Solutions to 4B21
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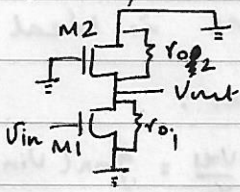
1.) (a) The primary advantage of CMOS is the high gain that is possible in a common source amplifier. (and subsequent amplifier families)

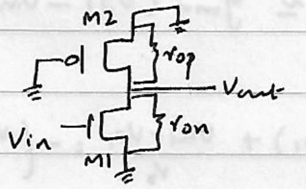
The reason for the high gain is that the output impedance of a CMOS common source is very high (Theoretically ∞ if we ignore channel length modulation).

As an example consider a non-CMOS and CMOS common source amplifier. for analysis.



$$g_{m1} V_{in} + \frac{V_{out}}{r_{o_1}} = \underbrace{-g_{m2} V_{out}}_{\text{This is the problem.}} - \frac{V_{out}}{r_{o_2}}$$

$$\frac{V_{out}}{V_{in}} = -g_{m1} \left(\underbrace{g_{m2}^{-1}}_{Low.} \,\|\, \underbrace{r_{o1} \| r_{o2}}_{High} \right) \Rightarrow \text{Gain is low.}$$



$$g_{m1} V_{in} + \frac{V_{out}}{r_{on}} = -\frac{V_{out}}{r_{op}}.$$

no current due to V_{out} in the PMOS.

$$\Rightarrow \frac{V_{out}}{V_{in}} = -g_{m1} \left(\underbrace{r_{on} \| r_{op}}_{High} \right) \Rightarrow \text{Gain is high.}$$

Marking method (see highlight below)

WHILE MARKING:-

I am looking for two statements: (i) High Gain (ii) High output impedance being the reason for the high gain.

(b) (i) Current through the MOSFETs $= \frac{\mu C_{ox}}{2} \frac{W}{L} \underbrace{(V_{ov})}_{\text{Gate overdrive voltage.}}{}^2$

for the NMOS M1 & M2, the current = 0.1mA, $V_{ov} = 0.2V$

$$\Rightarrow \frac{\mu_n C_{ox}}{2} \frac{W}{L} (0.2)^2 = 0.1e-3$$

$\mu_n = 1000\, cm^2/Vs$, $C_{ox} = 400e-9\, F/cm^2 \Rightarrow (W/L)_{m1} = (W/L)_{m2} = 12.5$

for the PMOS M3 & M4, current = 0.1mA, $V_{ov} = 0.2V$

$$\Rightarrow \frac{\mu_p C_{ox}}{2} \frac{W}{L} (0.2)^2 = 0.1e-3 =$$

$\mu_p = 250\, cm^2/Vs$, $C_{ox} = 400e-9\, F/cm^2 \Rightarrow (W/L)_{m3} = (W/L)_{m4} = 50.$

for the NMOS M5, current = 0.2mA as it sinks the current from Both Branches $\Rightarrow$ $\left(W/L\right)_{M5} = 25.$

(ii) We need to solve for the gain in the a.c circuit.

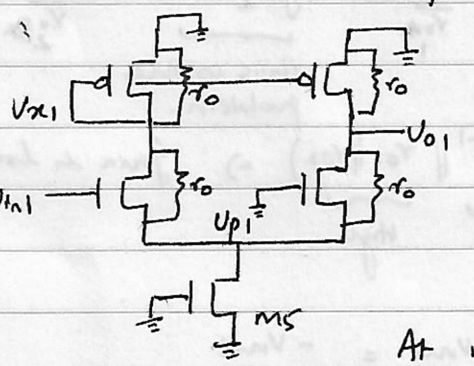
Differential Gain = $\dfrac{V_{out}}{(V_{in+}) - (V_{in-})}$

We will find output V_{o1} due to V_{in1} (i.e V_{in+}) alone
V_{o2} due to V_{in2} (i.e V_{in-}) alone

Then by superposition: $V_{out} = V_{o1} + V_{o2}$

We then calculate: $\dfrac{V_{out}}{V_{in1} - V_{in2}} = A_d =$ Differential Gain.

Consider in V_{in1} alone and ground V_{in2}. ($M5$ is ideal.)



At node V_{x1}:

$$-g_{mp} V_{x1} - \frac{V_{x1}}{r_o} = g_{mn}(V_{in1} - V_{p1}) + \frac{V_{x1} - V_{p1}}{r_o}$$

Since $(g_{mn} r_o, g_{mp} r_o) >> 1$

$$V_{x1} g_{mp} \simeq g_{mn}(V_{p1} - V_{in1}) \longrightarrow (1)$$

At node V_{p1}:

$$g_{mn}(V_{in1} - V_{p1}) + \frac{V_{x1} - V_{p1}}{r_o} = -\left(\frac{V_{o1} - V_{p1}}{r_o} - g_{mn} V_{p1}\right)$$

$$\Rightarrow g_{mn} V_{in1} - \frac{V_{x1}}{r_o} = -\frac{V_{o1}}{r_o} + V_{p1}\left(2g_{mn} + 2/r_o\right)$$

Using (1) & the fact $(g_{mn} r_o, g_{mp} r_o) >> 1$.

$$V_{p1} \cong \frac{V_{in1}}{2} + \frac{V_{o1}}{2 g_{mn} r_o} \longrightarrow (2)$$

At node V_{o1}:

$$-\frac{V_{o1}}{r_o} - g_{mp} V_{x1} = \frac{V_{o1} - V_{p1}}{r_o} - g_{mn} V_{p1}$$

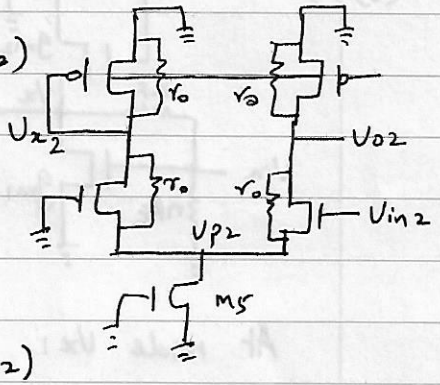
Using (1) $\Rightarrow$ $-\dfrac{V_{o1}}{r_o} - g_{mn} V_{p1} + g_{mn} V_{in1} = \dfrac{V_{o1}}{r_o} - \dfrac{V_{p1}}{r_o} - g_{mn} V_{p1}$

Using (2) $\Rightarrow$ $\left(g_{mn} + \dfrac{1}{2r_o}\right) V_{in1} = \left(\dfrac{-1}{2 g_{mn} r_o} + 2\right)\dfrac{V_{o1}}{r_o}$ $\Rightarrow$ $V_{o1} = \dfrac{g_{mn} r_o}{2} V_{in1} \rightarrow (3)$

Consider V_{in2} alone and ground V_{in1} ($M5$ is ideal)

At node V_{x2}:

$$-\frac{V_{x2}}{r_o} - g_{mp} V_{x2} = \frac{V_{x2} - V_{p2}}{r_o} - g_{mn} V_{p2} \Rightarrow V_{x2} g_{mp} = g_{mn} V_{p2} \rightarrow (4)$$



At node V_{p2}:

$$-\left(\frac{V_{x2}-V_{p2}}{r_o} - g_{mn}V_{p2}\right) = \frac{V_{o2}-V_{p2}}{r_o} + g_{mn}(V_{in2}-V_{p2})$$

Using (4).

$$\Rightarrow \quad 2g_{mn}V_{p2} = \frac{V_{o2}}{r_o} + g_{mn}V_{in2}. \longrightarrow (5)$$

At node V_{o2}:

$$-\frac{V_{o2}}{r_o} - g_{mp}V_{x2} = \frac{V_{o2}-V_{p2}}{r_o} + g_{mn}(V_{in2}-V_{p2})$$

using (4) $\Rightarrow$ $V_{o2} = -\frac{g_{mn}r_o}{2}V_{in2} \longrightarrow (6)$

from (3) & (6)

$$V_{o1} = +\frac{g_{mn}r_o}{2}V_{in1}$$

$$V_{o2} = -\frac{g_{mn}r_o}{2}V_{in2}$$

$$\Rightarrow V_{out} = \frac{g_{mn}r_o}{2}(V_{in1}-V_{in2})$$

superposition of V_{o1} & V_{o2}.

$$\Rightarrow A_d = \frac{V_{out}}{V_{in1}-V_{in2}} = \frac{g_{mn}r_o}{2}$$

$$g_{mn} = \mu_n C_{ox}\frac{W}{L}\underbrace{(V_{gs}-V_T)}_{V_{ov}} = 1000 \times 400e-9 \times (12.5) \times 0.2$$
$$= 1e-3 \text{ A/V}.$$

$r_o = 100e3$

$$A_d = \frac{g_m r_o}{2} = 50$$

Marking: this derivation is the key point that is being tested.

2) (a) The problem with a large overlap capacitance is that it provides a feedback pathway between the output and input. This results in the signal source seeing an effective capacitance that is larger than anticipated. The equivalent circuit becomes that shown in Fig 2.

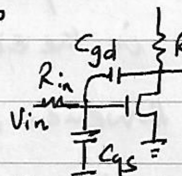


Fig 1.

This is called the "Miller Effect".
For a common source amplifier the gain is $-(g_m Z_{out})$, with Z_{out} the output impedance. The effective capacitance

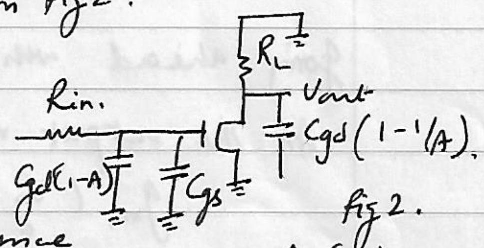


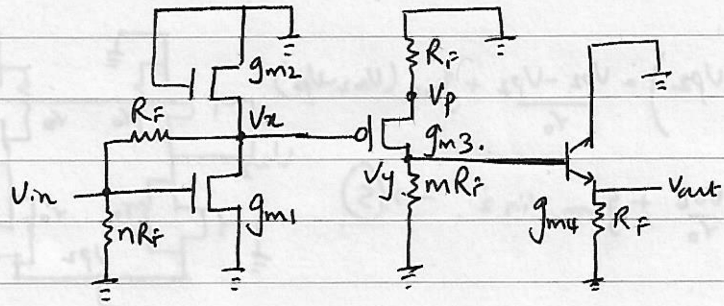
Fig 2.

A = gain.

capacitance becomes $C_{gd}(1-A) + C_{gs} = C_{gd}(1+g_m Z_{out}) + C_{gs}$.
In the example of Fig 1., $Z_{out} \cong R_L \| r_o$. This larger effective input capacitance makes the input pole $R_{in}(C_{gd}(1+g_m Z_{out})+C_{gs})$ more ...

MARKING: Looking for (i) Miller Effect (ii) $C_{gd}(1-A)$ (iii) $A = -g_m R_{out}$

(b) (i)



Marking:
The derivation is being marked.
Small careless errors: -1 point or sometimes forgiven.
Serious conceptual errors: -2 to -3 points.

At node V_x:

$$-g_{m2} V_x = \frac{V_x - V_{in}}{R_F} + g_{m1} V_{in} \Rightarrow -\left(g_{m2} + \frac{1}{R_F}\right) V_x = V_{in}\left(g_{m1} - \frac{1}{R_F}\right)$$

$$\Rightarrow V_x = -V_{in}\left(\frac{g_{m1}R_F - 1}{g_{m2}R_F + 1}\right)$$

At node V_p:

$$-\frac{V_p}{R_F} = -g_{m3}(V_x - V_p) \Rightarrow -V_p\left(\frac{g_{m3}R_F + 1}{R_F}\right) = -g_{m3} V_x$$

$$\Rightarrow V_p = \frac{g_{m3}R_F}{1 + g_{m3}R_F} V_x.$$

At node V_y:

$$\frac{V_y}{mR_F} = -\frac{V_p}{R_F} = -\frac{g_{m3}}{1 + g_{m3}R_F} V_x \Rightarrow V_y = -\frac{g_{m3} m R_F}{1 + g_{m3}R_F} V_x.$$

Note :- Here there is an assumption that the input impedance of the BJT is exceedingly high & is ignored. (This was stated as an assumptions the students could use in the context of the time constraints in the exam)

However more accurately:

$$-\frac{V_p}{R_F} = \frac{V_y}{mR_F} + \frac{V_y - V_{out}}{r_\pi}$$

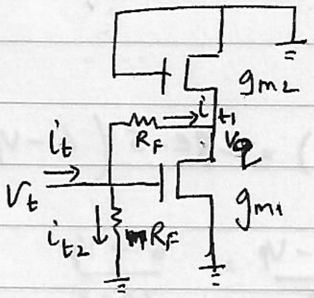
going ahead with the assumption that $\frac{V_y - V_{out}}{r_\pi} << \frac{V_y}{mR_F}$, $V_y = -\frac{g_{m3} m R_F \cdot V_x}{1 + g_{m3}R_F}$

At the output node:

$$g_{m4}(V_y - V_{out}) = \frac{V_{out}}{R_F} \Rightarrow V_{out}\left(\frac{g_{m4}R_F + 1}{R_F}\right) = g_{m4} V_y.$$

$$\therefore \frac{V_{out}}{V_{in}} = \left(\frac{g_{m4}R_F}{1 + g_{m4}R_F}\right)\left(\frac{g_{m3} m R_F}{1 + g_{m3}R_F}\right)\left(\frac{-1 + g_{m1}R_F}{1 + g_{m2}R_F}\right) = \text{Small signal gain.}$$

(ii)



To measure the input impedance, we apply a test voltage V_t.

$$Z_{in} = \frac{V_t}{i_t}$$

$$i_t = i_{t1} + i_{t2}$$

$$i_{t1} = \frac{V_t - V_q}{R_F}, \quad i_{t2} = \frac{V_t}{nR_F}.$$

$$g_{m1} V_t = -g_{m2} V_q - \frac{V_q - V_t}{R_F}$$

$$\Rightarrow \frac{-V_t(g_{m1}R_F - 1)}{g_{m2}R_F + 1} = V_q.$$

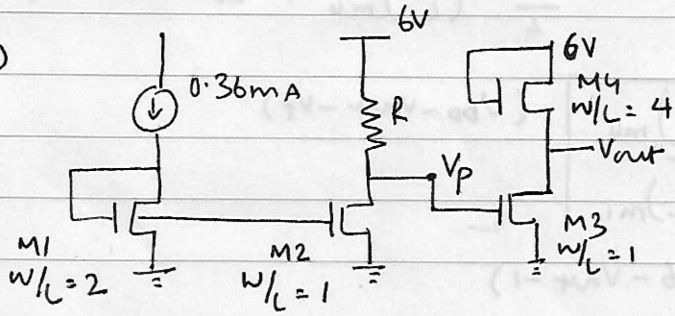
$$i_{t1} = \frac{V_t - V_q}{R_F} = \frac{V_t}{R_F}\left[1 + \frac{g_{m1}R_F - 1}{g_{m2}R_F + 1}\right] = \frac{V_t}{R_F}\left[\frac{g_{m2}R_F + g_{m1}R_F}{1 + g_{m2}R_F}\right].$$

$$i_{t2} = \frac{V_t}{nR_F}$$

$$i_t = \frac{V_t}{R_F}\left[\frac{1}{n} + \frac{g_{m2}R_F + g_{m1}R_F}{1 + g_{m2}R_F}\right].$$

$$\frac{V_t}{i_t} = \frac{R_F}{\left[\frac{1}{n} + \frac{g_{m2}R_F + g_{m1}R_F}{1 + g_{m2}R_F}\right]} = Z_{in}.$$

3.) (a)



(i) for M1 - M2 to be a good current mirror, current in M2 = 0.18 mA. & M2 must be in saturation.

Gate voltage of M2 = Gate voltage of M1 = $\sqrt{\frac{2 \times (0.36e{-3})}{\mu C_{ox} (W/L)_{M1}}} + V_T$

$\mu = 1000 cm^2/Vs$, $C_{ox} = 40$ ~~nF~~ $fF/\mu m^2$, $(W/L)_{M1} = 2$, $V_T = 1$

$\therefore$ Gate voltage of M2 = 4V.

for M2 to be in saturation $V_p \geq 4 - V_T = 3V$

& $\frac{6 - V_p}{R} = 0.18 mA \Rightarrow R \leq 16.67 k\Omega$.

(ii) $R = 20k\Omega \Rightarrow$ M2 is not in saturation.

$\therefore$ Current through M2 =

$$\mu C_{ox}\left(\frac{W}{L}\right)_{m2}\left((V_{GS,m2} - V_T)V_p - \frac{V_p^2}{2}\right) = 2e\text{-}5\,(6 - V_p)V_p.$$

Current through the resistor $R = \dfrac{V_{DD} - V_p}{R} = \dfrac{6 - V_p}{20e3}$.

The two currents are equal

$$\therefore (\cancel{8}^{2}e\text{-}5)(6 \cancel{- V_p})V_p = \frac{(6 \cancel{- V_p})}{20e3}$$

$$\Rightarrow \boxed{V_p = \frac{100}{20 \times \cancel{8}2} = 2.5V}.$$

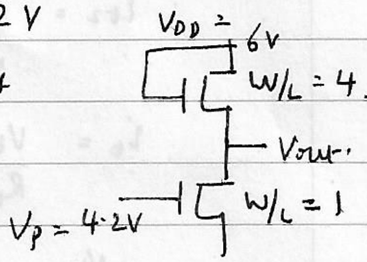
Cross-check $V_p = 2.5V$ is indeed $\leq (V_{GS,m2} - V_T)$

(iii) When $R = 10k\Omega$, M2 is in saturation.

$\therefore$ Current through M2 = 0.18 mA

$$V_p = 6 - (0.18e\text{-}3)(10k\Omega) = 4.2V$$

Considering the output stage, Both M3 & M4 are in saturation.



NOTE: M4 clearly is, but we assume $V_{out} \geq V_p - V_T$ & M3 is in saturation.

We will verify this assumption later.

Since current through M3 = Current through M4.

$$\frac{\mu C_{ox}}{2}\left(\frac{W}{L}\right)_{m3}(V_p - V_T)^2 = \frac{\mu C_{ox}}{2}\left(\frac{W}{L}\right)_{m4}(V_{DD} - V_{out} - V_T)^2$$

$$\Rightarrow V_p - V_T = \left[\sqrt{\frac{\left(\frac{W}{L}\right)_{m4}}{\left(W/L\right)_{m1}}}\right](V_{DD} - V_{out} - V_T)$$

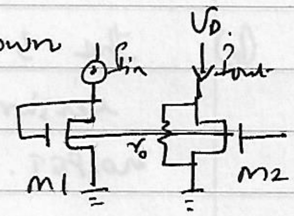
$$\Rightarrow 4.2 - 1 = 2\,(6 - V_{out} - 1)$$

$$\Rightarrow 3.2 = 10 - 2V_{out} \Rightarrow \boxed{V_{out} = 3.4\,V.}$$

4(a) ~~The corner frequency is the fre~~

MARKING: I am looking for (i) the argument on why V_x is practically insensitive to V_D due to the presence of M4. (ii) The role of M3

3)(b.) Consider the conventional 2 MOSFET current mirror shown

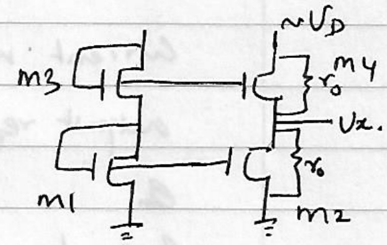


When both MOSFETs are in saturation, the current in M2 in the absence of channel length modulation would be I_{in}.

However, channel length modulation corrupts this current & the real current is $I_{in} + V_D/r_0 = I_{out}$. A fluctuation in V_D creates transients.

A cascode current mirror mitigates this problem.

The current now is corrupted by $\frac{V_x}{r_0}$.



If there is a small signal fluctuation in V_D, what is V_x/r_0?

$$-g_{m4}V_x + \frac{(V_D - V_x)}{r_0} = \frac{V_x}{r_0}$$

$$\Rightarrow V_x\left(\frac{2}{r_0} + g_{m4}\right) = \frac{V_D}{r_0} \Rightarrow V_x = \frac{V_D}{2 + g_{m4}r_0}.$$

$g_{m4}r_0 >> 1 \Rightarrow V_x$ ~~vary~~ varies very little and $\frac{V_x}{r_0} \to 0$.

$\therefore$ M4 protects the output current from variation in V_D.

M3 is used to bias M4 in saturation.

4)(a) The corner frequency is the frequency at which the thermal noise voltage equals the flicker noise voltage.

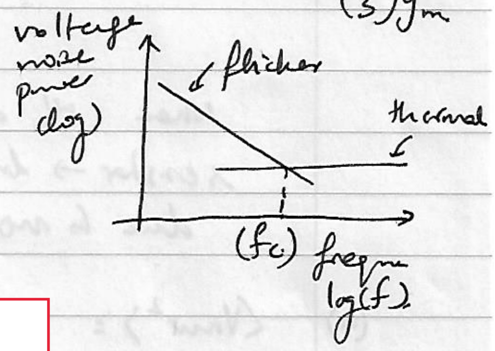
The flicker noise is dominant at low frequencies and $= \frac{\alpha}{C_{ox}WLf}$

The thermal noise is constant at all frequencies and $= 4kT\left(\frac{2}{3}\right)\frac{1}{g_m}$

at $f = f_c$.

$$4kT\left(\frac{2}{3}\right)\frac{1}{g_m} = \frac{\alpha}{C_{ox}WLf_c}$$

$$\Rightarrow f_c = \frac{3g_m\alpha}{8kT C_{ox}WL}.$$



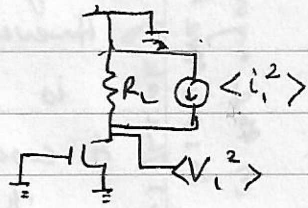
While marking:
I am looking for the definition, the figure and the equation.

While marking: I am seeing if the students considered all 3 noise sources: thermal in RL, thermal in MOSFET, flicker in MOSFET.
And then if the students scaled each one separately with the scaling of gm

(b) The 3 noise components are (i) thermal noise in the resistor (ii) thermal noise in the MOSFET (iii) Flicker noise in the MOSFET.

We consider them one at a time and apply superposition.

Gain: $-g_m R_L = A$
Zout: R_L



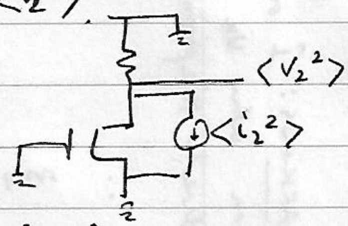
Current noise due to R_L = $\langle i_1^2 \rangle = \frac{4kT}{R_L}$

output reffered noise voltage due to R_L = $\langle v_1^2 \rangle = \langle i_1^2 \rangle Z_{out}^2 = 4kTR_L$.

Current noise due to MOSFET = $\langle i_2^2 \rangle = 4kT\left(\frac{2}{3}\right) g_m$

output ref. voltage noise due to MOSFET = $\langle i_2^2 \rangle Z_{out}^2 = \langle V_2^2 \rangle$.

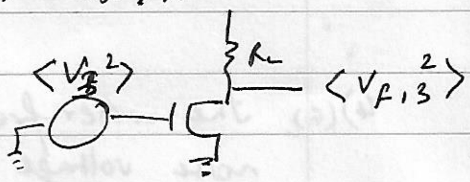
$$\langle V_2^2 \rangle = 4kT\left(\tfrac{2}{3}\right) g_m R_L^2$$



Flicker noise due to MOSFET = $\langle V_{f}^2 \rangle$

output noise voltage due to flicker = $\langle V_{f,3}^2 \rangle = \langle V_f^2 \rangle A^2$.

$$= \frac{\alpha}{C_{ox} W L f} \cdot g_m^2 R_L^2.$$



Total output referred voltage noise

$$\langle V_{out}^2 \rangle = 4kTR_L + 4kT\left(\tfrac{2}{3}\right) g_m R_L^2 + \frac{\alpha}{C_{ox} W L f} g_m^2 R_L^2.$$

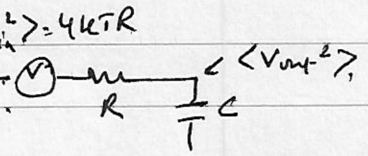
Total input ref voltage noise

$$= \langle V_{in}^2 \rangle = \frac{\langle V_{out}^2 \rangle}{A^2} = \frac{4kTR_L}{g_m^2 R_L^2} + \frac{4kT}{g_m}\left(\tfrac{2}{3}\right) + \frac{\alpha}{C_{ox} W L f}$$

When W/L doubles, g_m doubles and thermal noise due to resistor → drops 1/4, noise due to MOSFET drops 1/2, flicker noise due to MOSFET is invariant.

(c) $\langle V_{out}^2 \rangle = \langle v_{in}^2 \rangle |H(f)|^2$.

Across inf Bandwidth, $\langle V_{out}^2 \rangle_{f\to\infty} = \int_0^\infty \langle v_{in}^2 \rangle |H(f)|^2 . df$



$$\langle V_{out}^2 \rangle_f = \int_0^\infty 4kTR \cdot \frac{1}{1 + 4\pi^2 R^2 C^2 f^2} \cdot df$$

$$|H(f)|^2 = \frac{1}{1 + 4\pi^2 R^2 C^2 f^2}$$

$$= \frac{4kTR}{2\pi RC} \tan^{-1}(2\pi RCf) \Big|_0^\infty = \frac{2kT}{\pi C} \cdot \frac{\pi}{2} = \frac{kT}{C}.$$

∴ when capacitance doubles, the noise reduces by 1/2.