

1 (a) Large-scale wind power: Advantages: Abundant resource in the UK with the extensive coastline and wind conditions; technology is now mature; close to cost-parity with fossil-fuelled generation. Disadvantages: Variability of the resource means that back-up conventional generation is still needed; planning permission hard to obtain especially for land-based wind turbines; environmental issues; comparatively short lifetime (25 years).

Bio-mass: Advantages: Power can be controlled to meet demand (similar to fossil-fuelled generation); can convert existing coal-fired power stations to bio-mass. Disadvantages: cost of obtaining and transporting biomass; loss of arable land to grow biomass; time lag from planting biomass to harvesting it.

Tidal barrage: Advantages: Predictable (but variable) power output, depends on tides; considerable potential resource in the UK (Severn estuary estimated at 15% of total UK electrical energy); lifetime of these schemes is high. Disadvantages: Environmental impact; huge capital cost and long lead time from commissioning to generating electricity.

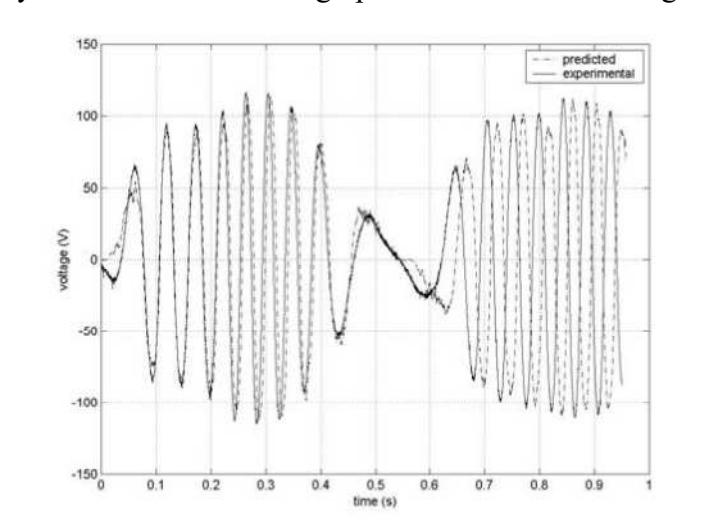
Hydroelectricity: Advantages: Long lifetime of these schemes; very cheap electricity in hindsight; controllable power to help with peak demand; rainfall in the UK means this is an abundant resource, especially in Scotland. Disadvantages: Limited undeveloped sites in the UK; environmental impact; high initial cost. [30%]

(b) Wave power is renewable and so no depletion of fossil fuels and no CO₂ emissions. With the UK's coastline the resource is estimated at around 25% of the UK's electrical energy demand. Wave power is fairly predictable compared to wind power, and so can reduce conventional generation capacity significantly.

Challenges: getting the power ashore; engineering against the hostile marine environment; immature technology so hard to attract investment.

Buoy-type systems utilise a buoy which will raise and lower in phase with the waves to drive a reciprocating generator, such as a vernier hybrid machine.

(c) The vernier hybrid machine produces an output voltage which has a fundamental component which is at the wave frequency and proportional to the wave amplitude, plus a higher frequency which is due to the high pole-number and slotting of the machine.



This is incompatible with a 50 Hz grid and so this output would need to be rectified and inverted before connection to the grid. [20%]

(ii) $P = (0.6 \times 2000 \times 1030 \times 9.81^2 / 32\pi) \times TH^2 = 1.18 \times 10^6 TH^2$.

For $H=1$ m, $T=4$ s this gives $P_{min} = 18.9$ MW and for $H=4$ m, $T=9$ s it gives $P_{max} = 170$ MW. It is also useful to know the power available for $H=2$ m, $T=6$ s which is 28.3 MW. [15%]

(iii) For devices spaced at 5 m intervals there will be $2000/5 = 400$ such devices. Assuming that they are rated for the maximum power available of 170 MW then each vernier hybrid machine has a $170/400 = 425$ kW rating. [5%]

(iv) Completing the table below we can find the annual electrical energy produced.

Wave height (m)	Power (MW)	No of days	No of hours	Energy(GW hr)
1	18.9	100	2400	45.4
2	28.3	200	4800	135.8
4	170	65	1560	265.2

Total annual energy is 446 GWhr. If operating at full capacity the system would produce $365 \times 24 \times 265.2 = 2323$ GWhr in one year giving a capacity factor of 0.192. This is low and suggests that the system should be de-rated whilst ensuring that measures are taken to avoid damage due to the high-amplitude waves. [20%]

Assessor's comment

A mainly well-attempted question that the majority of candidates had a go at. Excellent answers to parts (a) and (b), but surprisingly few candidates were successful with the numerical parts in section (c), the most common mistake being to forget that the expression for the mean power given was mean power per unit width of wavefront, so answers were a factor of 2000 out.

2 (a) The main reason for this limit is the intermittency of wind power, meaning that in periods of high winds excessive electrical power would be produced for which there is no load, possibly destabilising the grid. During periods of low winds, the extra generating capacity is still needed so there is little economic incentive to increase wind capacity. Finally, the existing grid is not rated to cope with the large predominantly north-south power flows that would occur with further integration of wind power.

If the production of electricity by wind could somehow be 'flattened out' then these issues would be reduced. Thus, increased energy storage schemes (pumped-storage, local battery storage e.g. electric vehicles, home/industrial energy storage schemes) would help. Also, greater grid integration enabling excess wind power to be exported and also power to be imported during periods of low wind power. Finally, grid upgrades to higher voltages or the use of FACTS helps with increasing the power transmission capacity of the grid. [25%]

(b) The equation shows that real power is controlled by controlling the load angle of the generator with respect to the load. To transmit more power, this angle has to increase, which is achieved in fossil-fuelled power generation by increasing the prime-mover power. This is done in response to a reduction in the system frequency, which means that the grid is taking energy from the kinetic energy stored in the rotors of the generators connected into it.

The equation also shows that reactive power is controlled by varying the excitation voltage of the generators, or by reducing the reactive power demand at the load by reactive power compensation schemes (FACTS, fixed capacitors etc). Thus, if the

voltage at a node of the power system falls below nominal, either the load power factor has to be corrected or the excitation voltage of the local generators is increased until the voltage is restored to nominal. [35%]

(c) Using the per-unit framework, choose a base MVA of 100 MVA. Thus, the per-unit impedances of the two transformers remain as 0.1 pu.

The base impedance for the transmission line is V_b^2/V_{Ab} ie $275^2/100 = 756 \Omega$ giving a pu reactance of $272/756 = 0.360$ pu.

Thus, the total pu series reactance impedance is $j0.056$.

The per-unit maximum power that can be transmitted is $1.1 \times 1 \times \sin(30)/0.56 = 0.98$ pu = 98 MW. This is well in excess of the maximum power output from the wind farm, so no problem there.

The per-unit average reactive power (meaning the average of the reactive power supplied at the generator and the reactive power absorbed at the load) is $(1.1^2 - 1^2)/(2 \times 0.56) = 0.1875 = 18.75$ MVar.

However, in the worst-case scenario of the load power factor being 0.8 lagging and the load real power being 40 MW, the load reactive power will be 30 MVar which is substantially greater than the capability of the transmission system to deliver.

One way to fix this issue is to reduce the load reactive power so that it is around 18 MVar when the load real power is 40 MW. Thus the capacitors would need to generate 12 MVar of reactive power. Equating to $3V^2/X_c$ in which V is the phase voltage of $33 \text{ kV}/\sqrt{3} = 19.05 \text{ kV}$ gives $X_c = 90.75 \Omega$ and so $C = 35 \mu\text{F}$.

Check: With a load reactive power of 18 MVar the load pu VA = 0.439 and so $I = 0.439$ pu. This gives $0.439^2 \times 0.56 = 0.108$ pu reactive power consumed in the transmission of the power and so the generator must produce 0.4 pu real power and $0.18 + 0.108 = 0.288$ pu reactive power. This gives an excitation voltage of 1.12 pu which is just slightly above the 1.1 pu limit.

(d) FACTS systems enable variable capacitors/inductances to be connected in series/parallel with power systems. In series they act to reduce the series reactance, enabling more power and reactive power to be transmitted. In parallel with loads, they

enable continuously-variable power factor correction which enables greater power flow by removing the need to transmit so much reactive power. [40%]

Assessor's comment

The least popular question on the paper, candidates demonstrated good knowledge for parts (a) and (b), but few obtained the correct answers in the numerical part (c). Most could convert the system into the pu framework, but few managed to find the maximum P and Q that can be transmitted and as a consequence couldn't find the capacitance to enable to grid to operate as required.

3 (a) The electricity supply industry has decades of experience with synchronous machines; synchronous generators can contribute reactive power by controlling their rotor field current; they are fairly reliable, especially brushless, permanent magnet ones (but these cannot contribute variable reactive power). Induction generators are cheap, robust and reliable, especially ones with a (squirrel) cage rotor; they can operate as variable speed devices over a limited speed range. Reliability is an issue for both synchronous/induction machines where brushes/slip rings are used (wound rotor).

Wind turbines have an optimal tip-speed ratio – the ratio of the speed of the blade tip, ωR , to the wind speed, v – at which they operate with maximum power coefficient, maximizing their power output. If the angular speed of the turbine remains in proportion to the wind speed, then the optimal tip-speed ratio is attained at all wind speeds, maximizing power output.

Synchronous generators can be used with a power electronic converter capable of handling the full rated output of the generator. The converter takes the ac power generated by the generator at any frequency, rectifies it to dc, then inverts it to produce 50/60 Hz ac matched to the grid. In this case, the generator is decoupled from the grid frequency. This allows variable speed operation, and large diameter, salient pole generators with many pole-pairs can be used without the need for a gearbox.

Induction generators can operate as variable speed devices over a very limited range, but this can be extended by using the same power electronic converters as above. A power electronic converter that feeds slip frequency power to the rotor via slip rings enables an adjustable speed range in a 2:1 or 3:1 speed ratio, for example. The converter in this case is fractionally rated (advantage). However, a gearbox is still needed and rotor winding is wound and requires slip-rings (disadvantages). The best of both worlds is possible with the brushless doubly-fed generator, where a fractionally-rated converter

supplies a control winding, enabling adjustable speed operation without brushes/slip-rings. [30%]

(b) Let the rated power be P_{\max} [kW]. We know that this occurs at a wind speed of 12 ms^{-1} , and we also know that at constant tip-speed ratio the output power varies as the wind speed cubed. Therefore, we can find the output power of the system at the 6, 10, 14 and 18 ms^{-1} wind speeds by scaling:

$$\text{At } 6 \text{ ms}^{-1} \text{ wind speed } P = (6/12)^3 P_{\max} = P_{\max}/8 = 0.4 \text{ MW}$$

$$\text{At } 10 \text{ ms}^{-1} \text{ wind speed } P = (10/12)^3 P_{\max} = 0.58 P_{\max} = 1.86 \text{ MW}$$

$$\text{At } 14 \text{ ms}^{-1} \text{ wind speed } P = (14/12)^3 P_{\max} = 1.59 P_{\max} = 5.09 \text{ MW (limited to 3.2 MW)}$$

$$\text{At } 18 \text{ ms}^{-1} \text{ wind speed } P = (18/12)^3 P_{\max} = 3.38 P_{\max} = 10.82 \text{ MW (limited to 3.2 MW)}$$

There is no power produced at the 2 ms^{-1} wind speed since it is below the cut-in wind speed, and no power produced at 22 ms^{-1} , since this is above the stall speed.

$\lambda = 8$, $C_p = 0.4$ for all wind speeds in variable speed operation.

Wind speed	MW	Days	Hours	MWhr
6 ms^{-1}	0.4	130	3120	1248
10 ms^{-1}	1.85	90	2160	3996
14 ms^{-1}	3.2	60	1440	4608
18 ms^{-1}	3.2	30	720	2304

Summing the total energy, we have 12.16 GWh. The capacity factor is $12156/(365*24*3.2) = 0.43$. [30%]

(c) (i) Using $\lambda = \omega_t R/v$ gives $\omega_t = 1.96 \text{ rads}^{-1}$

$$P = T\omega_t = 1.96T = 3.2 \text{ MW gives } T = 1633 \text{ kNm} \quad [5\%]$$

(ii) The induction generator operates on the steep part of the torque-speed curve, with a negative slip, at a speed just greater than the synchronous speed, $\omega_s = 2\pi f/p = 2\pi*50/4 = 78.5 \text{ rads}^{-1}$. At the rated speed (12 ms^{-1}), the turbine rotates at 1.96 rads^{-1} , so the gearbox ratio n_g is

$$n_g * 1.96 \geq 78.5, \text{ giving } n_g = 41 (\geq 40.05). \quad [5\%]$$

(iii) With the gearbox, the torque at the generator shaft is $1633 \text{ kNm} / 41 = 39.8 \text{ kNm}$.

Using simplified torque equation: $-39.8 \text{ kNm} = 3V^2s/(\omega_s R_2')$ where $V = 11\text{kV}/\sqrt{3} = 6.35 \text{ kV}$ (star-connected), $\omega_s = 78.5 \text{ rads}^{-1}$ and $R_2' = 0.4 \Omega$.

Rearranging gives $s = -0.01$. $\omega_r = (1-s)\omega_s = 79.29 \text{ rads}^{-1}$. [10%]

(iv) $I = 6.35 \text{ kV}/(R_1 + R_2'/s + j(X_1 + X_2')) = 6.35 \text{ kV}/(0.6 + 0.4/-0.01 + j2.5) =$

$6.35\text{kV}/(-39.4 + j2.5) = -160.52 - j10.19 = 160.84 \text{ A}$

$S = 3VI^* = 3 * 6350 * (-160.52 + j10.19) \rightarrow P = -3.058 \text{ MW}$, $Q = +194 \text{ kVAr}$ [10%]

(iv) $P_{\text{loss}} = 3(I_1^2 R_1 + I_2'^2 R_2') = 3 * ((160.84)^2 * 0.6 + (160.84)^2 * 0.4) = 77.61 \text{ kW}$

$P_{\text{in(mech)}} = P_{\text{out(elec)}} + P_{\text{loss}} = 3.058 + 0.07761 = 3.13561 \text{ MW}$ (3.14 MW)

Efficiency = $P_{\text{in(mech)}}/P_{\text{out(elec)}} = 3.058/3.13561 = 97.5\%$

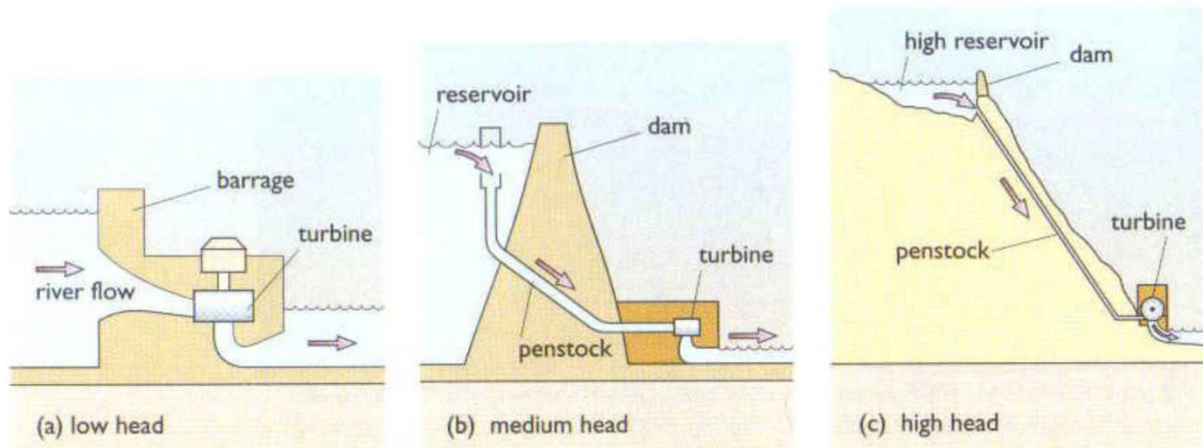
Candidates could also use $P_{\text{in(mech)}} = T\omega_r$ where $T = 3I_2'^2 R_2'/(s\omega_s)$, and then calculate P_{loss} and efficiency. [10%]

Assessor's comment

Not many candidates described the use of power converters well and many described their use for induction machines, but not synchronous machines. Part (c)(ii) many candidates did not round up gear ratio to 41. Part (c)(iii) some candidates found a positive slip, rather than negative (when generating), which affected calculations. Not many candidates used $s = 3vi^*$ in part (c)(iv) to easily calculate the real and reactive power. A number of candidates forgot this is a three-phase generator when calculating losses etc.

4 (a) η represents the system efficiency, taking into account power losses due to frictional drag and turbulence in the water, turbine losses and generator losses. g is the acceleration due to gravity (9.81 ms^{-2}). H is the head of the hydroelectric scheme (height of the water). ρ is the density of water (1000 kgm^{-3}). Q is the volumetric flow rate in m^3s^{-1} .

Hydroelectric schemes are primarily categorised by the head of water available: high head ($H > 100 \text{ m}$), low head ($H < 10 \text{ m}$) or medium head ($10 \text{ m} < H < 100 \text{ m}$).



The specific speed for a turbine for a hydroelectric scheme is given by $N_s = nP^{1/2}H^{5/4}$ where P is power expressed in kW and H in m. The turbines have an optimum specific speed at which they operate. Specific speed is rather like tip-speed ratio for wind turbines, and relates the system rpm, head of water and rated power through the optimum specific speed. Therefore, for a given value of P and H , and knowing the optimum specific speed of the turbine, the optimum rotational speed for the system may be found.

The optimum rotational speed of hydroelectric schemes are typically of the order of several hundred rpm. Ideally the generator will require no gearbox, and be able to connect directly to the grid. Because of their relatively low rotational speed, salient-pole synchronous generators fit both of these requirements and they can accommodate a large number of poles. They are also favoured because their excitation emf can be controlled so that they contribute to the reactive power demand of the power system. [30%]

(b) (i) For a head of 80 m and volumetric flow rate of $15 \text{ m}^3\text{s}^{-1}$, a Francis turbine would be most suitable for this scheme. [5%]

(ii) Using the power equation given in part (a), $P = 0.8 \times 9.81 \times 80 \times 1000 \times 15 = 9.42 \text{ MW}$ (9.418 MW)

Using $N_s = nP^{1/2}H^{5/4} \rightarrow n = 243.4 \times 80^{5/4} / 9418^{1/2} = 600 \text{ rpm} = 62.83 \text{ rad/s}$ [10%]

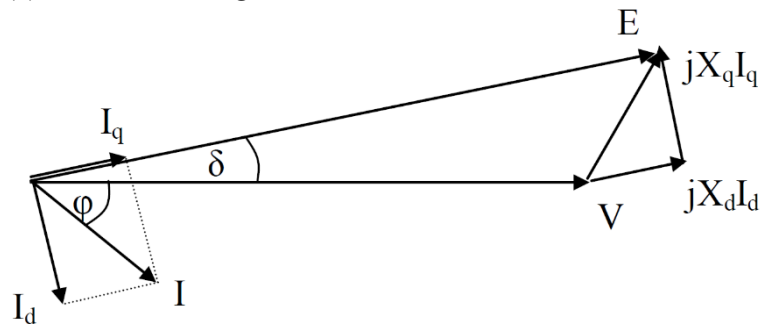
(iii) $P = T\omega \rightarrow T = 9418 / 62.83 = 149.9 \text{ kNm}$ [5%]

(iv) $N [\text{rpm}] = 60f/p \rightarrow p = 60 \times 50 / 600 = 5$ [pole-pairs]

VA rating at power factor 0.8 lagging: $S = P / 0.8 = 9418 / 0.8 = 11.77 \text{ MVA}$

[10%]

(c) Phasor diagram:



Star connected, so $V = 11 \text{ kV}/\sqrt{3} = 6.35 \text{ kV}$

$$P = 3VI\cos\phi \rightarrow I = 9.418 \text{ MW}/(3*6.35 \text{ kV}*0.8) = 618 \text{ A}$$

$$\phi = \cos^{-1}(0.8) = 36.9^\circ$$

From the phasor diagram:

$$I_q = I\cos(\phi+\delta) = I(\cos\phi\cos\delta - \sin\phi\sin\delta)$$

$$V\sin\delta = X_q I_q = X_q I(\cos\phi\cos\delta - \sin\phi\sin\delta)$$

Divide through by $\cos\delta$ and make $\tan\delta$ the subject of the equation:

$$\tan\delta = X_q I\cos\phi/(V+X_q I\sin\phi) = 0.9*618*0.8/(6350 + 0.9*618*0.6) = 444.96/(6350 + 333.72)$$

$$\rightarrow \text{load angle, } \delta = 3.81^\circ$$

$$I_d = I\sin(\phi+\delta) = 618*\sin(36.9^\circ + 3.81^\circ) = 618*\sin(40.71^\circ) = 403.1 \text{ A}$$

$$E = V\cos\delta + I_d X_d = 6350*\cos(3.81^\circ) + 403.1*1.5 = 6336 + 604.65 = 6940.65 \text{ kV} = 12.02 \text{ kV (line)} \approx 12 \text{ kV (line)} \quad [40\%]$$

Assessor's comment

Generally answered well. Some candidates incorrectly didn't use units of kw in specific speed equation, leading to errors. Deriving equations using phasor diagram was generally very good, but numerical errors led to many candidates getting the wrong answers numerically.