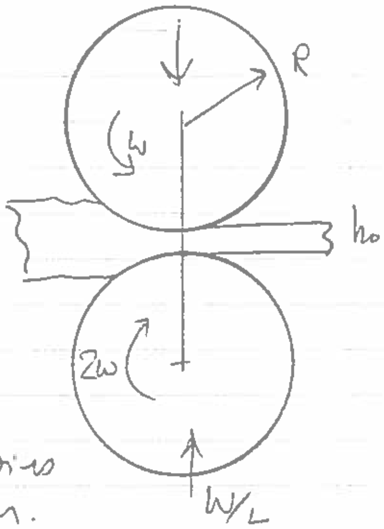
Dr D D Symons

1) For hydrodynamic support, need

i) convergent geometry
ii) positive entraining velocity
iii) viscous lubricant.



(i) Entraining velocity

$$\bar{u} = \frac{1}{2}\left\{(u_1 - u_c) + (u_2 - u_c)\right\}$$

u_1, u_2 are component surface velocities
u_c velocity of contact patch.

Since centres of discs do not translate $u_c = 0$

$$\therefore \quad \bar{u} = \frac{1}{2}(R\omega + R.2\omega) = \frac{3}{2}R\omega$$

Reynolds equation $\quad \frac{dp}{dx} = 12\eta\bar{u}\,\frac{h - h^*}{h^3}$

But $\quad h = h_0 + \frac{x^2}{R} \quad$ and $\quad p = \frac{Kx}{h^2}$

now $\quad \frac{dp}{dx} = \frac{\partial p}{\partial h}\cdot\frac{dh}{dx} + \frac{\partial p}{\partial x}$

$$\Rightarrow \; -\frac{2Kx}{h^3}\cdot\frac{2x}{R} + \frac{K}{h^2} = \frac{K}{h^3}\left\{h - \frac{4x^2}{R}\right\}$$

But $\frac{x^2}{R} = h - h_0 \quad \therefore \quad \frac{dp}{dx} = -\frac{3K}{h^3}\left\{h - \frac{4}{3}h_0\right\}$

$(h - 4h + 4h_0)$
$-(3h - 4h_0)$

Thus by comparing terms

$$\frac{3K}{h^3} = -\frac{12\eta\bar{u}}{h^3} \qquad K = -4\eta\bar{u} = -6\eta R\omega$$

and $\quad h^* = \frac{4}{3}h_0 \qquad h^*/h_0 = 4/3$

(ii) $\quad p = \frac{-6\eta R\omega x}{h^2} \quad$ has max at $h = \frac{4}{3}h_0 = h^*$

ie when $x^* = \sqrt{R(h-h_0)} = \underbrace{\sqrt{\frac{Rh_0}{3}}}$

$\sqrt{3}$ ~~4~~ 9

8 ~~$16\sqrt{3}$~~

So $p_{max} = \dfrac{6\eta R\omega x^*}{h^{*2}} = 6\eta R\omega \sqrt{\dfrac{Rh_0}{3}}\ \dfrac{9}{16h_0^2}$

1.95

$$p_{max} = \frac{9\sqrt{3}}{8}\ \frac{\eta R\omega\sqrt{Rh_0}}{h_0^2} = \underbrace{\frac{9\sqrt{3}}{8}\ \eta\omega\sqrt{\frac{R^3}{h_0^3}}}$$

To sketch distribution $p = -6\eta R\omega \dfrac{x}{h^2}$

Let ~~X~~ $H = h/h_0$

$X = x/\sqrt{Rh_0}$ then $H = 1 + X^2$ or $X = \sqrt{H-1}$

and $p = -\dfrac{6\eta R\omega\sqrt{Rh_0}}{h_0^2}\ \dfrac{X}{H^2}$

So let $P = \dfrac{p\ h_0^2}{6\eta R\omega\sqrt{Rh_0}}$ i.e. $\underbrace{P = \dfrac{X}{H^2}}$ & $H^* = 4/3$

H	1	1.1	1.2	1.333	1.4	1.5	1.6	1.8	2.0	3.0
X	0	.3162	.4472	.5773	.6325	.7071	.7746	.8944	1	1.414
P	0	.26	.31	.325	.323	.314	.3026	.276	.250	.157

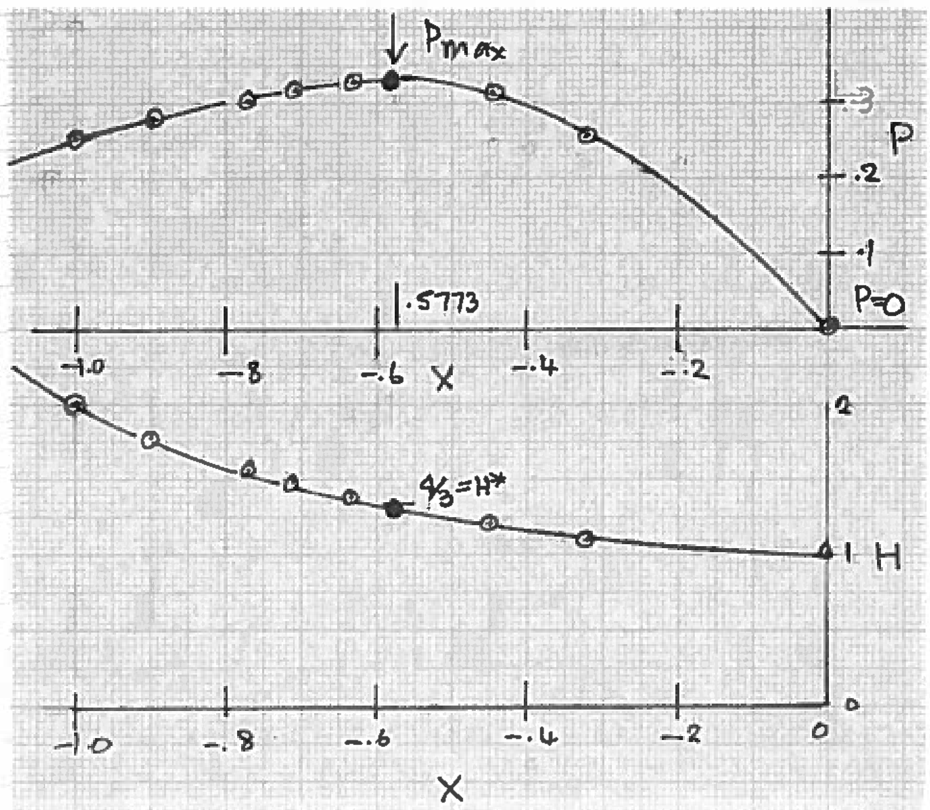
(iii) when $x = 0$ $\dfrac{dp}{dx} \neq 0$

for $x = 0^+$

so continuity infringed.

Sketches of

P and H vs X



(iv) Suppose $\eta = \eta_0 \exp(\alpha p)$

Reynolds eqn becomes

$$\frac{dp}{dx} = 12\eta_0 \exp(\alpha p)\,\bar{u}\,\frac{h-h^*}{h^3}$$

$$\Rightarrow \exp(-\alpha p)\frac{dp}{dx} = 12\,\eta_0\,\bar{u}\,\frac{h-h^*}{h^3}$$

But if $q = \dfrac{1-\exp(-\alpha p)}{\alpha}$

$$\alpha\frac{dq}{dp} = \alpha\exp(-\alpha p) \quad \text{i.e.} \quad \frac{dq}{dx} = \exp(-\alpha p)\frac{dp}{dx}$$

$$\frac{dq}{dx} = \frac{dq}{dp}\frac{dp}{dx}$$

∴ Reynolds eqn $\dfrac{dq}{dx} = 12\,\eta_0\,\bar{u}\,\dfrac{h-h^*}{h^3}$

But we know a solution for this in $q = \dfrac{Kx}{h^2}$

i.e. $q_{max} = \dfrac{9\sqrt{3}}{8}\,\eta_0 w\sqrt{\dfrac{R^3}{h_0^3}}$

But $q_{max} \to \dfrac{1}{\alpha}$ as $p \to \infty$ ∴ $\dfrac{1}{\alpha} \simeq \dfrac{9\sqrt{3}}{8}\,\eta_0 w\sqrt{\dfrac{R^3}{h_0^3}}$

or $h_0^3 \simeq \dfrac{81\times 3}{64}(\alpha\eta_0 w)^2 R^3$

or $h_0 \simeq \dfrac{9}{4\sqrt[3]{3}}(\alpha\eta_0 w)^{2/3} R$

$\dfrac{3^5}{2^6}$ $\dfrac{3^6}{2^6\,3}$ $\dfrac{9}{4\sqrt[3]{3}}$

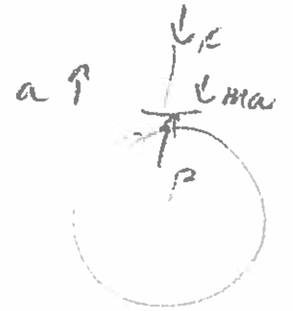
So if $\alpha = 10^{-8}$ Pa; $\eta_0 = .01$ Pas $w = 100\pi$

$$\frac{h_0}{R} \simeq \frac{9}{4\sqrt[3]{3}}\left(10^{-8}\times 10^{-2}\times 100\pi\right)^{2/3}$$

$= 9.95\times 10^{-6}$

So if $R = .025$m $h_0 = .24\times 10^{-7}$ m 24 nm //

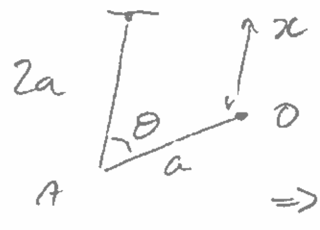
2(a)



Forces on follower

$$P - ma - F = 0$$
$$P = F + ma$$

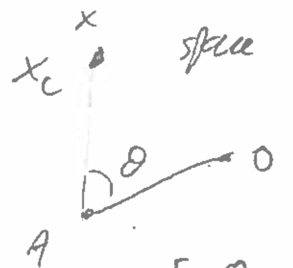
Equivalent mechanism

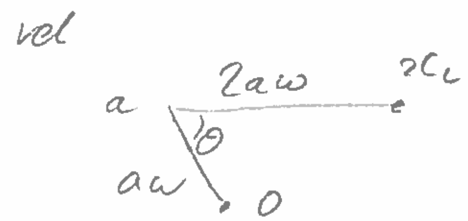


$$x = 2a - a\cos\theta$$
$$\ddot{x} = \omega^2 a \cos\theta$$
$$\Rightarrow P = F + m\omega^2 a\cos\theta$$

Critical $P = 0 \Rightarrow F = m\omega^2 a$ with $\cos\theta = -1$

(b) Entraining velocity $= \frac{1}{2}(v_1 + v_2 - 2v_p)$





$$v_p = \text{horizontal component of } A$$
$$= -a\omega\cos\theta$$
$$v_2 = -a\omega\cos\theta + 2a\omega$$

$$\Rightarrow \bar{u} = \frac{1}{2}(-a\omega\cos\theta + 2a\omega + 2a\omega\cos\theta)$$
$$= \frac{1}{2}a\omega(2 + \cos\theta)$$

Sliding velocity $= v_2 - v_1 = a\omega(2 - \cos\theta)$

(c) $\frac{h_{min}}{R} = 2.65 \times (2\alpha E^*)^{0.54} \left(\frac{\bar{u}\,\eta_0}{2E^*R}\right)^{0.7} \left(\frac{W/L}{2E^*R}\right)^{-0.13}$

Minimum film thickness for min $(\bar{u})^{0.7}(W)^{-0.13}$ as other terms are fixed.

$$W = F + m\omega^2 a \cos\theta$$
$$= 100 + 0.2 \times \left(\frac{1510}{60} \times 2\pi\right)^2 \times 10 \times 10^{-3} \cos\theta \quad [N]$$
$$= 100 + 50\cos\theta \quad [N]$$

So minimum $(2+\cos\theta)^{0.7} \times (100 + 50\cos\theta)^{-0.13}$

at minimum $2 + \cos\theta \Rightarrow \theta = 180°$.
with $W = 50N$

$R = 2a = 20 \times 10^{-3}$ m
$\eta_0 = 0.01$ Pa s
$\alpha = 2 \times 10^{-8}$ m²/N
$E^* = 115$ GPa
$\bar{u} = \frac{1}{2} a\omega \times 1 = \frac{1}{2} \times 10^{-2} \times \left(\frac{1510 \times 2\pi}{60}\right)$
$= 0.791$ m/s
$W = 50N$
$L = 8 \times 10^{-3}$ m

$$\Rightarrow h_{min} = 2.65 \times 20 \times 10^{-3} \underset{[4600]}{\left(2 \times 2 \times 10^{-8} \times 115 \times 10^{9}\right)^{0.54}}$$
$$\underset{[1.72 \times 10^{-12}]}{\times \left(\frac{0.791 \times 0.01}{2 \times 115 \times 10^9 \times 20 \times 10^{-3}}\right)^{0.7}}$$
$$\underset{[1.36 \times 10^{-6}]}{\times \left(\frac{50/8 \times 10^{-3}}{2 \times 115 \times 10^9 \times 20 \times 10^{-3}}\right)^{-0.13}}$$

$= 0.17\ \mu m$ – probably OK for a good ground surface.

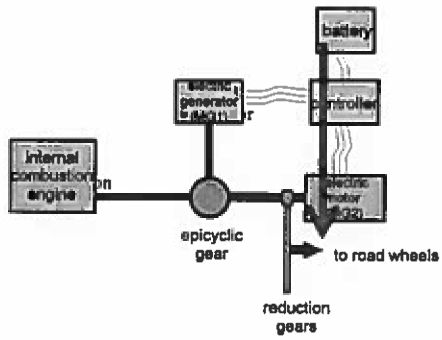
(d) see notes

3) (a)

Modes of operation

1. Accelerating from start and at low speeds

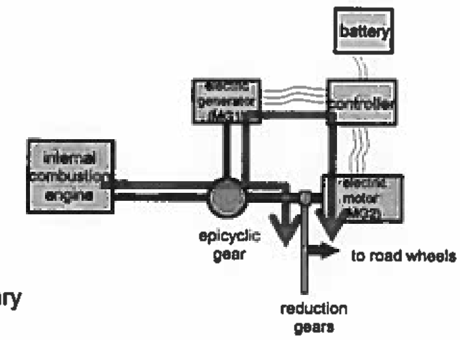
Vehicle runs entirely on energy from battery, delivered to wheels by MG2. The engine is not running.



2. Normal driving conditions

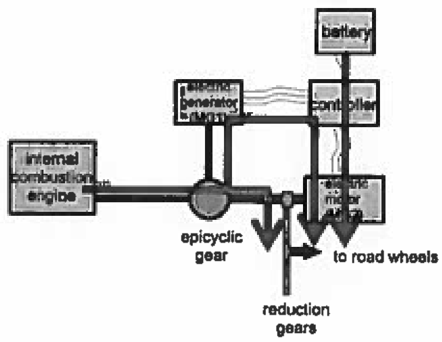
Vehicle runs entirely on energy from engine. Some of the engine power is delivered to the wheels via MG1 and MG2, which act as a CVT to provide the optimum speed ratio between engine and wheels. The system acts like an output-coupled split transmission.

Recirculation can also occur, if this is necessary to minimise the emissions / fuel consumption.



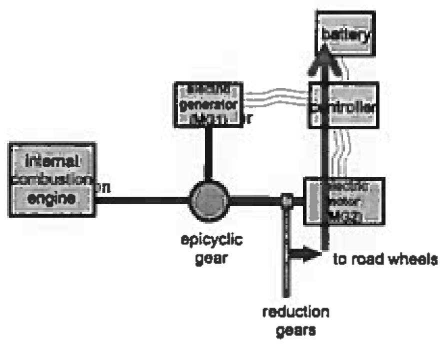
3. Rapid acceleration

Similar to mode 2, but additional power is supplied from the battery



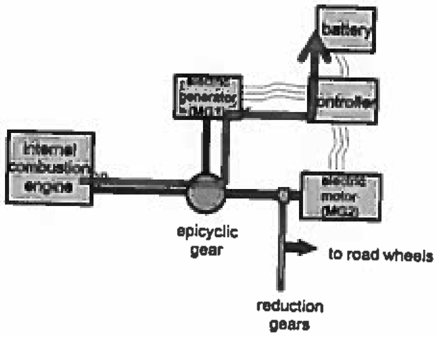
4. Deceleration

MG2 acts as a generator and puts energy into the battery. Conventional friction brakes also act when greater deceleration is required.



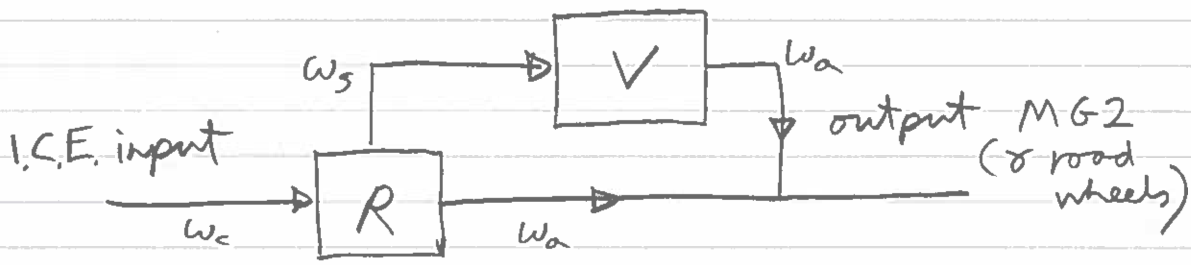
5. Battery charging

Generator puts energy into the battery in order to maintain state of charge within required range.



3) (b)

(i)



(ii) This is an output coupled transmission

(iii) Data sheet: $\omega_s = (1+R)\omega_c - R\omega_a$

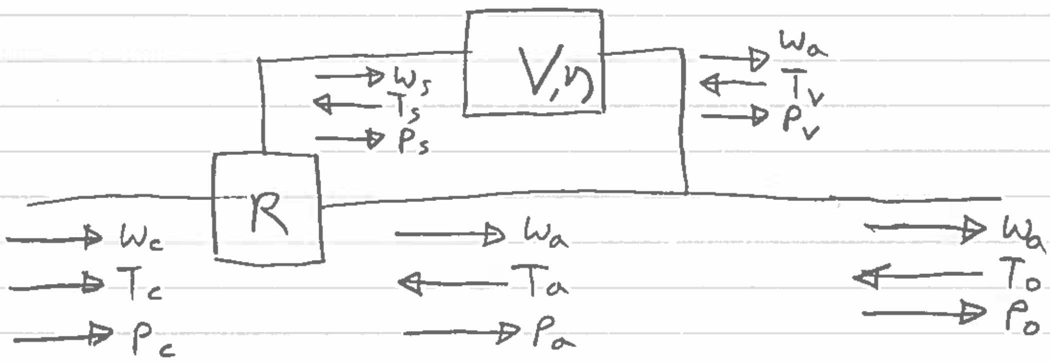
but $V = \dfrac{\omega_a}{\omega_s}$ $\therefore$ $\omega_s = \dfrac{\omega_a}{V}$

so $\dfrac{\omega_a}{V} = (1+R)\,\omega_c - R\omega_a$

$$\omega_a\left(\frac{1}{V} + R\right) = (1+R)\,\omega_c$$

$$\therefore \quad \frac{\omega_a}{\omega_c} = \frac{1+R}{\frac{1}{V}+R}$$

(iv) Torques, choose signs to be consistent with powers & speeds shown on figure



3) (b) (iv) (cont.)

need to find $\frac{T_v}{T_a} = \frac{T_s}{T_a} \times \frac{T_v}{T_s}$

for $\frac{T_v}{T_s}$ consider power through V

$$\eta = \frac{P_v}{P_s} = \frac{w_a T_v}{w_s T_s}$$ (assuming power flows from left to right)

$$\therefore \frac{T_v}{T_s} = \eta \frac{w_s}{w_a} = \frac{\eta}{V}$$

for $\frac{T_s}{T_a}$ use virtual power on epicyclic

$$P_c = P_s + P_a$$
$$T_c w_c' = T_s w_s' + T_a w_a'$$

Set $w_c' = 0$ to eliminate T_c $\therefore \frac{T_s}{T_a} = -\frac{w_a'}{w_s'} = \frac{1}{R}$

$$\therefore \frac{T_v}{T_a} = \frac{T_s}{T_a} \times \frac{T_v}{T_s} = \frac{1}{R} \times \frac{\eta}{V}$$

Now consider powers: $P_v = \eta P_s$

$$\bar{\eta} = \frac{P_{out}}{P_{in}} = \frac{P_o}{P_c} = \frac{P_v + P_a}{P_s + P_a} = \frac{w_a T_v + w_a T_a}{\frac{w_a T_v}{\eta} + w_a T_a}$$

$$= \frac{\frac{T_v}{T_a} + 1}{\frac{T_v}{T_a}\frac{1}{\eta} + 1} = \frac{\frac{\eta}{RV} + 1}{\frac{1}{RV} + 1}$$

$$= \frac{\eta + RV}{1 + RV}$$

ENGINEERING TRIPOS PART IIB 2016
ASSESSOR'S REPORT, MODULE 4C16

The examination was taken by 16 candidates for Part IIB, plus 1 graduate taking the Progress Examination.

The three questions were each marked out of 15 to give a raw total out of 45.

No scaling was required.

Question 1 Hydrodynamic lubrication of two rollers
Most candidates could identify three conditions for hydrodynamic lubrication for part (a) but part (b) was very poorly answered. Candidates were unable to correctly differentiate the expression for pressure as they seemed unable to cope with the fact that it was given as a function of x and h (when h was itself also a function of x).

Question 2 Cylindrical cam
The acceleration and force calculations for part (a) were generally well done. Many fell down in (b) by not finding the velocities of the two parts of the cam, and also the velocity of the contact, in order to find the entraining and sliding speeds. A common mistake in (c) was to look for the maximum load to find the minimum film thickness, when in fact the change in entraining speed (as given in (b)) is important. Relatively easy marks were missed in (d) by not giving sensible comments relating to the Ford Zetec cam train.

Question 3 Hybrid transmission
This question was answered well by most of the candidates. Generally good descriptions given of modes of operation of the hybrid transmission (although many failed to explain how the two motor-generators could act as a CVT). Part (b) was answered well with only the last part (finding the overall efficiency) causing any difficulty.

Digby Symons
12th May 2016