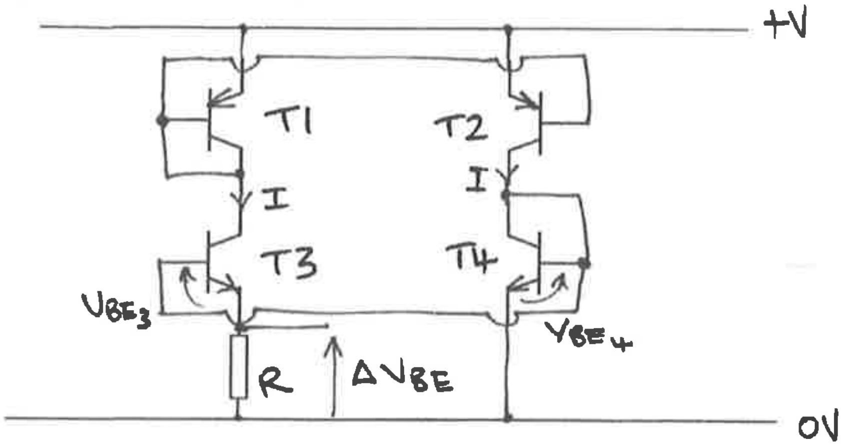
1(a)

4B13
2015

P. Robertson



It is a pair of current mirrors connected together. T1 and T2 are matched transistors which source equal current I to T3 and T4. T3 is arranged to have a collector area of a fixed ratio larger than T4 such that its current density is lower (by a factor of r).

Then, $J_{C3} = J_{S3}\, e^{V_{BE3}/V_t}$ and $J_{C4} = J_{S4}\, e^{V_{BE4}/V_t}$

J = current density, A/m

$I_{C3} = I_{C4}$ $\therefore$ $r J_{C3} = J_{C4} = J$ and $J_{S4} = J_{S3} = J_S$ ⇐ material property – not geometry dependent

$\therefore\ r \cancel{J_S}\, e^{V_{BE3}/V_t} = \cancel{J_S}\, e^{V_{BE4}/V_t}$ $\therefore$ cancel J_S ☺

$\therefore\ \dfrac{kT}{q}\ln r + V_{BE3} = V_{BE4}$ as $V_t = \dfrac{kT}{q}$

$\Rightarrow\ \Delta V_{BE} = \dfrac{kT}{q}\ln r = CT$ ⇐ absolute temperature

constant

$\therefore$ for 0.2mV/K, $k\ln r/q = 2\times10^{-4}$

$\therefore\ \ln r = 2.319$

$\therefore\ r = 10.16$ area ratio

[25%]

(b) @ 1MPa $\epsilon = 0.1\%$, with metal strain gauge G.F. = 2

$\therefore\ \dfrac{\delta R}{R} = 0.2\%$ for 1MPa, with a 4-element full bridge,

the output voltage $= V_s \times \dfrac{\delta R}{R} = 10\text{ mV}$

worst error is when the temp. co. adds to each $\dfrac{\delta R}{R}$ for an extreme case ie: $5\times10^{-6}\times 200 = 0.1\%$

So error could be up to ½ of the 1MPa reading = 500kPa

[15%]

1(c)(i) $R = Ae^{(\beta'/T)}$ $\therefore 10^4 = A e^{\left(\frac{4000}{273+25}\right)} \Rightarrow A = 0.01481$

$\therefore$ @ $60°C$ $R = 0.01481\, e^{\left(\frac{4000}{273+60}\right)} = 2.44\, k\Omega$

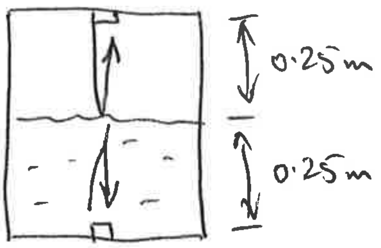
$\therefore V_o = 2.44 \times 10^3 \times 10^{-4} = \underline{0.244\, V}$ [15%]

(ii) @ $60°C$ power dissip. in thermistor $\approx (10^{-4})^2\, 2.44 \times 10^3$
$= 0.0244\, mW$

heat flux

$$P = \frac{kA\,\Delta t}{t} = \frac{0.27 \cdot 2\times10^{-6} \cdot \Delta t}{50\times10^{-6}} = 2.44\times10^{-5}$$

$\therefore \underline{\Delta t = 0.00226\ °C}$ [15%]

d) 

= 0.5 m path

—— " ——

$Z_{air} = 340$ Rayls
$Z_{fuel} = 1168$ Rayls
$Z_{PZT} = 30\times10^6$ Rayls

$$P_{coeff.\ Fuel \to PZT} = \frac{4 \cdot 1168 \cdot 30\times10^6}{(1168 + 30\times10^6)^2} = 1.56\times10^{-4}$$

$$P_{coeff.\ Air \to PZT} = \frac{4 \cdot 340 \cdot 30\times10^6}{(340 + 30\times10^6)^2} = 4.53\times10^{-5}$$

$$Ref.\ coeff.\ Fuel-Air = \frac{(1168-340)^2}{(1168+340)^2} = 0.301$$

Air atten. over 0.5 m = 0.5 dB $= \times 0.891$
Fuel atten. over 0.5 m = 1 dB $= \times 0.794$

$\frac{Fuel}{Air}$ $\therefore$ Relative amplitude as $P = V^2/R$ $= \sqrt{\left(\frac{(1.56\times10^{-4})^2 \times 0.794 \times 0.301}{(4.53\times10^{-5})^2 \times 0.891 \times 0.301}\right)}$ of 0.308 $= \underline{3.25}$ [30%]

Pulse-echo fuel = 313 μs, Pulse-echo air = 1.47 ms $\therefore$ $\underline{diff = 1.16\, ms}$

2(a) Answer should include descriptions of :-

photolithography : mask, expose, develop, etch, remove resist

sacrificial layers : SiO_xN_y, photoresist

crystal plane etching + doping for etch-stop = diaphragms

deposition : plasma Si and SiO_xN_y layers

evap. + sputtering.

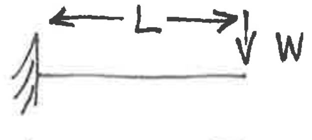
Example geometries : surface μ-fab. accelerometer
bulk pressure sensor

[35%]

(b)(i) Mass of proof mass $= 750\times10^{-6} \times 250\times10^{-6} \times 5\times10^{-6} \times 2330$ kg
$= 2.18\times10^{-9}$ kg

$F = ma$, $F = kx$ — k: spring constant, x: deflection

From structures databook:



$$\delta = \frac{WL^3}{3EI} \qquad \theta = \frac{WL^2}{2EI}$$

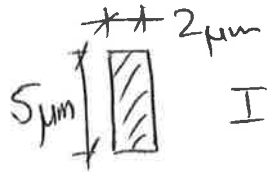
$$\delta = \frac{-ML^2}{2EI} \qquad \theta = \frac{-ML}{EI}$$

Hence, for a double-end clamped beam:

for $\theta = 0$ at end

$$\therefore M = \frac{WL}{2} \qquad \therefore \delta = \frac{WL^3}{3EI} - \frac{WL^3}{4EI} = \frac{WL^3}{12EI} \qquad \therefore \text{stiffness} = \frac{12EI}{L^3}$$

So for springs: $L = 250\,\mu m$
$E = 140$ GPa



$$I = \frac{1}{12}\, 5\times10^{-6}\,(2\times10^{-6})^3 = 3.33\times10^{-24}\ m^4$$

$\therefore$ Stiffness, per spring $= 0.358$ Nm^{-1}

[20%]

$\therefore k_{total} = 4\times0.358 = 1.432$ Nm^{-1}

$\therefore$ resonant freq. $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = 4.08$ kHz

(ii) $$C = \frac{A\epsilon_0}{d} = \frac{250\times10^{-6} \cdot 5\times10^{-6} \cdot 8.854\times10^{-12}}{2\times10^{-6}} = 5.53\times10^{-15}\ F$$

2 (b) (iii) contd.

Consider capacitor plates

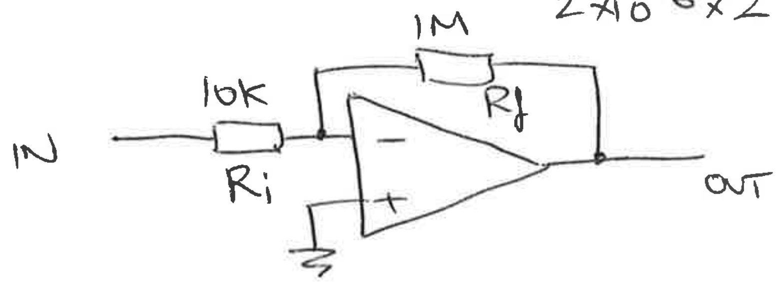
Voltage, v

$$E = \tfrac{1}{2} C v^2 \qquad \text{Force} = \frac{dE}{dx} = -\frac{1}{2}\frac{A\epsilon_0 v^2}{x^2} = \frac{-Cv^2}{2x}$$

$\therefore$ with $k_{total} = 1.432\ \mathrm{Nm^{-1}}$, to deflect $1\,\mu m$ requires $1.432\,\mu N$, but with a Q-factor of 150 at resonance, this falls to $\frac{1.432}{150} \times 10^{-6}\,N = 9.55 \times 10^{-9}\,N$

$$\therefore\ 9.55 \times 10^{-9} = \frac{5.53 \times 10^{-15}\, v^2}{2 \times 10^{-6} \times 2} \Rightarrow \underline{v = 2.63\,V} \quad [25\%]$$

(c)



resistor $v_n = \sqrt{4kTRB}$

$B = 100 \times 10^3\ \mathrm{Hz}$

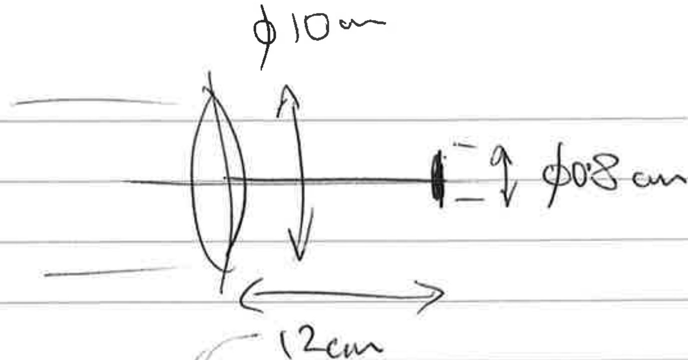
Source (μV_{rms})	gain	noise @ output (μV_{rms})
Thermal Ri 4.07	×100	407
thermal Rf 40.7	×1	40.7
i noise amp. $i_n \times \sqrt{B} \times R_i$ 6.32	×100	632
v noise amp. $v_n \times \sqrt{B}$ 0.95	×100	95

$$\sqrt{\sum v_n^2} = 759\ \mu V_{rms}$$

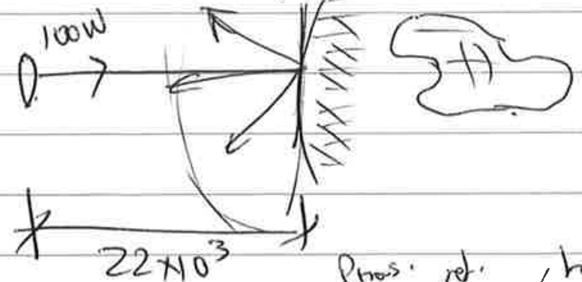
$\therefore$ @ input $= 7.59\ \mu V_{rms}$

$\therefore$ signal must be $7.59\,\mu V \times 10^{\frac{10}{20}} = \underline{24\,\mu V}$ [20%]

3(a)



Ref. = 0.5
ϵ = 0.85



Ptros. ref. hand.

$$P_r = \frac{100 \times 0.5 \times 2}{2\pi R^2} \cdot \frac{\pi\, 0.1^2}{4} = \underline{2.58 \times 10^{-10}\ W}$$

iso into hem.

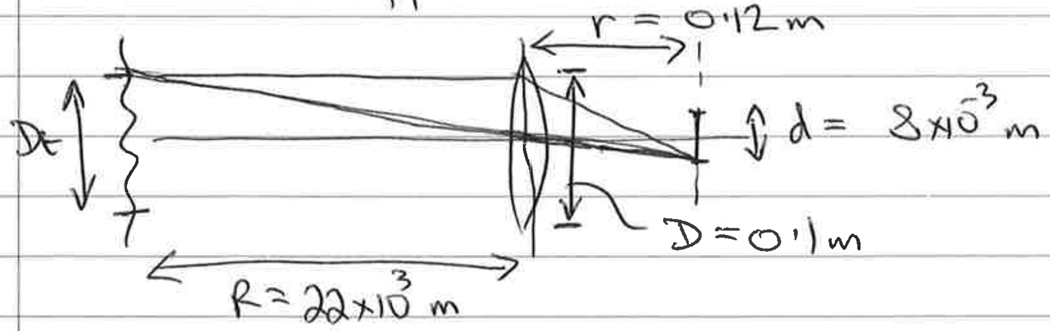
[25%]

3(b)

$$W = \overset{0.85}{\epsilon}\ \overset{5.67\times10^{-8}}{\sigma_{SB}}\ T^4$$

assume = 1

$$\delta W = \frac{W \cos\theta}{\pi} A\, \delta\omega$$



$$\delta W = \frac{W}{\pi} \cdot \underbrace{\frac{\pi D_t^2}{4}}_{A} \cdot \underbrace{\frac{\pi D^2}{4R^2}}_{\delta\omega}$$

$$\delta\omega = \frac{\frac{\pi D^2}{4}}{\cancel{4\pi} R^2} \cdot \cancel{4\pi}$$

$\therefore$ by similar triangles $\frac{D_t}{R} = \frac{d}{r}$ $\therefore$ $\delta W = \epsilon\, \sigma_{SB}\, T^4 \frac{\pi}{16} \frac{d^2 D^2}{r^2}$

3(b) contd. @ $-50°C$, $T = 223\,K \Rightarrow \delta W = 4.21 \times 10^{-13}\, T^4 = 1.04\,mW$

@ $10°C$, $T = 283\,K \Rightarrow \delta W = 2.70\,mW$

$\therefore \Delta(\delta W) = 1.66\,mW$

$D_E = \frac{Rd}{r} = \frac{22\times10^{3} \cdot 3\times10^{-3}}{0.12} = 1467$ m diameter area

(1.69 km²) [25%]

3 C(i)

640 nm $\qquad E = h\nu = \frac{hc}{\lambda}$ ($h = 6.626\times10^{-34}$, $c = 3\times10^{8}$, $\lambda = 640\times10^{-9}$) $= 3.106\times10^{-19}$ J/photon.

$\Rightarrow$ producing 0.5 electrons.

$\therefore 1W = 3.22\times10^{18}$ photons/sec.

$\times 0.5 \times 1.6\times10^{-19} = 0.256$ A/W.

$\therefore 6.65\times10^{-11}\,A = 66.5\,pA$.

$\times 10^{5}\,\Omega \Rightarrow 6.65\,\mu V$ [25%]

(Cii)

assume 300K

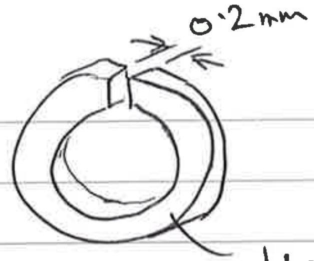
$$V_n = \left[\Big(\underbrace{10^{-13} \times B^{1/2} \times 10^{5}}_{10^{-16}B}\Big)^2 + \Big(\underbrace{10^{-9} B^{1/2}}_{10^{-18}B}\Big)^2 + \Big(\underbrace{4kT\, 10^{5}\, B}_{1.656\times10^{-15}B}\Big) \right]^{1/2}$$

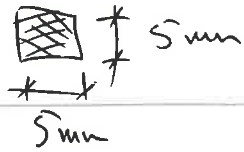
$\therefore V_n = \sqrt{B}\; 4.19\times10^{-8}$ Vrms.

$\therefore 6.65\,\mu V \geq 5 V_n = 5 \times 4.19\times10^{-8} \sqrt{B}$ [25%]

$\therefore B \leq 1.00\,kHz$

4 (a)





$\mu_r = 5800$, $0{\cdot}1\%\ K^{-1}$

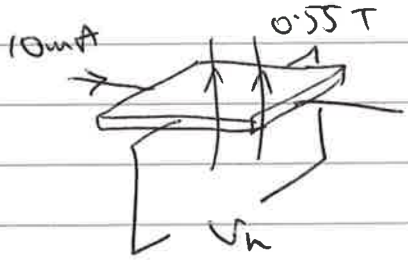
$$\int H.dl = NI = 100 \quad \therefore H_a \times 0{\cdot}2\times10^{-3} + H_p \times \pi \times 50\times10^{-3} = 100$$

$$B_a = \mu_0 H_a = B_p = \mu_0\, 5800\, H_p$$

$$\therefore \quad H_p = \frac{H_a}{5800}$$

$$\therefore 100 = 2\times10^{-4} H_a + \frac{50\pi}{5800}\times10^{-3} H_a = 2{\cdot}27\times10^{-4} H_a$$

$$\therefore H_a = 440\times10^{3}\ A/m \Rightarrow B = 0{\cdot}55\ T$$



$$R = \frac{\rho l}{A} = \frac{0{\cdot}05 . 10^{-3}}{10^{-3} . 5\times10^{-5}} = 1k\Omega$$

$\therefore$ 10mA $\Rightarrow$ 10V supply

Force balance $\quad B q v_d = \frac{v_h}{w} q \quad \therefore \quad$ ~~and~~ $v_d = \frac{10}{L}\mu_{Si}$ $\uparrow 0{\cdot}16$

$$\therefore v_h = w B v_d = 10^{-3} . 0{\cdot}55 . 1600 \qquad \frac{1}{10^{-3}} = 1600\ m/s$$

$$v_h = 0{\cdot}88V$$

[30%]

(b) Neglect thermal expansion.

μ_r $5800 \rightarrow 1{\cdot}025 \times 5800 = 5945$

$$\therefore \quad 100 = 2\times10^{-4} H_a + \frac{50\pi}{5945}\times10^{-3} H_a = 2{\cdot}264\times10^{-4} H_a$$

$\Rightarrow$ 0·3% change

$= 0{\cdot}3A$ @ 100A [20%]

(c) 500m depth $\Rightarrow$ 5·05 MPa $\Rightarrow$ 0·0505 mm gap change in 0·2mm.

$= -25\%$

4(c) contd. $L = \frac{N\Phi}{I}$ where $\Phi = BA$

So for 200 turns @ 0.5A = 100 A turns.

$B = 0.55\,T$ as before

$\therefore L = \frac{200 \cdot 0.55 \cdot (5\times10^{-3})^2}{0.5} = 5.5\,mH$ @ sea level.

As inductance is mainly set by the air gap.
then drop in gap by 25% causes inductance to rise by ~25% ⇒ 6.875 mH
or + 1.375 mH

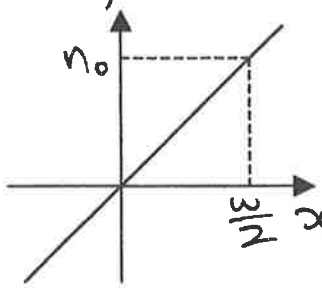
(or 7.06 mH by full analysis)

[25%]

4(d). $\tau = \frac{L}{r} = \frac{20\times10^{-6}}{0.1} = 0.2\,ms$ for inductance of motor.

The response bandwidth may be estimated from considering the charge carrier transit time constant across the device, by diffusion. For simplicity, we shall assume a linear carrier concentration gradient.

n, excess carrier conc.



x, dist. across width from centre line

$n = \frac{2n_0 x}{w}$

$\therefore \frac{dn}{dx} = \frac{2n_0}{w}$

Fick's Law:–

carrier flux $f = -D\frac{dn}{dx}$, $D = \frac{\mu k T}{q}$

N = total excess carriers 1 side:

$N = Ld\int_0^{w/2} \frac{2n_0}{w} x\,dx = \frac{n_0 w L d}{4}$

consider 1 side: $\frac{dN}{dt} = f\,Ld = -D\frac{2n_0}{w}Ld = -D\frac{2\cancel{Ld}}{w}\frac{4N}{w\cancel{Ld}}$

$\therefore \frac{dN}{dt} = -\frac{8DN}{w^2}$ ⇒ soln: $N = N_0 e^{-t/\tau}$ $\therefore -\frac{N_0}{\tau}e^{-t/\tau} = \frac{-8DN_0 e^{-t/\tau}}{w^2}$

$\therefore$ time constant $\tau = \frac{w^2}{8D}$ so, $t_{rise} = 2.2\tau$ and $f_{-3dB} = \frac{1}{2\pi\tau}$

$\therefore \tau = \frac{(10^{-3})^2}{8\times4\times10^{-3}} = 31\,\mu s$.

$D = \frac{0.16 \cdot 1.38\times10^{-23} \cdot 300}{1.6\times10^{-19}} = 4\times10^{-3}$

[25%]

$\therefore$ limit is set by motor ⇒ $f_{-3dB} = \frac{1}{2\pi\tau} = 796\,Hz$

Q1 Thermometry, strain sensing, ultrasonic ranging

A popular question which was well answered by most. The first twin transistor circuit for temperature sensing was correctly recalled by most, as was the thermistor response. However, the thermal error due to self heating evaded a number of people. The ultrasonic section was also quite straightforward and most attempts were along the right lines, although some misinterpreted the beam paths.

Q2 MEMS fabrication, device resonance & noise

A quite well-answered question on MEMs fabrication (surface & bulk micro-machining), with most candidates knowing the main steps, however process details were often missing. The estimate of resonant frequency was also generally well done, but few calculated the beam stiffness accurately. The final part on circuit noise was also reasonably well attempted.

Q3 Pyrometer and optical rangefinder system

A very popular question, which was well answered in most cases. Some neglected to assume the lens area, but used the sensor area instead, for calculating the optical power collected. The pyrometer analysis was generally correctly done. The final part on detector quantum efficiency and useable bandwidth stumped a few candidates, but most had the right idea.

Q4 Inductive & Hall effect sensors

Another quite popular question, which attracted some high quality attempts although some candidates failed to calculate the flux density and self-inductance correctly. Many recalled the Hall effect sensor equations – or at least remembered the end result to use for the bandwidth estimate. Only a few recalled that the time constant for an inductor+ resistor circuit is L/R.

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