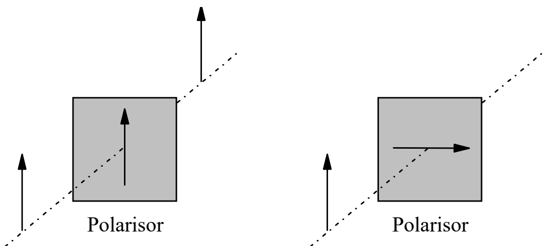
Other values of ϕ lead to elliptical polarisation states.

The main limitation of Jones matrices is that they only analyse the forward propagating waves, hence any reflections or boundary conditions are not propagated. This can be fixed with the use of extended Jones, Muller or Berremann matrices.

b) [30%] Polarisors pass a single polarisation state whilst blocking all others.
Two types: linear and circular.

The polarisor can be written as a Jones matrix. If the direction of the polarisor is such that is passes vertically polarised light. $P_y = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$

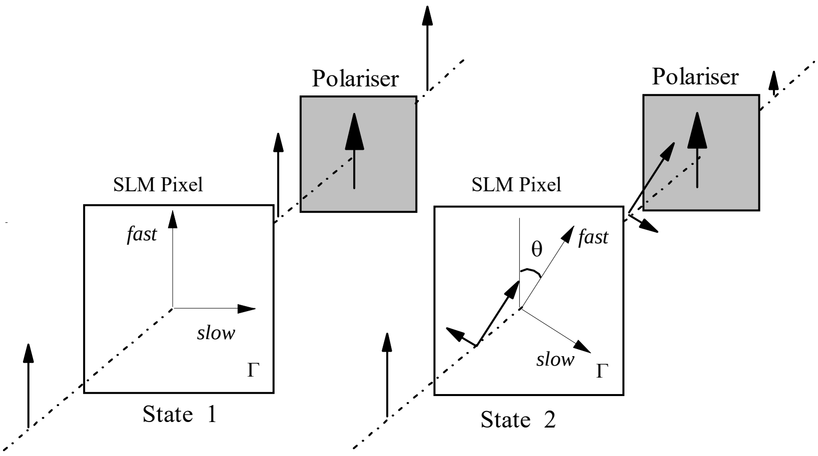
Similarly, the polarisor can be rotated about the z axis by an angle ψ such that. $P = R(-\psi)\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} R(\psi)$ Giving a generalised polarisor.



$$P = \begin{pmatrix} \sin^2\psi & -\frac{1}{2}\sin 2\psi \\ -\frac{1}{2}\sin 2\psi & \cos^2\psi \end{pmatrix}$$

Assumptions – that the origin is at $x = y = z = 0$. The angle ψ angle is measured with respect to the y axis and that a clockwise rotation about the z axis is defined as being negative.

c) [40%] Binary Intensity Modulation



If the light is polarised so that it passes through an FLC pixel parallel to the fast axis in one state, then there is no change due to the birefringence and the light will pass through a polarisor which is also parallel to the fast axis. If the pixel is then switched into state two, the fast axis is rotated by θ and the light now undergoes some birefringent action. We can calculate the retardance Γ of the liquid crystal layer for a given cell thickness d, birefringence Δn and wavelength λ. $\Gamma = \dfrac{2\pi d \Delta n}{\lambda}$. We can use Jones matrices to represent the optical components from left to right.

State 1
$$\begin{pmatrix} V_x' \\ V_y' \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} e^{-j\Gamma/2} & 0 \\ 0 & e^{j\Gamma/2} \end{pmatrix} \begin{pmatrix} 0 \\ V_y \end{pmatrix} = \begin{pmatrix} 0 \\ V_y e^{j\Gamma/2} \end{pmatrix}$$

State 2
$$\begin{pmatrix} V_x' \\ V_y' \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} e^{-j\Gamma/2}\cos^2\theta + e^{j\Gamma/2}\sin^2\theta & -j\sin\frac{\Gamma}{2}\sin(2\theta) \\ -j\sin\frac{\Gamma}{2}\sin(2\theta) & e^{j\Gamma/2}\cos^2\theta + e^{-j\Gamma/2}\sin^2\theta \end{pmatrix} \begin{pmatrix} 0 \\ V_y \end{pmatrix} = \begin{pmatrix} 0 \\ V_y\left(e^{j\Gamma/2}\cos^2\theta + e^{-j\Gamma/2}\sin^2\theta\right) \end{pmatrix}$$

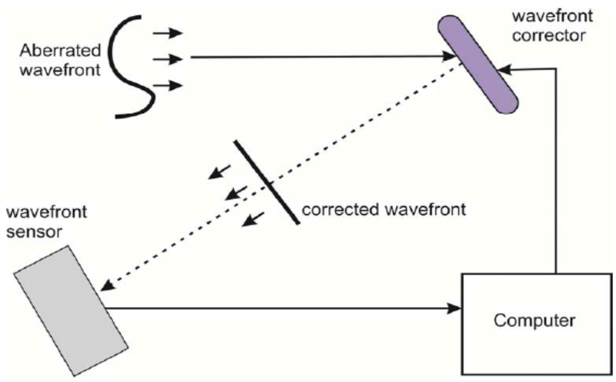
If the thickness of the FLC is set so that $\Gamma = \pi$, then the light in the direction of the slow axis will be rotated by 180º. This leads to a rotation of the polarisation after the pixel, which is partially blocked by the following polarisor. Maximum contrast ratio will be achieved when state 2 is at 90º to the polarisor and the resulting horizontal polarisation is blocked out. This will occur when

$$V_y\left(e^{j\Gamma/2}\cos^2\theta + e^{-j\Gamma/2}\sin^2\theta\right) = V_y\left(j\cos^2\theta - j\sin^2\theta\right) = 0$$

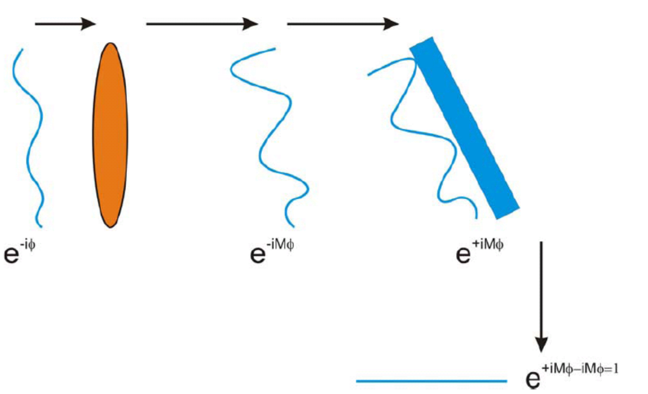
Hence, the optimum FLC switching angle for a FLC is $\theta = 45^o$ and $d = \lambda/2\Delta n$.

Q4 A popular question well answered. Most knew the adaptive optics and conjugation book work well and it was well answered. The Zernike section was also well answered, but a few tried to draw the whole table! c) was well answered with most getting a few of the aberration sources, however a lot of candidates drew a functional diagram rather than an optical layout. Part d) was not so well answered with many thinking of holograms instead of the BPOMF

a) [30%]
The basic AO principle is to determine in a rapid timescale how an image is being distorted by the medium or environment through which it passes and from the measurement of the distortions apply a correction to the wavefront and then use a "closed loop" feedback to generate a distortion-free image. Adaptive optics are heavily used in astronomy to correct for the aberrations introduced by the earth's atmosphere. Adaptive optic systems remove the phase aberrations that arise in propagating wavefronts to ensure that a high quality image is obtained. Generally, adaptive optic systems use the principle of phase conjugation to correct the distortions in the wavefronts. This is done by adding aberrations of equal amplitudes of the opposite sign.

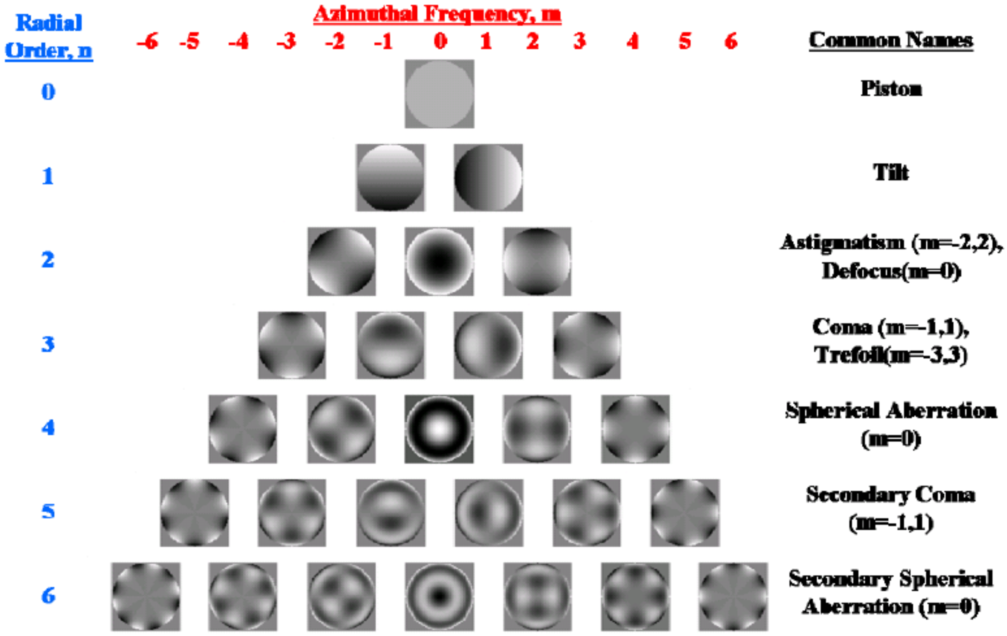


b) [30%] For optical systems aberrations can be defined mathematically by characterizing the aberrations in terms of power series expansions. As most optical systems have an axis based on circular symmetry, it is useful to expand the wave aberration in terms of a complete set of basis functions that are orthogonal over the interior of a circle. Zernike polynomials



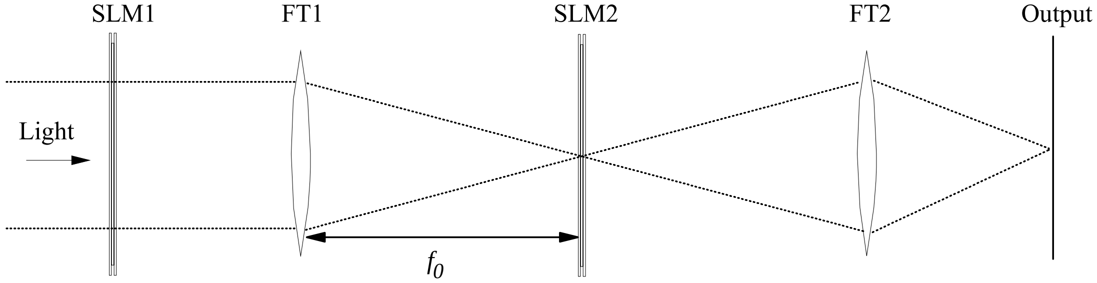
satisfy this condition as they form a complete set of functions modes that are orthogonal over a circle of unit radius.

We use Zernike polynomials because they have a number of beneficial mathematical properties. They are orthogonal over the continuous unit circle and all their derivatives are continuous. They are efficient at representing errors that are common in optics such as coma, spherical aberration, and astigmatism. Finally, they form a complete set, meaning that they can represent arbitrarily complex continuous surfaces given enough terms. Examples of Zernike polynomials are shown in below. The lowest order is generally referred to as Piston and corresponds to a movement of the wavefront up and down.



In order to represent an aberration optically, we calculate a series of Zernike polynomials that best match the shape of the aberration. In most applications, the dominant effects of the aberration can be corrected for using only the lowest few Zernike orders, greatly simplifying the correction task. Optical modulators such as deformable mirrors are well suited to the use of Zernikes as they match the modal shapes formed when the membrane in the mirror is deformed.

c) [25%]



There are many different sources of optical aberration in this switch architecture.
1) There may be wavefront distortion from the optical souces and the apodisation of the optical fibre plus distorion to the intensity profile (non-Gaussian) or non flat laser phase profile.
2) Aberrations from the collimation lens before SLM1. Errors from the edges of the lens due to fabrication. Plus mis-alignmnet ot the input beam (shift/tip/tilt)
3) Aberrations from the lens before FT1, especially of the focal length f0 is compressed with further optics. This will lead to parabolic wavefronts or spherical aberrations.
4) There may be imperfections in both SLMs, especially due to non flatness or non uniformity of the FLC cell gap
5) Aberrations from the second lens FT2. Errors from the edges of the lens due to fabrication. Plus mis-alignmnet ot the input spectrum(shift/tip/tilt)..
5) Further optics may also be needed to scale the FT2 replay field to fit the detection camera at the output.

d) [15%] There are two inherent features of the FLC SLMs, specifically SLM2, which can be harnessed as part of an AO system. The matched filter displayed on SLM2 is calculated offline and could be adapted to include the aberrations within the system. This works well when the filter is calculated via some form of optimisation algorithm which can be adapted to include the complex wavefront distortion. This includes phase conjugation before and after the matched filter plane. The main problem is how to estimate the aberrations within the system. Some may come from ray tracing the system, but others such as

SLM flatness and misalignments are not so simple to estimate in the final system. It should be possible to optimise the filter in situ which would then compensate for all of the distortions.