1. (a) (i)

S- parameters


Figure 4.5
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$$
S_{i j}=\left.\frac{V_{i}^{-}}{V_{j}^{+}}\right|_{V_{k}^{+}=0 \text { for } k \neq j}
$$

ABCD paramters:

(a)

$$
\left[\begin{array}{l}
V_{1} \\
I_{1}
\end{array}\right]=\left[\begin{array}{ll}
A & B \\
C & D
\end{array}\right]\left[\begin{array}{l}
V_{2} \\
I_{2}
\end{array}\right]
$$

This was generally well answered, although a common mistake was to not define the directions of the currents and or voltages.
(ii)

By inspection:

$$
\begin{gathered}
{\left[\begin{array}{ll}
A & B \\
C & D
\end{array}\right]=\left[\begin{array}{cc}
1 & 0 \\
1 / Z & 1
\end{array}\right]} \\
S_{11}=\frac{A+\frac{B}{Z_{0}}-C Z_{0}-D}{A+\frac{B}{Z_{0}}+C Z_{0}+D}=-\frac{Z_{0}}{2 Z+Z_{0}}
\end{gathered}
$$

$$
S_{12}=\frac{2(A D-B C)}{A+\frac{B}{Z_{0}}+C Z_{0}+D}=\frac{2 Z}{2 Z+Z_{0}}
$$

$S_{22}, S_{21}$ found by symmetry.

$$
1+S_{11}=\frac{2 Z}{2 Z+Z_{0}}=S_{21}
$$

(b) (i)
$Z_{\text {in }}=Z_{0} \frac{Z_{L}+Z_{0} j \tan (\beta l)}{Z_{0}+Z_{L} j \tan (\beta l)}, \beta=\frac{2 \pi}{\lambda}$ (databook)

$$
40=Z_{1} \frac{(200+100 j)+Z_{1} j \tan (\beta l)}{Z_{1}+(200+100 j) j \tan (\beta l)}
$$

Let $t=\tan (\beta l)$

$$
40\left(Z_{1}+(200+100 j) j t=200 Z_{1}+100 j Z_{1}+Z_{1} j t\right.
$$

Equate real and imaginary parts:

$$
\begin{aligned}
& \text { Re: } 40 Z_{1}-4000 t=200 Z_{1}->Z_{1}=-25 t \\
& \text { Im: } 8000 t=Z_{1}\left(100+Z_{1} t\right) \\
& \qquad 8000 t=-25 t\left(100-25 t^{2}\right) \\
& \quad t= \pm \sqrt{16.8}= \pm 4.10
\end{aligned}
$$

Use -4.10 so that $Z_{1}>0 . Z_{1}=102.5 \Omega$
$\beta l=\tan ^{-1}(-4.10)=-76.3^{\circ}=104^{\circ} \rightarrow l=0.288 \lambda$
This was poorly done by many candidates who tried to find a solution with the Smith chart, this is not suited to the problem since the characteristic impedance of the line is an unknown to be found.
(ii) More generally:
$Z_{\text {in }}=Z_{0} \frac{Z_{L}+Z_{0} j \tan (\beta l)}{Z_{0}+Z_{L} j \tan (\beta l)}$
Let $t=\tan (\beta l)$ This can take any real value while giving positive $l$
We also know that $Z_{i n}$ is real and positive, $Z_{1}$ is real and positive and $Z_{L}=A+B j$ where $A$ must also be positive.

$$
\begin{gathered}
Z_{i n}=Z_{1} \frac{(A+j B)+Z_{1} j t}{Z_{1}+(A+j B) j t} \\
Z_{i n}\left(Z_{1}+(A+j B) j t\right)=Z_{1}\left((A+j B)+Z_{1} j t\right)
\end{gathered}
$$

Equating real parts:

$$
\begin{gathered}
Z_{\text {in }} Z_{1}-Z_{\text {in }} B t=Z_{1} A \\
t=\frac{Z_{\text {in }} Z_{1}-Z_{1} A}{Z_{\text {in }} B}
\end{gathered}
$$

Equating imaginary parts:

$$
Z_{\text {in }}(A t)=Z_{1}\left(B+Z_{1} t\right)
$$

Sub for $t$

$$
\begin{gathered}
Z_{\text {in }} A \frac{Z_{\text {in }} Z_{1}-Z_{1} A}{Z_{\text {in }} B}=Z_{1}\left(B+Z_{1} \frac{Z_{i n} Z_{1}-Z_{1} A}{Z_{\text {in }} B}\right) \\
A \frac{Z_{\text {in }}-A}{B}=\left(B+Z_{1} \frac{Z_{\text {in }} Z_{1}-Z_{1} A}{Z_{\text {in }} B}\right) \\
Z_{\text {in }} A\left(Z_{\text {in }}-A\right)=Z_{\text {in }} B^{2}+Z_{1}^{2}\left(Z_{\text {in }}-A\right) \\
Z_{1}=\sqrt{\frac{Z_{\text {in }} A\left(Z_{\text {in }}-A\right)-Z_{i n} B^{2}}{\left(Z_{\text {in }}-A\right)}}
\end{gathered}
$$

For $Z_{1}$ to be real and positive, sqrt must be positive

$$
Z_{\text {in }} A>\frac{Z_{\text {in }} B^{2}}{Z_{\text {in }}-A}
$$

2. (a) (i) Non-linear distortion. All active components show some degree of non-linearity results in the production of other frequency terms.
In a narrowband transmitter, harmonics can be filtered, but $3^{\text {rd }}$ (and higher) order intermodulation components remain close to signal May breach spectral mask.

Most candidates could say something about distortion, although many missed the point that for a narrowband transmitter, most harmonics can be filtered and it is only really $3^{\text {rd }}$ order intermods which pose a problem. The detrimental impact of this (adjacent channel interference) was also often missed.
(ii)

$$
\begin{gathered}
P_{w_{1}}=P_{\text {in }}+b 1 \\
P_{2 \omega_{1}-\omega_{2}}=3 P_{\text {in }}+b 2
\end{gathered}
$$

Subtract:

$$
\Delta P=P_{\omega_{1}}-P_{2 \omega_{1}-\omega_{2}}=-2 P_{i n}+b 1-b 2
$$

At $I I P 3=P_{\text {in }}, \Delta P=0$

$$
\begin{gathered}
0=-2 I I P 3+b 1-b 2 \\
b 1-b 2=2(I I P 3)
\end{gathered}
$$

So $\Delta P=-2 P_{\text {in }}+2(I I P 3)$

$$
\begin{aligned}
I I P 3 & =P_{\text {in }}+\frac{\Delta P}{2} \\
O I P 3 & =P \omega_{1}+\frac{\Delta P}{2}
\end{aligned}
$$

This was generally done well, although some attempted to start from the Taylor expansion and work in
(ii) $\mathrm{OIP} 3=21 \mathrm{dBm}$

Noise floor $=-140 \mathrm{dBm} / \mathrm{Hz}$

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SFDR = 2/3(OIP3-Noise) = 107.3dB/Hz^2/3
```

Alternatively the noise could be left in dBm, which is what most did.
(b) (i)

$$
\text { F for filter }=\mathrm{IL}=1.5 \mathrm{~dB}
$$

$$
\begin{gathered}
F_{c a s}=F_{1}+\frac{1}{G_{1}}\left(F_{2}-1\right)+\frac{1}{G_{1} G_{2}}\left(F_{3}-1\right) \\
F=1.41+(1.41)(1.58-1)+\left(\frac{1.48}{10}\right)(1.58-1) \\
= \\
=2.31=3.64 d B
\end{gathered}
$$

Noise Power at output:

$$
\begin{aligned}
& P_{n}=G_{c a s} k T_{e_{c a s}} B=k\left(F_{c a s}-1\right) T_{0} B G_{c a s} \\
& P_{n}=7.42 \times 10^{-10} \mathrm{~W}=-61.3 \mathrm{dBm}
\end{aligned}
$$

So P_out for 3dB SNR = 58.3dBm
So at input -86.5 dBm
(ii) To reduce F, maximise gain of early stages.

$F=2 d B$
Filter may prevent overload of amplifiers due to adjacent channels. By moving it the compression on the amplifiers may need to be considered.

Also potential problem if amplifiers are not unconditionally stable.
Some recognised that moving the filter may be a bad idea, so credit was also given for that arrangement with justification.
3. (a)


Antenna height will impact on the spatial separation of the nulls and also the point at which the slope changes. Greater antenna height will give a longer range in the first part of the plot where fading occurs, but the average path loss follows freespace.

Frequency has the opposite effect - higher frequency leads to shorter separations.
For a simple bookwork question answers to this were disappointing with several using free space propagation rather than flat earth
(b) Contributions from background noise which is received by the antenna and thermal noise generated in the antenna loss.

Background noise is from other sources and may be natural or man-made. Overall brightness temperature is the weighted average of the noise sources multipled by antenna gain in that direction, so for the pattern shown where there are significant side lobes, large noise sources in these bands may contribute significantly to the overall brightness temperature.

$$
T_{A}=\eta_{r a d} T_{b}+\left(1-\eta_{r a d}\right) T_{p}
$$

Again simple bookwork, but some missed that radiation efficiency would lead to a thermal noise contribution.
(b) (i) Carrier power $=\mathrm{Si}+\mathrm{Ga}-\mathrm{L}$

All noise referred to amplifier input:
$T_{e}=T_{A}+(F-1) T_{0}+\frac{(L-1) T_{0}}{G}$
Noise power at receiver $=\mathrm{kTeG} / \mathrm{L}=\mathrm{C} /(\mathrm{c} / \mathrm{n})$

$$
\begin{aligned}
& T_{e}=\frac{C L}{k\left(\frac{C}{N}\right) G}=\frac{S_{i} G_{A}}{k\left(\frac{C}{N}\right)} \\
& F=1+\frac{T_{e}}{T_{0}}-\frac{T_{A}}{T_{0}}-\frac{L-1}{G}=1+\frac{S_{i} G_{A}}{k\left(\frac{C}{N}\right)}-\frac{T_{A}}{T_{0}}-\frac{L-1}{G}
\end{aligned}
$$

(iii) QPSK modulation - implies that both I and Q channels need to be recovered.

1 MHz bandwidth - ADC sample rates will be ok for both Low IF and Zero IF approaches (either direct conversion or superheterodyne)

Direct sampling at 1.5 GHz isn't very practical ADCs possible but very expensive.
Satellite application low signal levels, probably need a high gain so multi-stage heterodyne might be attractive to reduce oscillation potential with high gain.
(ii) $P_{r}=\frac{P_{t} G_{t} G_{r} \lambda^{2}}{16 \pi^{2} R^{2}}$

$$
\begin{gathered}
\frac{P_{r} 16 \pi^{2} R^{2}}{G_{t} G_{r} \lambda^{2}}=\frac{P_{t}}{} \\
P_{r}=4 \times 10^{-13} \mathrm{~mW}, R=20 \times 10^{6} \mathrm{~m} G_{t}=25.11, \mathrm{Gr}=4, \lambda=0.19 \mathrm{~m} \\
P_{t}=7 \mathrm{~W}
\end{gathered}
$$

Overall this was a very unpopular question with bipolar answers,
4. (a) Network may not be matched at input and output, so some power from source is reflected back to source and some power from the network is reflected back into the network without been dissipated in the load.

Power gain - $G=\frac{P_{L}}{P_{i n}}$ ratio of power dissipated in load to power delivered to input of 2-port network - independent of $Z_{S}$

Available gain $-G_{A}=\frac{P_{\text {avn }}}{P_{\text {avs }}}$ ratio of power available from 2 port network to power available from source - assumes conjugate matching of source and load, depends on $Z_{S}$ but not on $Z_{L}$

Transducer gain $G_{T}=\frac{P_{L}}{P_{\text {avs }}}$ ratio of power delivered to load to the power available from the source. Depends on $Z_{S}$ and $Z_{L}$

For a conjugate matched amplifier $G=G_{A}=G_{T}$
A well answered bookwork question
(b) Stability:
$K=\frac{1-\left|S_{11}\right|^{2}-\left|S_{22}\right|^{2}+|\Delta|^{2}}{2\left|S_{12} S_{21}\right|}>1$
and
$|\Delta|=\left|S_{11} S_{22}-S_{12} S_{21}\right|<1$
$|\Delta|=0.684$
$K=0.384$
So both conditions for stability are broken.

$$
\begin{aligned}
& C_{L}=\frac{\left(S_{22}-\Delta S_{11}^{*}\right)^{*}}{\left|S_{22}\right|^{2}-|\Delta|^{2}} R_{L}=\left|\frac{S_{12} S_{21}}{\left|S_{22}\right|^{2}-|\Delta|^{2}}\right| \\
& C_{S}=\frac{\left(S_{11}-\Delta S_{22}^{*}\right)^{*}}{\left|S_{11}\right|^{2}-|\Delta|^{2}}, R_{S}=\left|\frac{S_{12} S_{21}}{\left|S_{11}\right|^{2}-|\Delta|^{2}}\right|
\end{aligned}
$$

$$
\begin{gathered}
C_{L}=13.14 \angle 66.4^{\circ}, R_{L}=12.7 \\
C_{S}=2.98 \angle 125^{\circ}, R_{S}=2.45
\end{gathered}
$$

These are large circles at some distance from the centre of the smith, so the centres are off the chart. To plot, the point of closest approach needs to be taken (for load $=0.44$ angle 66.4) then curve can be approximated as straight.

Since S11 and S22 < 1 centre is in the stable region.
Well answered, the size and position of the solutions threw some, and some omitted to state which regions were stable.
(ii) $G_{T}, \max <G_{m s g}=\left|\frac{S_{21}}{s_{12}}\right|$

$$
=42.5
$$

Since the amplifier is not unilateral, the input impedance depends on the output matching and output impedance depends on the source matching. Therefore it is required to simultaneously solve for the input and output matching together if the gain is to be optimised.

Since the amplifier is unstable, it may be necessary to consider the impedance matching of the input and output networks at other frequencies as well.

## Again generally well answered

(ii) unilateral figure of merit:

$$
u \equiv \frac{\left|S_{11} S_{12} S_{21} S_{22}\right|}{\left(1-\left|S_{11}\right|^{2}\right)\left(1-\left|S_{22}\right|^{2}\right)}
$$

$\mathrm{U}=1.3350$
$0.1834<\mathrm{GT} / \mathrm{GU}<8.91$
Can't use the unilateral assumption, so the input and output impedances will have to be simultaneously solved. It will not be possible to use constant gain circles in this case.

