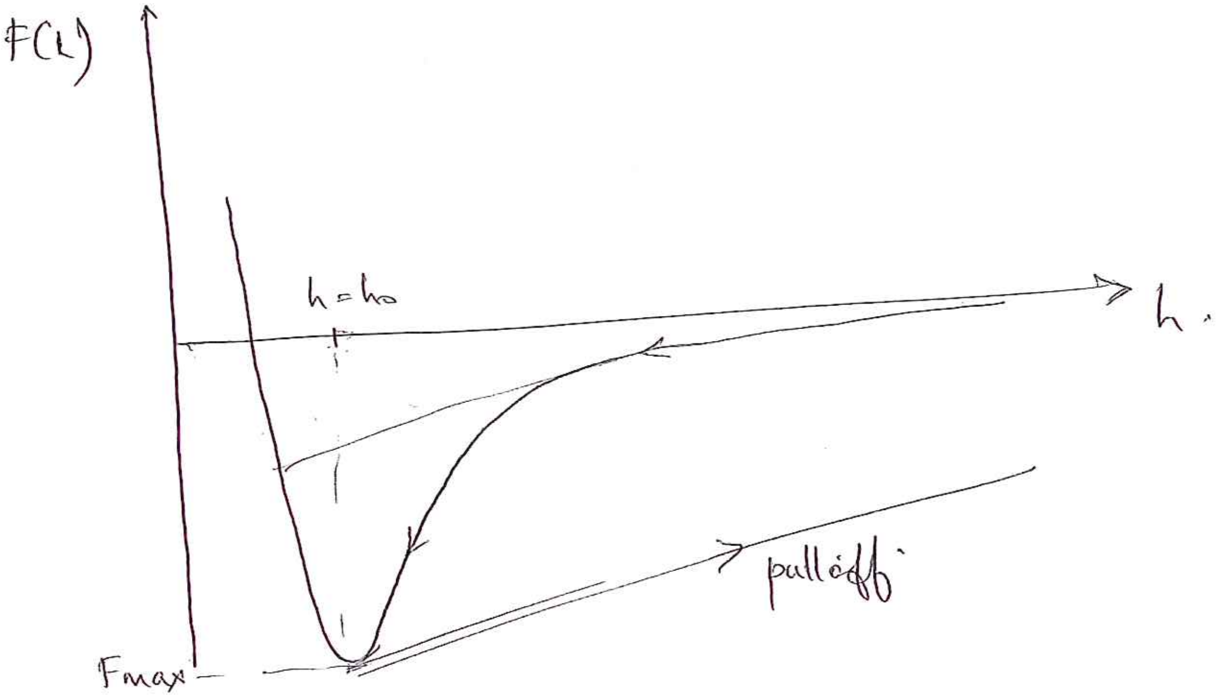
4C15 MEMS DESIGN

Q1

$$F(h) = \frac{8\pi R w}{3}\left\{ \underbrace{\left(\frac{h}{h_0}\right)^{-2}}_{\text{attraction}} - \underbrace{\frac{1}{4}\left(\frac{h}{h_0}\right)^{-8}}_{\text{repulsion}} \right\}.$$



$F(h) = 0$ when

$$\left(\frac{h}{h_0}\right)^{-2} = \frac{1}{4}\left(\frac{h}{h_0}\right)^{-8}$$

$$\frac{h}{h_0} = \sqrt[6]{\frac{1}{4}} = 0.79$$

$$\frac{dF(h)}{dh} = \frac{8\pi R w}{3}\left\{ \frac{-2}{h_0}\left(\frac{h}{h_0}\right)^{-3} + \frac{2}{h_0}\left(\frac{h}{h_0}\right)^{-9} \right\}.$$

$$= 0 \quad \text{when} \quad h = h_0.$$

At instability:

$$\frac{dF}{dh} = \frac{-16\pi R w}{3}\left\{ \left(\frac{h}{h_0}\right)^{-3} - \left(\frac{h}{h_0}\right)^{-9} \right\} = -k.$$

occurs when $h/h_o > 1$ and $\therefore \left(\frac{h}{h_o}\right)^{-9} << \left(\frac{h}{h_o}\right)^{-3}$.

$$\frac{16\pi R w}{3 h_o}\left(\frac{h}{h_o}\right)^{-3} \simeq k.$$

$$\frac{h}{h_o} = \left(\frac{16\pi R w}{3 h_o k}\right)^{1/3}.$$

$$\therefore h = \left(\frac{16\pi R w h_o^2}{3k}\right)^{1/3}.$$

$$\therefore h = \left(\frac{16\pi \times 0.1\times 10^{-6} \times 1000\times 10^{-3} \times (0.2\times 10^{-9})^2}{3\times 10}\right)^{1/3}$$

$$= 1.88 \text{ nm}$$

Distance travelled on pull-off

maximum force $= 2\pi R w$ substituting $h = h_o$ in the expression for $F(h)$.

$$\text{displacement} = \frac{2\pi R w}{k} = \frac{2\pi \times 0.1\times 10^{-6} \times 1}{10}$$

$$= 62.8 \text{ nm}.$$

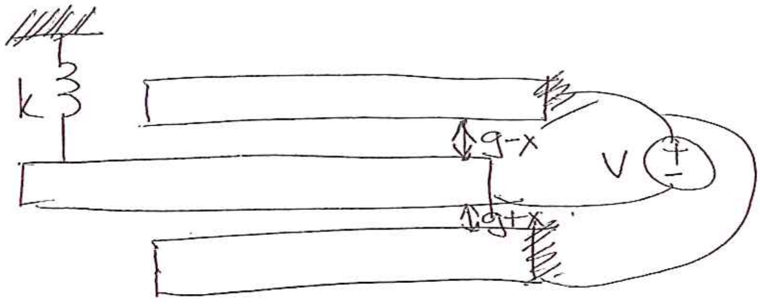
reduction in pull-off force

$$= 2\pi R (w_{old} - w_{new}).$$

$$= 2\pi \times 10^{-7} \times (1 - 0.01)$$

$$= 6.22 \times 10^{-7} \text{ N}$$

Q2



(a) $$F_{net} = \frac{\epsilon_0 A V^2}{2}\left[\frac{1}{(g-x)^2} - \frac{1}{(g+x)^2}\right] - kx$$

Equilibrium:

$F_{net} = 0$

Stability criterion violated when:

(b) $\dfrac{\partial F_{net}}{\partial x} \geq 0$ (electrical spring dominates)

$$\frac{\epsilon_0 A V^2}{2}\left(\frac{1}{(g-x)^2} - \frac{1}{(g+x)^2}\right) = kx$$

$$\frac{\epsilon_0 A V^2}{2}\left[\frac{2}{(g-x)^3} + \frac{2}{(g+x)^3}\right] = k$$

or $$\frac{1}{2}\left[\frac{4gx}{(g-x)^2(g+x)^2}\right] = \frac{2}{2}\left[\frac{2g^3 + 6gx^2}{(g-x)^3(g+x)^3}\right]x$$

$$\frac{4}{2}gx(g-x)(g+x) = 4g(g^2+3x^2).x$$

$x = 0$ or $g^2 - x^2 = g^2 + 3x^2$

$$4x^2 = 0$$

$$\therefore x_{PI} = 0$$

$$\therefore \epsilon_0 A V_{PI}^2\left[\frac{2}{g^3}\right] = k \text{ or } V_{PI} = \sqrt{\frac{kg^3}{2\epsilon_0 A}}$$

compared to the standard expression (see datasheet)

$$V_{PI,standard} = \sqrt{\frac{8}{27}\frac{kg^3}{\epsilon_0 A}}$$

The analysis would not change if the area A is increased by arraying a large number of electrodes together.

(c) The voltage V_1 is reduced to zero. At some point (assuming no adhesion between the surfaces), the restoring force (kx) will be large enough to pull the beam off the electrode. Damped oscillations will then result resulting in the beam finally arriving at the nominal rest position.

(d) $$V_{2PI} = \sqrt{\frac{8}{27}\frac{kg^3}{\epsilon_0 A}}$$

is the pull-in voltage for a single parallel-plate electrode system. Compared to the expression in (b)

$$\frac{V_{2PI}}{V_{1PI}} = \frac{\sqrt{\frac{8}{27}\frac{kg^3}{\epsilon_0 A}}}{\sqrt{\frac{1}{4}\frac{kg^3}{\epsilon_0 A}}} = \sqrt{\frac{2}{27}}$$

Q5

(a) $b_d = \dfrac{\eta A}{h}$

$= \eta \left[\dfrac{A_{mass}}{h_{mass}} + \dfrac{A_{comb}}{h_{comb}} \right]$

$= 1.8\times10^{-5} \left[\dfrac{5\times5\times10^{-6}}{2\times10^{-6}} + \dfrac{500\times10^{-5}\times10^{-4}}{1\times10^{-6}} \right]$

$= 2.34\times10^{-4}$

$Q = \dfrac{25\times10^{-6}\times10^{-4}\times2330\times2\pi\times2\times10^{3}}{2.34\times10^{-4}}$

$= 312$

(b) $F_{comb} = N\epsilon_0 \dfrac{t}{g} V_p V_{ac}$

$x_d = \dfrac{F_{comb}}{m\omega_0^2} \cdot Q = N\epsilon_0 \dfrac{t}{g} V_p V_{ac} \cdot \dfrac{1}{\omega_0 \cdot b}$

$= \dfrac{500\times8.85\times10^{-12}\times100\times10\times0.1}{2\pi\times2\times10^{3}\times2.34\times10^{-4}}$

$= 0.151\ \mu m$

(c) $b_s = \dfrac{\eta A_{mass}}{h_{mass}} + \dfrac{96\eta L W^3}{\pi^4 h^3}\times N$

$= \dfrac{1.8\times10^{-5}\times25\times10^{-6}}{2\times10^{-6}} + \dfrac{96\times1.8\times10^{-5}\times500\times(10^{-5})^3(10^{-4})}{\pi^4 \quad (10^{-6})^3}$

$= 1.8\times10^{-5}(12.5 + 88.9) = 1.82\times10^{-3}$

$Q_s = \dfrac{25\times10^{-6}\times10^{-4}\times2330\times2\pi\times2.1\times10^{3}}{1.82\times10^{-3}} =$ 42.2

(d) $\left|\frac{k y_s}{F_c}\right|$ (from databook) $= \frac{1}{\left\{\left[1-\left(\omega/\omega_n\right)^2\right]^2+\left(\frac{\omega}{\omega_n Q_y}\right)^2\right\}^{1/2}}$

$\therefore\; y_s = \frac{1}{\left\{\left[1-\left(\frac{2}{2.1}\right)^2\right]^2+\left(\frac{2}{2.1\times 42.2}\right)^2\right\}^{1/2}} \frac{\times \Omega_z \times 2\times10^3 \times 0.151\times10^{-6}}{\pi \times (2.1\times10^3)^2}$

$= \frac{10.45 \times 2\times2\times10^3 \times 0.151\times10^{-6}}{360\,(2.1\times10^3)^2} = 3.97\ \text{pm}$

(e) $\Omega_{zn} = \sqrt{\frac{k_B T \omega_s}{Q_y m \omega_d^2 X_d^2}}$

$= \sqrt{\frac{1.38\times10^{-23}\times300\times2\pi\times2.1\times10^3}{42.2\times(2\pi\times2\times10^3)^2\times(0.151\times10^{-6})^2\times5\times5\times0.1\times10^{-9}\times2330}}$

$= 2.44\times10^{-4}$ rad/s/$\sqrt{Hz}$

$= 0.014$ deg/s/$\sqrt{Hz}$

~~(f)~~ .

Q4

(a) $F = \dfrac{\epsilon_0 \cdot A \cdot 2 V_p v_{ac}}{2 g^2}$

$= \dfrac{8.85\times10^{-12} \times 200\times25\times10^{-12} \times 0.2\times10}{(2\times10^{-6})^2}$

$= 22.13$ nN.

(b) $i = \dfrac{d}{dt}(CV_p) = V_p \omega_0 x \cdot \dfrac{dC}{dx} = 10\times2\pi\times400\times10^3\times x\times\dfrac{\epsilon_0 A}{g^2}$

$x = \dfrac{22.13\times10^{-9} \times 50000}{0.139\times10^{-9}\times(2\pi\times400\times10^3)^2}$

$\therefore i = \dfrac{10\times22.13\times10^{-9}\times50000\times8.85\times10^{-12}}{0.139\times10^{-9}\times2\pi\times400\times10^3}\times\dfrac{200\times25}{4}$

$\therefore i =$ 0.35 μA

(c) Equivalent motional parameters obtained by drawing analogies between mechanical and electrical domains

$\dfrac{\dot{x}}{F} = \dfrac{1}{sm+b+k/s}$.

$\Downarrow$

$\dfrac{i/\eta}{\eta V_{ac}} = \dfrac{1}{sm+b+k/s}$

$\dfrac{i}{V_{ac}} = \dfrac{1}{sL_m+R_m+\frac{1}{sC_m}}$

$\therefore \dfrac{i}{V_{ac}} = \dfrac{\eta^2}{sm+b+k/s}$.

where $\eta =$ electromechanical transduction coefficient

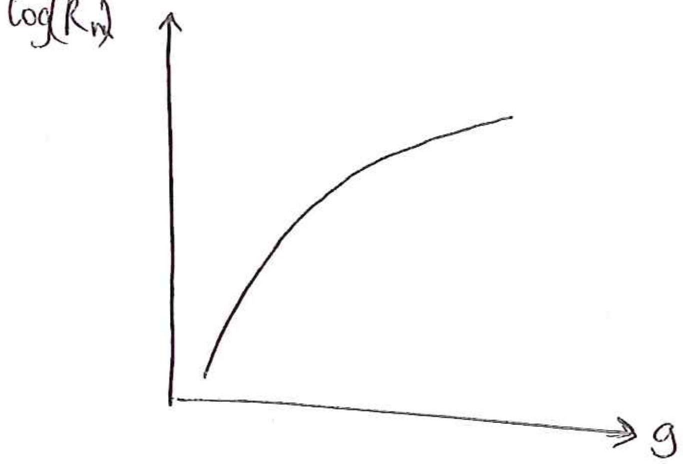
$= \dfrac{\epsilon_0 A}{g^2}\times V_p$ for this case $= 0.111\times10^{-6}$

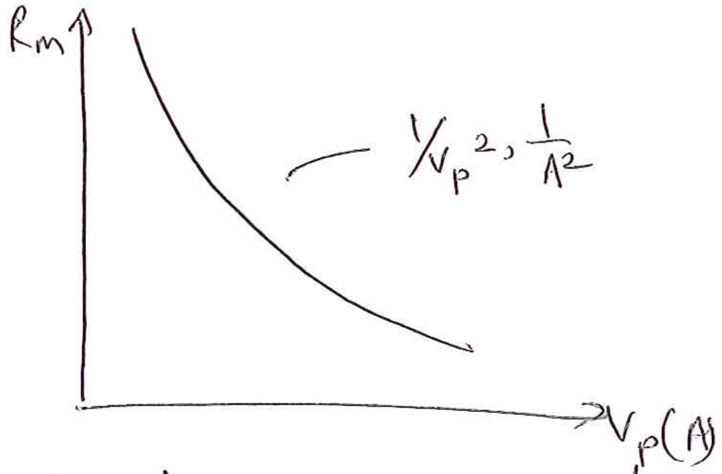
From the above we get :-

$C_m = \dfrac{\eta^2}{k}$, $L_m = \dfrac{m}{\eta^2}$, $R_m = \dfrac{b}{\eta^2}$

$\Rightarrow C_m = 14.1\,aF$, $L_m = 11.28\,kH$, $R_m = 567.3\,k\Omega$

(d) $$R_m = \frac{b}{\eta^2} = \frac{b\,g^4}{(\epsilon_0 A V_p)^2}$$





Approaches to minimise motional resistance include

→ reduce gap, subject to lithography, etch control + process constraints
introduces issues relating to dynamic range and pull-in

→ increase V_p, introduces issues relating to pull-in

→ increase A, subject to process constraints + design parameters (length of beam).

(e) Three sources of nonlinear behaviour include:

(1) spring hardening nonlinearity - geometric stiffening due to clamped-clamped boundary condition

(2) material nonlinearity of silicon.

(3) electrostatic nonlinearity (softening) due to the nonlinear force-displacement characteristic of parallel-plate transducers.

Q1 This question was well done. Most students demonstrated a good understanding of underlying concepts. Some mistakes were made in obtaining an estimate for gap spacing when the cantilever snaps into the substrate in (b) but other parts were generally well done.

Q2 Electrostatic actuator

This question was generally well done. Mistakes were generally made in part (a) (ii) requiring a derivation of the pull-in displacement and pull-in voltage. The case of the standard parallel-plate actuator in (b) was recognised by students who attempted this question.

Q3 Gyroscope

In part (a), the damping due to Couette drag by the comb fingers was often neglected without explanation. Most students were able to complete part (b) correctly but neglected to add the Couette damping of the mass for (c). Errors were made in numerical calculations in (d) and (e) though the majority of students who attempted the question appeared to understand how to set up the calculations for these parts.

Q4 MEMS Resonator

Numerical errors were common in parts (a) and (b) with a factor of 2 often missed out in the force expression in (a). A full derivation of the motional parameters was often not shown in (c). Parts (d) was generally well done but partial / incorrect responses received for (e).