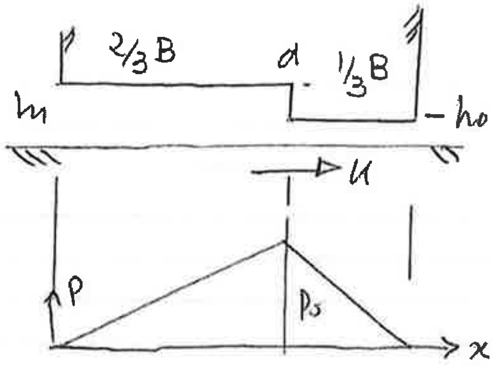
(a) Reynolds' equation from Data Sheet

$$\frac{dp}{dx} = 12\eta\,\bar{u}\,\frac{h-h^*}{h^3}$$

In this case $\bar{u} = U/2$



Since in ~~both~~ inlet & outlet regions h is constant then so is pressure gradient $\frac{dp}{dx}$

Setting pressure at step as p_s

inlet $\qquad \left.\frac{dp}{dx}\right)_1 = \frac{3p_s}{2B} = 6\eta U\,\frac{h_1 - h^*}{h_1^3}$

i.e. $\qquad \frac{6\eta U h^*}{h_1^3} = \frac{6\eta U}{h_1^2} - \frac{3p_s}{2B}$

Outlet $\qquad \left.\frac{dp}{dx}\right)_o = -\frac{3p_s}{B} = 6\eta U\,\frac{h_o - h^*}{h_o^3}$

$$\frac{6\eta U h^*}{h_o^3} = \frac{6\eta U}{h_o^2} + \frac{3p_s}{B}$$

hence $\qquad 6\eta U h_o + \frac{3p_s h_o^3}{B} = 6\eta U h_1 - \frac{3p_s h_1^3}{2B}$

$$\frac{\cancel{3}p_s}{2B}\left(h_1^3 + 2h_o^3\right) = \overset{2}{\cancel{6}}\eta U\,(h_1 - h_o)$$

But $h_1 = h_o + d \qquad p_s\left[(h_o+d)^3 + 2h_o^3\right] = 4\eta U B d$

i.e. $\qquad p_s = \frac{4\eta U B d}{\left[(h_o+d)^3 + 2h_o^3\right]}$

But load per unit width $P' = \frac{1}{2}p_s B$

i.e. $\qquad P' = \frac{2\eta U B^2 d}{\left[(h_o+d)^3 + 2h_o^3\right]}$

Equally well equate volume flow in inlet & outlet again from data sheet $q_x\big)_i = -\frac{h_1^3}{12\eta}\frac{3p_s}{2B} + \frac{h_1 U}{2}$

$$q_x\big)_o = \frac{h_o^3}{12\eta}\frac{3p_s}{B} + \frac{h_o U}{2}$$

equating $\quad \frac{U}{2}(h_1 - h_o) = \frac{3p_s}{12\eta B}\left\{\frac{h_1^3}{2} - h_o^3\right\}$

as before.

(b) Put $D = d/h_o$ then $\quad P' = \frac{2\eta U B^2}{h_o^2}\left[\frac{D}{(D+1)^3+2}\right]$

for given η, U, B, h_o maximising P' requires

$$\frac{d[\;]}{dD} = \frac{(D+1)^3 + 2 - 3(D+1)^2 D}{[\quad]^2} \Rightarrow 0$$

i.e. $\quad D^3 + 3D^2 + \cancel{3D} + 1 + 2 - 3D^3 - 6D^2 - \cancel{3D} = 0$

$$2D^3 + 3D^2 - 3 = 0$$

If $D = 0.8064 \qquad LHS = -.0004$ ✓ then $h_o = \frac{d}{.8064} = 1.24h_o$

(c) Consider shear stresses τ acting on runner; at any position $\tau = \eta \times$ velocity gradient

Velocity gradient dominated by Couette or shear term so to a reasonable approximation

in inlet region $\tau_1 = \eta\frac{U}{h_1}$ and outlet $\tau_2 = \eta\frac{U}{h_2}$

$\therefore F'$, tangential force /unit width $= \frac{2}{3}B\tau_1 + \frac{B}{3}\tau_2$

$$\Rightarrow \frac{2}{3}B\frac{U\eta}{h_1} + \frac{1}{3}\frac{BU\eta}{h_o}$$

$$= \frac{\eta U B}{3h_o}\left\{\frac{2}{D+1} + 1\right\}$$

$$F' = \frac{\eta U B}{3h_o}\left\{\frac{D+3}{D+1}\right\}$$

then $\text{CoF} = \dfrac{F'}{P'} = \dfrac{\eta u B}{3h_0}\dfrac{D+3}{D+1} \cdot \dfrac{h_0^2}{2\eta u B^2}\dfrac{(D+1)^3+2}{D}$

$$\Rightarrow \frac{h_0}{B} \cdot \frac{1}{6}\frac{D+3}{D+1}\frac{(D+1)^3+2}{D}$$

So if $\dfrac{h_0}{B} = \dfrac{1}{1000}$ & $D = 0.806$

$$\text{COF} = \frac{1}{1000} \times \frac{1}{6} \times \frac{3.806}{1.806} \times \frac{1.806^3+2}{.806} = \underline{0.0034}$$

[Aside, allowing for Poisseuille flow provides an additional term

so that $F' = \dfrac{\eta u B}{3h_0}\left\{\dfrac{D+3}{D+1} + \dfrac{6D^2}{(D+1)^3+2}\right\}$

and $\text{COF} = \dfrac{h_0}{B} \times \dfrac{1}{6} \times \left\{\dfrac{D+3}{D+1} + \dfrac{6D^2}{(D+1)^3+2}\right\} \times \dfrac{(D+1)^3+2}{D}$

$$\Rightarrow 0.0042 \;]$$

(d) we have $P' = \dfrac{2\eta B^2 u}{h_0^2} \cdot \dfrac{D}{(D+1)^3+2}$

But $h_0 = d/D$, so interms of d

$$P' = \frac{2\eta B^2 u}{d^2}\frac{D^3}{(D+1)^3+2} = \frac{2\eta B^2 u f(D)}{d^2}$$

If now u is doubled while P' remains constant then

$f(D)$ must halve, ie. $\dfrac{D^3}{(D+1)^3+2} = \dfrac{1}{2} \times \dfrac{.806^3}{1.806^3+2}$

ie. $\dfrac{D^3}{(D+1)^3+2} = 0.0332$

or $30.14\,D^3 = D^3 + 3D^2 + 3D + 3$

ie. $27.14\,D^3 - 3D^2 - 3D - 3 = 0$

Using Solver on calculator gives $D = 0.583$

So that new value of $h_0 = \dfrac{d}{.583} = 1.72d$

ie increase by factor $\dfrac{1.72}{1.24} = 1.39$

and COF becomes

$$1.39 \times \frac{1}{1000} \times \frac{1}{6} \times \frac{3.583}{1.583} \times \frac{1.583^3+2}{.583}$$

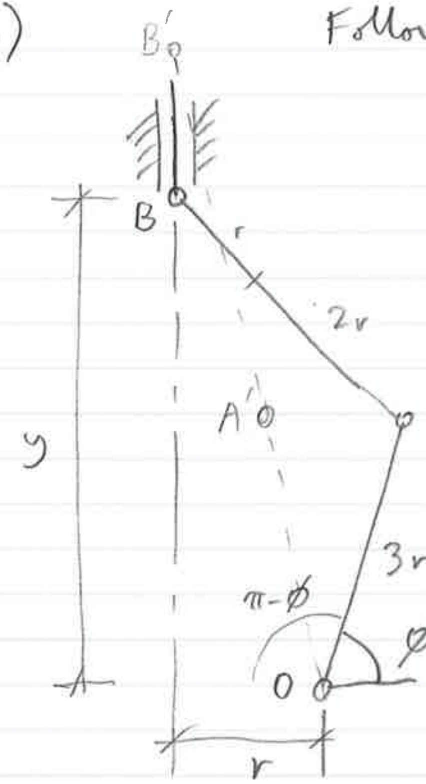
$$= .0054 \quad \text{ie increase by } \frac{54}{34} = 1.56$$

Examiner's comments

Parts (a) & (b) generally well done – there is no need to evaluate h^* explicitly (it comes out at $1.204\,h_0$ if done). Mixed success with part (c) often because of algebraic errors – introducing D is recommended. Part (d) not well done. When the velocity changes so will the minimum film thickness h_0: the dimension that must stay unchanged is step d.

2) (a) Follower contact on tip circle

(i)



Max. lift occurs when OAB are co-linear

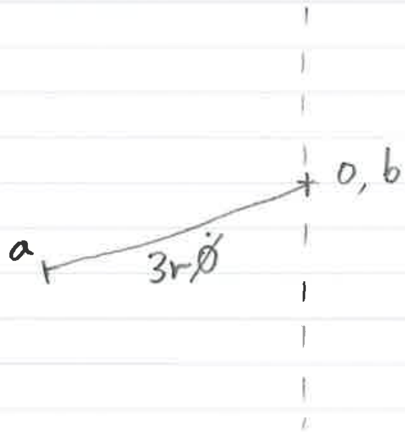
i.e. $\cos(\pi-\phi) = \frac{r}{6r} = \frac{1}{6}$

$\therefore \pi - \phi = \cos^{-1}\frac{1}{6}$

$\underline{\phi = 99.6^\circ}$

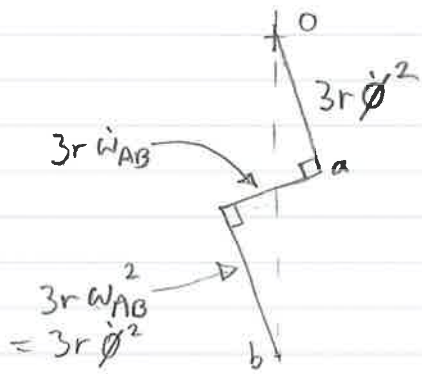
$y = \sqrt{(6r)^2 - r^2} = \sqrt{35}\,r = 5.92\,r$

(ii) Velocity diagram at max. lift:



$\omega_{AB} = \frac{ab}{AB} = \frac{3r\dot{\phi}}{3r} = \dot{\phi}$ clockwise

Acceleration diagram at max. lift



$\therefore$ Acc of b $= \ddot{y}$

$\ddot{y} = -2 \times 3r\dot{\phi}^2 \times \frac{6}{\sqrt{35}}$

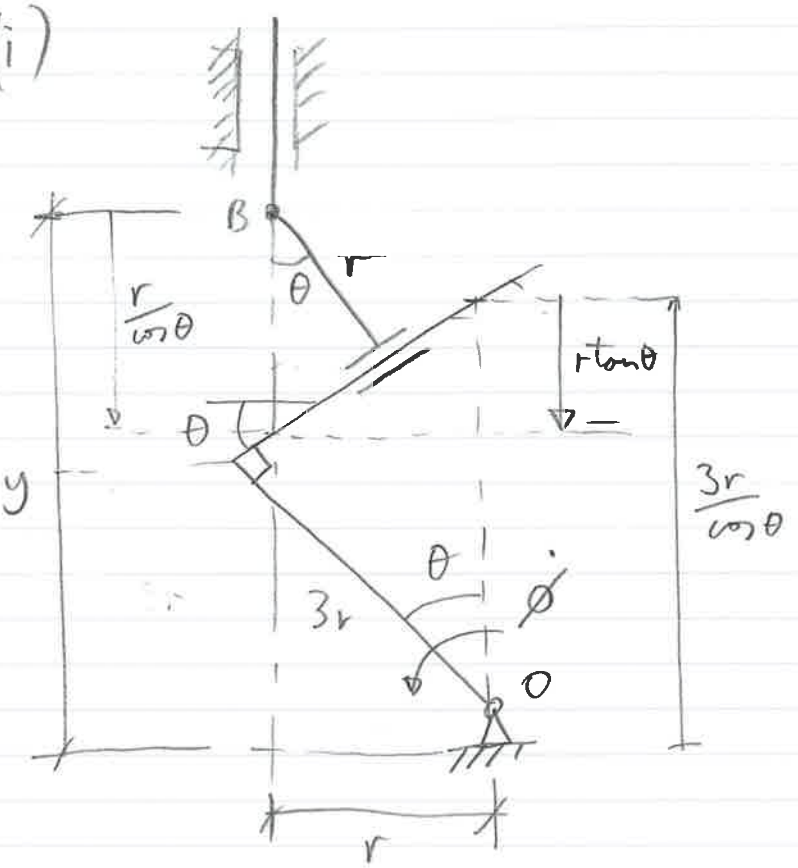
$= -\frac{36}{\sqrt{35}} r\dot{\phi}^2$

$= -6.09\, r\dot{\phi}^2$

Acceleration at minimum lift is $\underline{\text{zero}}$

2) (cont.) (b)

(i)



$$y = \frac{3r}{\cos\theta} - r\tan\theta + \frac{r}{\cos\theta}$$

$$= \frac{4r}{\cos\theta} - r\tan\theta$$

$$= r\left(\frac{4}{\cos\theta} - \tan\theta\right)$$

(or: $y = r\cos\theta + 3r\cos\theta + \tan\theta(3r\sin\theta - r + r\sin\theta)$

$= 4r\cos\theta + \tan\theta\,(4r\sin\theta - r)$

$\cos^2\theta + \sin^2\theta = 1$ $\quad = 4r\cos\theta + \frac{4r\sin^2\theta}{\cos\theta} - r\tan\theta$

$= \frac{4r}{\cos\theta} - r\tan\theta$ ✓)

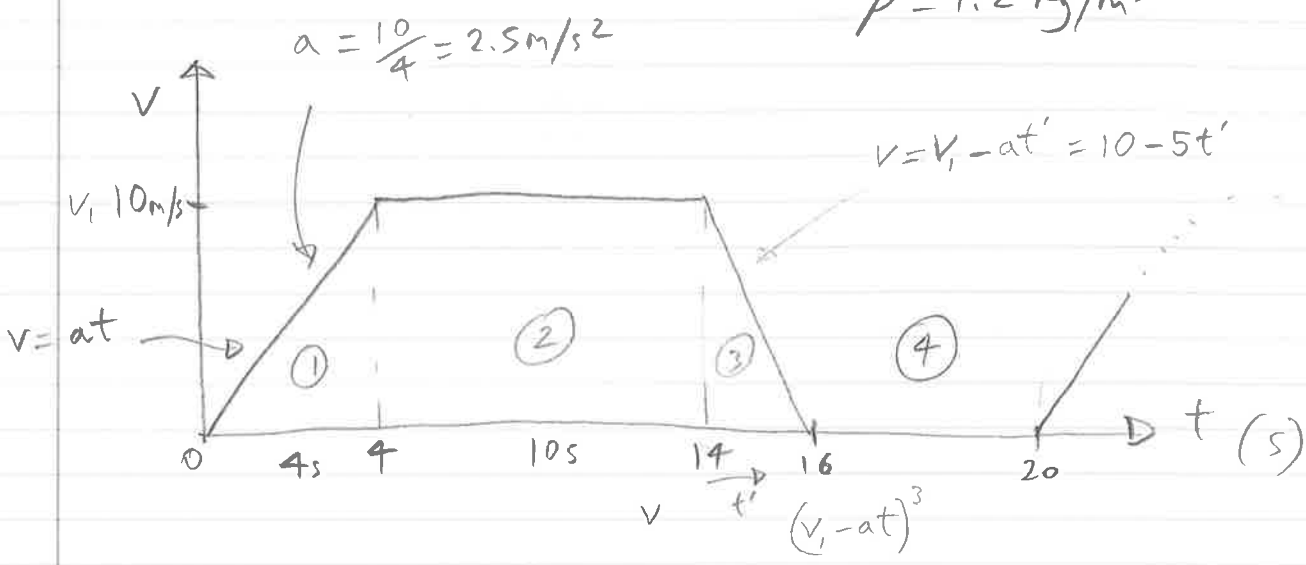
COMMENTS:
Well answered by most. Almost all found the max. lift but there were often errors in obtaining the acceleration. Equivalent mechanisms generally well drawn but some mistakes in analysing geom.

3)

$M = 1000\,kg \quad A = 2\,m^2 \quad C_D = 0.4 \quad C_{RR} = 0.01$

(a)

$\rho = 1.2\,kg/m^3$



$$P = mav + \tfrac{1}{2}C_D A \rho v^2 v + C_{RR} m g v$$
$$= ma^2 t + \tfrac{1}{2}C_D A \rho (at)^3 + C_{RR} m g (at)$$

(i) Peak power =

$$1000 \times 2.5 \times 10 + \tfrac{1}{2} \times 0.4 \times 2 \times 1.2 \times 10^3 + 0.01 \times 1000 \times 9.81 \times 10$$
$$= 25{,}000 + 480 + 981 \ \text{W}$$
$$= 26.5\ \text{kW}$$

(ii)

① Acceleration

$$\text{Energy} = \tfrac{1}{2} m v^2 + \tfrac{1}{2} C_D A \rho a^3 \frac{t^4}{4} + C_{RR} m g a \frac{t^2}{2}$$

$$= \tfrac{1}{2} \times 1000 \times 10^2 + \tfrac{1}{2} \times 0.4 \times 2 \times 1.2 \times 2.5^3 \times \frac{4^4}{4} + 0.01 \times 1000 \times 9.81 \times 2.5 \times \frac{4^2}{2}$$
$$= 50{,}000 + 480 + 1962 \ \text{J}$$
$$= 52.4\ \text{kJ}$$

② Cruise

$$\text{Energy} = 480 \times 10 + 981 \times 10$$
$$= 4800 + 9810 \ \text{J}$$
$$= 14.6\ \text{kJ}$$

③ Deceleration

$$\text{Energy} = -\tfrac{1}{2} m v^2 + \tfrac{1}{2} C_D A \rho \ldots$$
$$-50{,}000 + 240 + 981 \ \text{J}$$
$$= -48.8\ \text{kJ}$$

(by inspection)

④ Stop

Energy $= 0$

Check of work done against resistances in phase (3):

$$P = \tfrac{1}{2} C_D A \rho (10-5t')^3 + C_{RR} mg (10-5t')$$

$$\text{Energy} = \int_0^2 P\,dt'$$

$$= \tfrac{1}{2} C_D A \rho \int_0^2 10^3 - 3\times10^2\times5t' + 3\times10\times(5t')^2 - (5t')^3 \,dt'$$

$$+ C_{RR} mg \int_0^2 10 - 5t' \,dt'$$

$$= \tfrac{1}{2} C_D A\rho \left[1000t' - \frac{1500t'^2}{2} + \frac{750t'^3}{3} - \frac{125t'^4}{4} \right]_0^2$$

$$+ C_{RR} mg \left[10t' - \frac{5t'^2}{2} \right]_0^2$$

$$= \tfrac{1}{2}\times0.4\times2\times1.2 \{2000 - 3000 + 2000 - 500\}$$

$$+ 0.01\times1000\times9.81\times\{20 - 10\}$$

$$= 240 \qquad + \qquad 981$$

(iii) $\therefore$ Mean power $= \dfrac{(1) + (2) + (3) + (4)}{20}$

$= 914\,W \simeq 1\,kW$

(Note: less than cruise power of 1461 W!)

(iv) Flywheel motive power = Peak − mean

$= 25{,}547\,W = 25.5\,kW$

IC engine cannot meet power demand for all of phase (1) & (2) so:

Need to store at least: $52442 - 914\times14$
$+14610$

$= 54256\,J$

to end of cruise phase!

(v) So need to run engine for $\dfrac{54256}{914} = 59.4\,s$

$\simeq 1\,min$

before setting off.

3) (b)

Flywheel $U = \frac{1}{2} J \omega^2$

$J = \frac{1}{2} m R^2 \qquad m = \pi R^2 t \rho$

Maximum stress: $\sigma \simeq \frac{1}{2} \rho R^2 \omega^2 \quad \therefore R^2 \omega^2 = \frac{2\sigma}{\rho}$

Thus $U = \frac{1}{4} m R^2 \omega^2 = \frac{1}{4} m \times \frac{2\sigma}{\rho}$

$$\therefore \quad m = \frac{2U\rho}{\sigma} = \frac{2 \times 54256 \times 1600}{1 \times 10^9}$$

$$= 0.174 \text{ kg}$$

COMMENTS:

Although many candidates found correct expressions for the driving power a number failed to recognise that C_D and C_{RR} were dimensionless. Some also thought that rolling resistance would scale with speed. Energy used in each phase of the drive cycle was done well although some had difficulty with phase 3 (deceleration). Mean power and energy storage done correctly by almost all. Flywheel sizing was done very well by those who recalled their lecture notes, but less well by those who tried to derive an expression for stress.