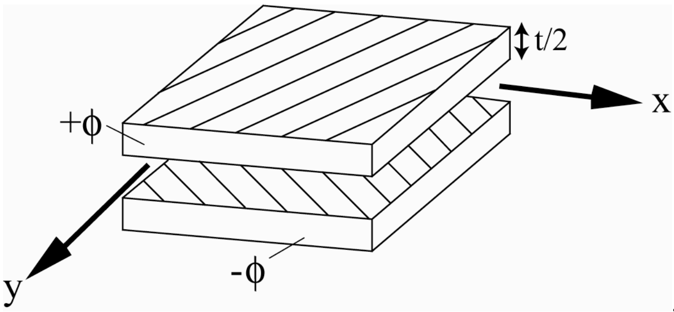
CRIBS

Question 1

(a) A balanced laminate ($A_{16} = A_{26} = 0$) is one in which the laminate as a whole exhibits no tensile-shear interactions i.e. the tension-shear interaction terms contributed by the individual laminae all cancel out each other (a tensile stress induces no shear straining and a shear stress induces no normal strain).

Consider a $\pm\phi$ angle ply laminate with a ply thickness $t / 2$, as illustrated in the figure below.



It can be easily found that the tension-shear interaction terms for the $-\phi$ ply have the opposite sign from the corresponding terms for the $+\phi$ ply.

$$\left(\overline{Q}_{16}\right)_{+\phi} = -\left(\overline{Q}_{16}\right)_{-\phi}$$

$$\left(\overline{Q}_{26}\right)_{+\phi} = -\left(\overline{Q}_{26}\right)_{-\phi}$$

The laminate stiffness matrices A_{16} and A_{26} can be written as

$$A_{16} = t/2\left(\overline{Q}_{16}\right)_{+\phi} + t/2\left(\overline{Q}_{16}\right)_{-\phi} = 0$$

$$A_{26} = t/2\left(\overline{Q}_{26}\right)_{+\phi} + t/2\left(\overline{Q}_{26}\right)_{-\phi} = 0$$

This laminate is balanced when loaded along the x -direction, or along the y-direction.

(b) (i)

$$\frac{\nu_{12}}{E_1} = \frac{\nu_{21}}{E_2} \Rightarrow \nu_{21} = 0.06$$

Calculate $[Q]$ in principal material axes (1, 2)

$$Q_{11} = \frac{E_1}{1-\nu_{12}\nu_{21}} = \frac{E_1}{1-0.3\times 0.06} = 1.02E_1$$

$$Q_{22} = \frac{E_2}{1-\nu_{12}\nu_{21}} = \frac{0.2E_1}{1-0.3\times 0.06} = 0.20E_1$$

$$Q_{12} = \frac{\nu_{12}E_2}{1-\nu_{12}\nu_{21}} = \frac{0.3\times 0.2E_1}{1-0.3\times 0.06} = 0.06E_1$$

$$Q_{66} = G_{12} = 0.09E_1 \qquad Q_{16} = Q_{26} = 0$$

$$\left[Q\right] = E_1\begin{bmatrix} 1.02 & 0.06 & 0 \\ 0.06 & 0.20 & 0 \\ 0 & 0 & 0.09 \end{bmatrix}$$

(ii) Calculate the transformed stiffness matrix $[\overline{Q}]$ in the global x-y axes. The transformed stiffness matrix for the +45° plies is given by

$$\left(\overline{Q}_{11}\right)_{45^\circ} = E_1\left[1.02\; c^4 + 0.20\; s^4 + 2\left(0.06 + 2\cdot 0.09\right)s^2c^2\right] = 0.43E_1$$
$$\left(\overline{Q}_{12}\right)_{45^\circ} = E_1\left[\left(1.02 + 0.20 - 4\cdot 0.09\right)s^2c^2 + 0.06\left(c^4 + s^4\right)\right] = 0.25E_1$$
$$\left(\overline{Q}_{22}\right)_{45^\circ} = E_1\left[1.02\; s^4 + 0.20\; c^4 + 2\left(0.06 + 2\cdot 0.09\right)s^2c^2\right] = 0.43E_1$$
$$\left(\overline{Q}_{16}\right)_{45^\circ} = E_1\left[\left(1.02 - 0.06 - 2\cdot 0.09\right)c^3s - \left(0.20 - 0.06 - 2\cdot 0.09\right)cs^3\right] = 0.21E_1$$
$$\left(\overline{Q}_{26}\right)_{45^\circ} = E_1\left[\left(1.02 - 0.06 - 2\cdot 0.09\right)cs^3 - \left(0.20 - 0.06 - 2\cdot 0.09\right)c^3s\right] = 0.21E_1$$
$$\left(\overline{Q}_{66}\right)_{45^\circ} = E_1\left[\left(1.02 + 0.20 - 2\cdot 0.06 - 2\cdot 0.09\right)s^2c^2 + 0.09\left(s^4 + c^4\right)\right] = 0.28E_1$$

where $c = \cos 45,\; s = \sin 45$

$$\left[\overline{Q}\right]_{45^\circ} = E_1\begin{bmatrix} 0.43 & 0.25 & 0.21 \\ 0.25 & 0.43 & 0.21 \\ 0.21 & 0.21 & 0.28 \end{bmatrix}$$

The transformed lamina stiffness matrix $[\overline{Q}]$ for the -45° plies is given by

$$\left[\overline{Q}\right]_{-45^\circ} = E_1\begin{bmatrix} 0.43 & 0.25 & -0.21 \\ 0.25 & 0.43 & -0.21 \\ -0.21 & -0.21 & 0.28 \end{bmatrix}$$

The laminate extensional stiffness matrix $[A]$

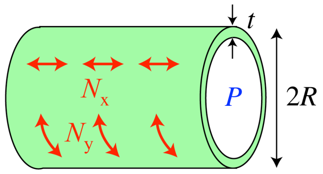
$$A_{11} = t/2\left[\cdot\left(\overline{Q}_{11}\right)_{+45} + \left(\overline{Q}_{11}\right)_{-45}\right] =$$
$$= t/2\cdot\left(0.43 + 0.43\right)\, E_1$$
$$= 0.43\; tE_1$$

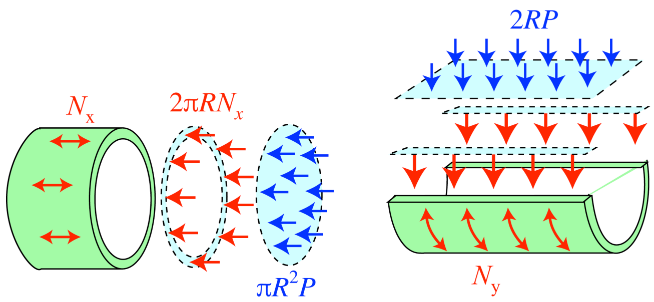
Similarly

$$A_{12} = t/2\cdot\left(0.25 + 0.25\right)\, E_1 = 0.25\; tE_1$$
$$A_{22} = t/2\cdot\left(0.43 + 0.43\right)\, E_1 = 0.43\; tE_1$$
$$A_{16} = t/2\cdot\left(0.21 - 0.21\right)\, E_1 = 0$$
$$A_{26} = t/2\cdot\left(0.21 - 0.21\right)\, E_1 = 0$$
$$A_{66} = t/2\cdot\left(0.28 + 0.28\right)\, E_1 = 0.28\; tE_1$$

$$\left[A\right] = \begin{bmatrix} 0.43 & 0.25 & 0 \\ 0.25 & 0.43 & 0 \\ 0 & 0 & 0.28 \end{bmatrix} \mathrm{t}E_1$$

(iii) Using the radius R of the cylinder (D= 2R), the stress resultants from internal pressure P are given by





$$\pi R^2 P = 2\pi R N_{\text{x}}$$

$$\Rightarrow N_{\text{x}} = \frac{PR}{2} = 0.03 \text{ MN m}^{-1}$$

$$2N_{\text{y}} = 2RP \Rightarrow N_{\text{y}} = PR = 0.06 \text{ MN m}^{-1}$$

Stress resultant from bending

$$\frac{\sigma}{y} = \frac{M}{I} \Rightarrow N_x = \frac{RM}{\pi R^3} = \frac{M}{\pi R^2} = 0.16 \text{ MN m}^{-1}$$

$$N_{\text{xy}} = 0$$

Net hoop and axial forces:
$N_{\text{x}} = 0.19$ MN m^{-1}, $N_{\text{y}} = 0.06$ MN m^{-1}, $N_{\text{xy}} = 0$

$$\begin{pmatrix} \varepsilon_{\text{x}} \\ \varepsilon_{\text{y}} \\ \gamma_{\text{xy}} \end{pmatrix} = [A]^{-1} \begin{pmatrix} N_{\text{x}} \\ \text{N}_{\text{y}} \\ \text{N}_{\text{xy}} \end{pmatrix}$$

$$[A] = \begin{bmatrix} 0.43 & 0.25 & 0 \\ 0.25 & 0.43 & 0 \\ 0 & 0 & 0.28 \end{bmatrix} \text{t}E_1$$

$$[A]^{-1} = \frac{1}{0.034tE_1} \begin{bmatrix} 0.12 & -0.07 & 0 \\ -0.07 & 0.12 & 0 \\ 0 & 0 & 0.12 \end{bmatrix}$$

$$\begin{pmatrix} \varepsilon_{\text{x}} \\ \varepsilon_{\text{y}} \\ \gamma_{\text{xy}} \end{pmatrix} = [A]^{-1} \begin{pmatrix} N_{\text{x}} \\ \text{N}_{\text{y}} \\ \text{N}_{\text{xy}} \end{pmatrix} = \frac{1}{0.034tE_1} \begin{bmatrix} 0.12 & -0.07 & 0 \\ -0.07 & 0.12 & 0 \\ 0 & 0 & 0.12 \end{bmatrix} \begin{pmatrix} 0.19 \\ 0.06 \\ 0 \end{pmatrix}$$

$$= \frac{1}{tE_1} \begin{pmatrix} 0.54 \\ -0.18 \\ 0 \end{pmatrix} \text{MN m}^{-1} = \frac{1}{tE_1} \begin{pmatrix} 5.4 \times 10^{-4} \\ -1.8 \times 10^{-4} \\ 0 \end{pmatrix}$$

2 (a) $\varepsilon_x = \sigma_x / E_x$, $\varepsilon_y = -\nu_{xy} \varepsilon_x$

(0_4) $\varepsilon_x = 1600 \times 10^6 / 138 \times 10^9 = 0.0101 \approx 0.010$

→ matches ε_L^+ ← critical

$\varepsilon_y = -0.3 \times 0.0101 = -0.003$ $|\varepsilon_y| < \varepsilon_L^-$ (not critical)

(90_4) $\varepsilon_x = 45 \times 10^6 / 9 \times 10^9 = 0.005$ matches ε_T^+ ← critical

$\varepsilon_y = -\varepsilon_x \nu_{xy} = -1 \times 10^{-4}$ less than ε_L^-

$(0,90)_s$ $\varepsilon_x = 370 \times 10^6 / 74 \times 10^9 = 5 \times 10^{-3}$ matches ε_T^+ ← critical, less than ε_L^+

$\varepsilon_y = -1.85 \times 10^{-4}$ less than ε_L^- and ε_T^-

0_4 – Tensile failure along fibres

90_4 – Tensile failure transverse to fibres

$(0,90)_s$ – Tensile transverse failure in 90 plies

(b) (0_4) $\varepsilon_x = \dfrac{\sigma}{E_x} - \nu_{yx}\dfrac{\sigma}{E_y} = \sigma\left(\dfrac{1}{138} - \dfrac{0.02}{9}\right) = 5.0 \times 10^{-3} \dfrac{\sigma}{GPa}$

($\nu_{yx} = \nu_{xy}$ for 90_4) $\varepsilon_y = \left(\dfrac{\sigma}{E_y} - \dfrac{\nu_{xy}\sigma}{E_x}\right) = 0.109 \dfrac{\sigma}{GPa}$

$\sigma_{failure} = \min\left\{ \dfrac{0.01}{5 \times 10^{-3}}, \dfrac{0.005}{0.109} \right\} GPa = \min\{2, 0.0459\} GPa$ (ε_L^+, ε_T^+)

$= 46$ MPa

(90_4) Same answer as for $[0_4]$ as same pattern of stress

$[0,90]_s$ $\varepsilon_x = \varepsilon_y = \left(\dfrac{\sigma}{74} - \dfrac{0.037\sigma}{74}\right) = 0.013 \dfrac{\sigma}{GPa}$

$\sigma_{failure} = \min\{\varepsilon_L^+, \varepsilon_T^+\} \times \dfrac{GPa}{0.013} = 380$ MPa

2 (c)

$$\varepsilon_x = \varepsilon_y = 0.013 \frac{\sigma}{GPa}$$

$$Q_{11} = 138/(1-0.3\times0.02) = 138.8\ GPa \qquad Q_{22} = \frac{9}{1-0.006} = 9.05\ GPa$$

$$Q_{12} = 0.3\times 9/(1-0.006) = 2.72\ GPa$$

$$\sigma_1 = Q_{11}\varepsilon_1 + Q_{12}\varepsilon_2 = 141.5\times 0.013\sigma = 1.84\sigma$$

$$\sigma_2 = Q_{12}\varepsilon_1 + Q_{22}\varepsilon_2 = 0.153\sigma$$

Tsai - Hill: $\frac{\sigma_1^2}{s_L^2} + \frac{\sigma_1\sigma_2}{s_L^2} + \frac{\sigma_2^2}{s_T^2} = 1$

use tensile strength in both cases

$$\Rightarrow \quad \frac{\sigma^2}{MPa^2}\left(\frac{1.84^2}{1448^2} + \frac{1.84\times0.153}{1448^2} + \frac{0.153^2}{48.3^2}\right) = 1$$

$$\Rightarrow \quad \sigma = 290\ MPa$$

(d) See next page.

2(d) Coupon tests for lamina/laminate stress-based failure
Strength-based test configurations aim to have a uniform stress in the test section and to avoid stress concentrations.

Both uni-and multi-directional laminate lay-ups are used.

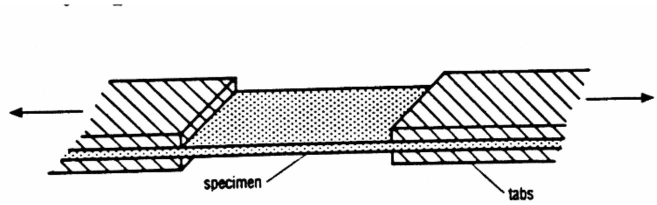
Tension
Tabs may be bonded on.

May have a waisted test section.

Alignment important

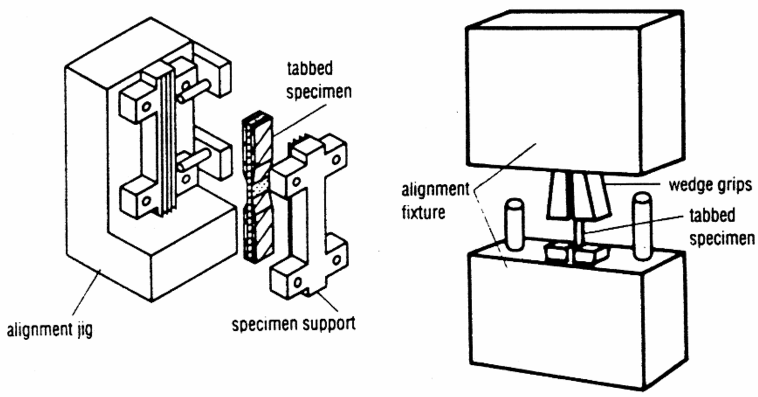
Strain or clip gauges or laser extensometers

may be used to monitor the test



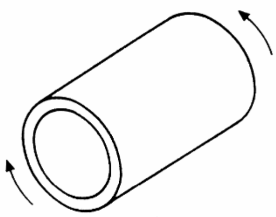
Compression
Need to prevent macro-buckling of the specimen.

Either use very squat specimens, or have anti-buckling guides.



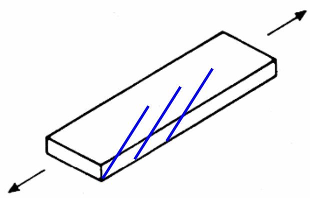
In-plane shear

Tube tests



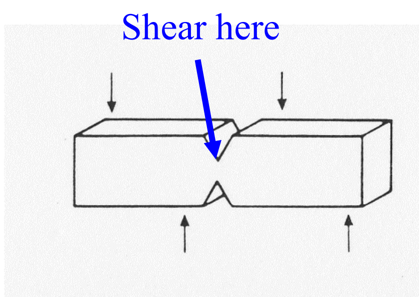
Difficult to prepare specimens
Difficult to perform test

Off-axis tests



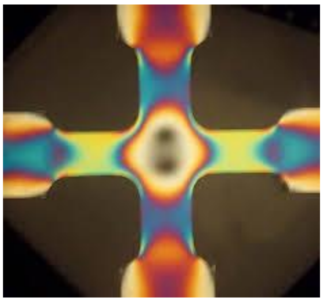
Interpretation of data required
End-effects important

Iosipescu shear test



Specialised rig and specimen required
Some questions about data interpretation

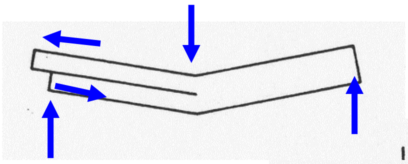
Biaxial tests



Requires specialised rigs
Tubes tests common
Careful specimen preparation
Lots of testing required
to characterise failure

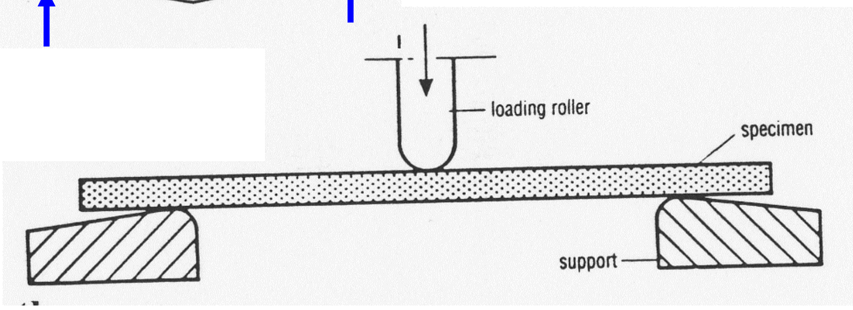
Interlaminar shear

A squat beam is used to generate interlaminar shear
Data is semi-quantitative
Good for quality control purposes

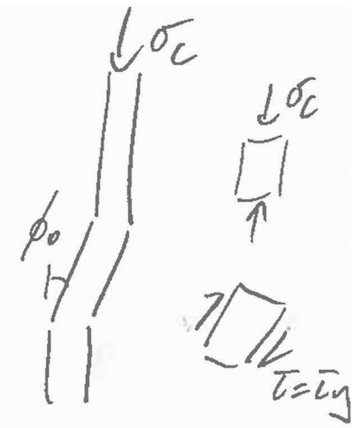


Flexure

A long span is used to avoid interlaminar shear
Interpretation required
Used for quality control purposes



Q3 (a) Assume waviness in the form as sketched.



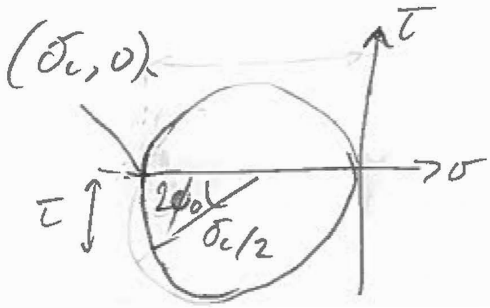
Then failure occurs when the shear stress in the matrix in the band of tilted fibres equals τ_y

From the Mohrs circle:

$$\tau = \frac{\sigma_c}{2} \sin 2\phi_0$$

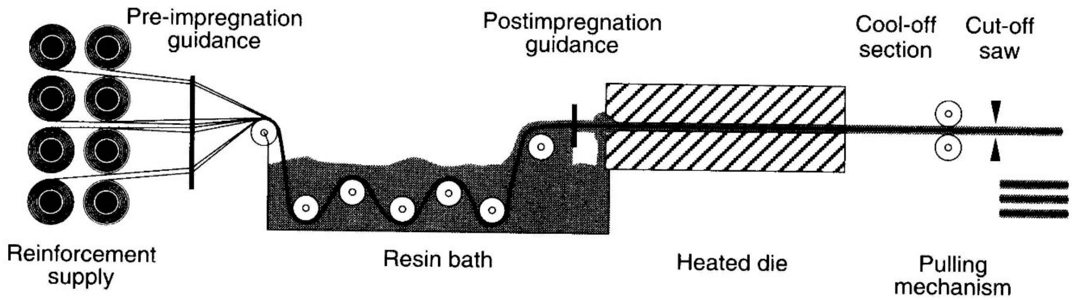
$$\simeq \sigma_c \phi_0 \text{ for small } \phi_0$$

Hence $\sigma_c = \tau_y/\phi_0$



(b) Pultrusion

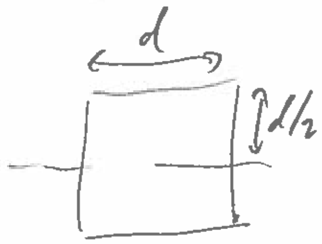
- Similar to metal extrusion, but fibres are pulled through die
- Principally thermoset resin
- Cost effective mass production of prismatic shapes



(c) Pultrusion, by pulling on the fibres, must reduce the waviness of the fibres and hence the compressive strength by the equation above. Moulding tends to allow deformation and waviness.

(d) - Cost will be important - GFRP is relatively cheap and pultrusion a relatively cheap composite process.
- Possible to make long beam-like prismatic sections
- Able to include a mix of lay-ups for 'flange' and 'web' type elements
- Durable - resistant to moisture
- Lightweight for installation and to reduce self-weight loads
- Good mechanical properties (stiffness and strength)

3(e)



In bending, neglecting contribution of sides $I = 2dt \times (d/2)^2 = \frac{d^3 t}{2}$

End deflection '$\delta = \frac{WL^3}{3EI}$' $\Rightarrow \delta = \frac{FL^3}{3E_x} \frac{2}{d^3 t}$

Constraint $\Rightarrow$ $E_x t > \frac{2}{3}\frac{FL^3}{\delta d^3} = \frac{2}{3} \times \frac{30 \times 3^3}{2\times10^{-3} (0.1)^3} = 2.7\times10^8 \, Nm^{-1}$

In torsion $T = G \frac{4 A_e^2}{\oint \frac{ds}{t}} \phi$ $\quad \oint \frac{ds}{t} = \frac{4d}{t}$, $A_e = d^2$

$\Rightarrow \phi = \frac{Q \, 4d/t}{G_{xy} . 4 . d^4} = \frac{Q}{G_{xy} d^3 t}$

Constraint $\Rightarrow$ $G_{xy} t > \frac{Q}{\phi d^3} = \frac{80}{\frac{0.2\times\pi}{180\times3} \times 0.1^3} = 6.9\times10^7 \, Nm^{-1}$

ϕ - rad/m

Aim to balance constraints with $E_x / G_{xy} = 3.9$

Retain 10% 90.

% 45	E_x	G_{xy}	E_x/G_{xy}
50	26.5	8.7	3
40	30	8	3.75
35	30.6	7.9	← 38 by interpolation
30	33	7.3	6.5

$t = 2.7\times10^8 / 30.6\times10^9 = 8.8mm$

check $t = 6.9\times10^7 / 7.9\times10^9 = 8.7mm$

So choose 10% 90°
38% 45° with total thickness 8.8mm
52% 0°

4 (a) Cost ⇒ use GFRP and a moulding process
→ probably compression moulding with SMC (sheet moulding compound) will give a reasonable compromise between quality and cost.

Performance – stiffness and strength are both likely to be relevant.
Need to use oriented fibres where stiffness comes into play (eg in legs)
In other areas a random mat will probably give adequate performance, perhaps some unidirectional stiffeners could be included

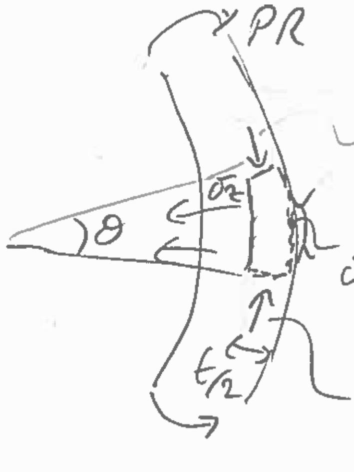
Joints – these are likely to be critical components
To keep cost down and to simplify the design the component should be moulded in one piece if possible – may require a radical redesign (curving shapes) to avoid highly stressed regions between legs and other parts.

Durability – GFRP will be good for moisture

Testing – this will be critical, including weathering and fatigue testing

4(b)

(i)



$\frac{\sigma}{y} = \frac{M}{I} \Rightarrow$

$\sigma_m = \frac{t}{2} \times \frac{PR}{\frac{1}{12} b t^3}$

Equilibrium on dashed box $\rightarrow$

$\Rightarrow -\sigma_z \times \theta R + 2 \times \frac{\sigma_m t}{4} \sin\left(\frac{\theta}{2}\right) = 0$

$\Rightarrow \sigma_z = \frac{\sigma_m t}{4R}$

$= \frac{6PRt}{bt^3} \cdot \frac{t}{4R} = \frac{3P}{2bt}$

(ii) Compare σ_m and σ_z

For σ_m, weakest in compression (σ_m compressive on outside of beam)

$\Rightarrow \sigma_L^- = \sigma_m = \frac{6PR}{bt^2} \Rightarrow P = \frac{1172 \times 60 \times (20)^2}{6 \times 800} \frac{N mm^{-2} \times (mm)^3}{(mm)}$

$= 5860 N$

For σ_z, need to estimate from given properties.

Through thickness tension (in this case) most nearly like σ_T^T with a failure σ of 48.3 MPa

ie $P = \sigma_z \times \frac{2bt}{3} = \frac{48.3 \times 2 \times 60 \times 20}{3} = 38 \times 10^3 N$ (units: $N mm^{-2}$, mm, mm)

In plane failure is critical $\Rightarrow$ $P = 5900 N$

(c) Critical for curved & thick composites.

Other locations where 3D stress effects occur such as ply drops, joints, holes, edges, attachment points. Also 3D shapes rather than thin shells.

Examiner's Comments

Question 1: Elastic Deformation
A very popular question, very well answered by all candidates. Marks were lost mainly because of errors in estimating the stress resultants in part b(iii).

Question 2: Stress-based lamina failure
This was the least popular question on the paper. Most students seemed to run out of time. Part (a) was answered reasonably well, but part (b) was answered less well with many students not including the Poisson effect when estimating the tensile stresses. Only a few students attempted part (c), presumably due to time pressure. Almost all candidates attempted part (d) which was answered poorly due to lack of details.

Question 3: Microbuckling, design (carpet plots)
Part (a) was answered reasonably well, but only a few answers had good sketches and stated clearly the assumptions. Parts (b) and (c) were answered well. Part (d) answers lacked detail. In part (e), several candidates didn't estimate correctly the constraint due to the torsional load and a large number of them did not have time to use the carpet plots to optimize the design.

Question 4: Interlaminar failure, design
Part (a) was answered reasonably well, with loss of marks due to lack of details. Part (b) was answered less well. In part b(i), only a few candidates had accurate drawings and applied correctly the force balance. Part b(ii) was answered poorly as the majority of candidates focused on tensile failure rather than compressive, which required higher failure loads. A significant number of candidates didn't realise that the outside of the beam was under compression. Part (c) was answered poorly; very few candidates mentioned that through-thickness stresses are critical for composites with high thickness and curvature.

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