EGT3, PART IIB 2018

CRIB for 4C2: DESIGNING WITH COMPOSITES.

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Q1 /4C2

CRIBS
Question 1
(a)

$$\frac{\nu_{12}}{E_1} = \frac{\nu_{21}}{E_2} \Rightarrow \nu_{21} = 0.02461$$

Calculate $[Q]$ in principal material axes (1, 2)

$$Q_{11} = \frac{E_1}{1-\nu_{12}\nu_{21}} = \frac{76}{1-0.34\times 0.02461} = 76.64 \text{ GPa}$$

$$Q_{22} = \frac{E_2}{1-\nu_{12}\nu_{21}} = \frac{5.5}{1-0.34\times 0.02461} = 5.54 \text{ GPa}$$

$$Q_{12} = \frac{\nu_{12}E_2}{1-\nu_{12}\nu_{21}} = \frac{0.34\times 5.5}{1-0.34\times 0.02461} = 1.88 \text{ GPa}$$

$$Q_{66} = G_{12} = 2.3 \text{ GPa} \qquad Q_{16} = Q_{26} = 0$$

$$[Q] = \begin{bmatrix} 76.64 & 1.88 & 0 \\ 1.88 & 5.54 & 0 \\ 0 & 0 & 2.3 \end{bmatrix} \text{ GPa}$$

(b) Calculate the transformed stiffness matrix $[\overline{Q}]$ in the global x-y axes. The transformed stiffness matrix for the +30° plies is given by

$$\left(\overline{Q}_{11}\right)_{30^\circ} = 76.64\, c^4 + 5.54\, s^4 + 2(1.88 + 2\times 2.3)s^2c^2 = 45.89 \text{ GPa}$$

$$\left(\overline{Q}_{12}\right)_{30^\circ} = (76.64 + 5.54 - 4\times 2.3)s^2c^2 + 1.88(c^4 + s^4) = 14.86 \text{ GPa}$$

$$\left(\overline{Q}_{22}\right)_{30^\circ} = 76.64\, s^4 + 5.54\, c^4 + 2(1.88 + 2\times 2.3)s^2c^2 = 10.34 \text{ GPa}$$

$$\left(\overline{Q}_{16}\right)_{30^\circ} = (76.64 - 1.88 - 2\times 2.3)c^3s - (5.54 - 1.88 - 2\times 2.3)cs^3 = 22.89 \text{ GPa}$$

$$\left(\overline{Q}_{26}\right)_{30^\circ} = (76.64 - 1.88 - 2\times 2.3)cs^3 - (5.54 - 1.88 - 2\times 2.3)c^3s = 7.90 \text{ GPa}$$

$$\left(\overline{Q}_{66}\right)_{30^\circ} = (76.64 + 5.54 - 2\times 1.88 - 2\times 2.3)s^2c^2 + 2.3(s^4 + c^4) = 15.28 \text{ GPa}$$

where $c = \cos 30$, $s = \sin 30$

$$\left[\overline{Q}\right]_{30^\circ} = \begin{bmatrix} 45.89 & 14.86 & 22.89 \\ 14.86 & 10.34 & 7.90 \\ 22.89 & 7.90 & 15.28 \end{bmatrix} \text{ GPa}$$

The transformed lamina stiffness matrix $[\overline{Q}]$ for the -30° plies is given by

$$\left[\overline{Q}\right]_{-30^\circ} = \begin{bmatrix} 45.89 & 14.86 & -22.89 \\ 14.86 & 10.34 & -7.90 \\ -22.89 & -7.90 & 15.28 \end{bmatrix} \text{ GPa}$$

The transformed lamina stiffness matrix $[\overline{Q}]$ for the 90° plies is given by

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The transformed lamina stiffness matrix $[\overline{Q}]$ for the 90° plies is given by

$$[\overline{Q}]_{90^\circ} = \begin{bmatrix} 5.54 & 1.88 & 0 \\ 1.88 & 76.64 & 0 \\ 0 & 0 & 2.3 \end{bmatrix} \text{GPa}$$

Set t (=0.25 mm) for lamina thickness

$$A_{16} = nt\cdot\left[(\overline{Q}_{16})_{+30} + (\overline{Q}_{16})_{-30} + (\overline{Q}_{16})_{90}\right] = nt\cdot[22.89-22.89+0] = 0$$

$$A_{26} = nt\cdot\left[(\overline{Q}_{26})_{+30} + (\overline{Q}_{26})_{-30} + (\overline{Q}_{26})_{90}\right] = nt\cdot[7.90-7.90+0] = 0$$

Since A_{16}=A_{26}=0. the laminate is balanced. This means that the laminate as whole does not exhibit any tensile-shear interactions. Tensile-shear interactions are tensile strains arising from applied shear stresses and visa versa and result in in-plane distortion of the laminate.

(c)

Similarly

$$A_{11} = nt\cdot\left[(\overline{Q}_{11})_{+30} + (\overline{Q}_{11})_{-30} + (\overline{Q}_{11})_{90}\right] = n\cdot 0.25\cdot[45.89+45.89+5.54] = 24.33n \text{ MNm}^{-1}$$

$$A_{12} = nt\cdot\left[(\overline{Q}_{12})_{+30} + (\overline{Q}_{12})_{-30} + (\overline{Q}_{12})_{90}\right] = n\cdot 0.25\cdot[14.86+14.86+1.88] = 7.9n \text{ MNm}^{-1}$$

$$A_{22} = nt\cdot\left[(\overline{Q}_{22})_{+30} + (\overline{Q}_{22})_{-30} + (\overline{Q}_{22})_{90}\right] = n\cdot 0.25\cdot[10.34+10.34+76.64] = 24.33n \text{ MNm}^{-1}$$

$$A_{66} = nt\cdot\left[(\overline{Q}_{66})_{+30} + (\overline{Q}_{66})_{-30} + (\overline{Q}_{66})_{90}\right] = n\cdot 0.25\cdot[15.28+15.28+2.3] = 8.215n \text{ MNm}^{-1}$$

$$[A] = n\begin{bmatrix} 24.33 & 7.90 & 0 \\ 7.90 & 24.33 & 0 \\ 0 & 0 & 8.215 \end{bmatrix} \text{MNm}^{-1}$$

(d)

$$\begin{pmatrix} N_x \\ N_y \\ N_{xy} \end{pmatrix} = [A]\begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{pmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{21} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix}\begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{pmatrix}$$

$\therefore N_x = N_y \ (\varepsilon_x = \varepsilon_y),\ N_{xy} = 0$ since $\gamma_{xy} = 0$

$\therefore N_x = A_{11}\varepsilon_x + A_{12}\varepsilon_y = (A_{11}+A_{12})\varepsilon_x = (24.33n+7.9n)\cdot(2.3\times10^{-3}) = 74.129n \text{ kNm}^{-1}$

By balancing the forces exerted by the pressure and the stress

$\pi R^2 P = 2\pi R N_x$

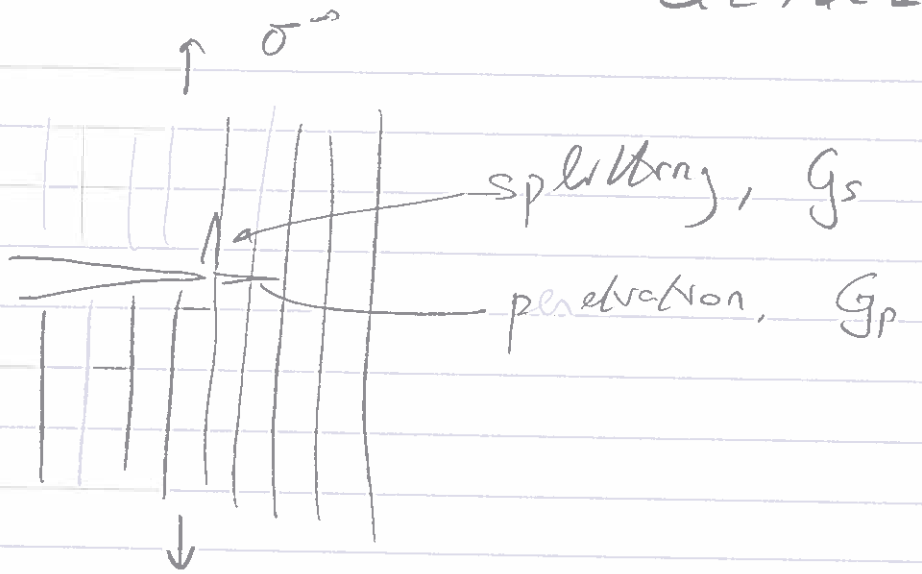
$$\Rightarrow N_x = \frac{PR}{2} = \frac{2\cdot 0.5}{2} = 0.5 \text{ MPa} = 500 \text{ kN m}^{-1}$$

$N_x^{design} \geq N_x = 500$

$74.129n \geq 500 \Rightarrow n \geq 6.74$

$\therefore n = 7$ is minimum value.

Q2. (a)



Under remote tension, the mode I parent crack may either continue to propagate straight ahead (penetration mode) with an energy release rate G_p or by splitting at 90°, with an energy release rate $G_s \approx \frac{1}{4} G_p$.

If the toughness Γ_s in the splitting direction is sufficiently small compared to the penetration toughness, Γ_p, i.e.

$$\frac{\Gamma_s}{\Gamma_p} < \frac{G_s}{G_p} \approx \frac{1}{4}$$

Then splitting will occur in preference to penetration.

The ±45° plies in a quasi-isotropic laminate will bridge any splits and lead to an increase in Γ_s. Thus, splitting is less likely.

Q2/4C2

Q2. (b) Poisson ratio for a unidirectional 0° ply or a 90° ply is almost zero. Poisson ratio for a ±45° laminate is −1.

Q2. (c) Composites have a low out-of-plane strength and a low out-of-plane toughness. Joints often generate out-of-plane stresses. Also CFRP and GFRP have a low ductility in all directions and consequently only a limited amount of stress relaxation can occur at joints such as holes or loading pins or bolts.

Q2. (d) Longitudinal tensile strength is fibre-dominated with little contribution from the matrix. Transverse tensile strength is matrix governed & voids can act as stress raisers.

Q3/4C2

Question 3

(a) (i)

$$S_{11} = \frac{1}{E_1} = \frac{1}{39} = 0.025 \text{ GPa}^{-1}, \qquad S_{22} = \frac{1}{E_2} = \frac{1}{8.3} = 0.12 \text{ GPa}^{-1},$$

$$S_{12} = -\frac{\nu_{12}}{E_1} = -\frac{0.3}{39} = -0.0076 \text{ GPa}^{-1}, \qquad S_{66} = \frac{1}{G_{12}} = \frac{1}{4.1} = 0.24 \text{ GPa}^{-1}$$

$$[S] = \begin{bmatrix} 0.025 & -0.0076 & 0 \\ -0.0076 & 0.12 & 0 \\ 0 & 0 & 0.24 \end{bmatrix} \text{GPa}^{-1}$$

(ii) From Datasheet

$$\bar{S}_{11} = S_{11}c^4 + S_{22}s^4 + (2S_{12} + S_{66})c^2s^2$$

so that, for $\phi = 25^\circ$,

$$\bar{S}_{11} = 0.025\cos^4(25) + 0.12\sin^4(25) + (0.24 - 0.0152)\cos^2(25)\sin^2(25) = 0.054 \text{ GPa}^{-1}$$

This is about twice the on-axis value ($= 0.025$ GPa^{-1} at $\phi = 0^\circ$).

The maximum value of $\bar{S}_{11}$ is found from the condition

$$\frac{\partial \bar{S}_{11}}{\partial \phi} = -S_{11}4c^3s + S_{22}4s^3c + (2S_{12} + S_{66})(-2cs^3 + 2sc^3) = 0$$

c = 0 or s = 0 implies
zeros at 0 and 90 degrees (min and max)
(mpfs)

Divide through by 4cs

$$\frac{\partial \bar{S}_{11}}{\partial \phi} = -S_{11}c^2 + S_{22}s^2 + (2S_{12} + S_{66})\left(\frac{-s^2 + c^2}{2}\right) = 0$$

$$\therefore c^2 = \frac{S_{12} - S_{22} + \frac{S_{66}}{2}}{-S_{11} - S_{22} + 2S_{12} + S_{66}}$$ (after substituting for $s^2 = 1 - c^2$)

$$= \frac{-0.0076 - 0.12 + \frac{0.24}{2}}{-0.025 - 0.12 - 2 \cdot 0.0076 + 0.24} = \frac{0.0076}{0.0798} = 0.095$$

This should be -0.095, so no maximum between 0 and 90

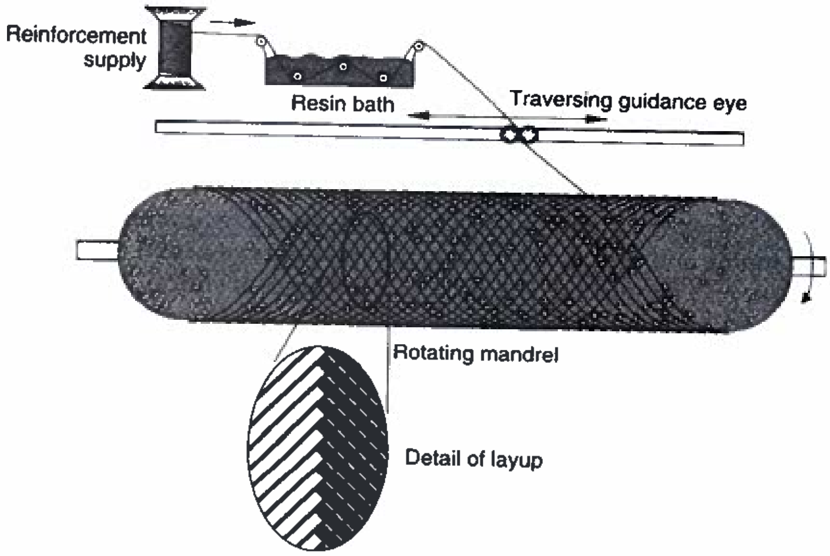
$\therefore c = 0.3$, ie $\phi = 72^\circ$

$$\bar{S}_{11} = 0.025\cos^4(72) + 0.12\sin^4(72) + (0.24 - 0.0152)\cos^2(72)\sin^2(72) = 0.12 \text{ GPa}^{-1}$$

(b) (i) Filament winding is a process suited to automation, although limited to certain components shapes (tubes). Fibre tows i.e. bundles of fibres, are drawn through a bath of resin, before wound onto a mandrel or former of the required shape.

The process involves: (a) A creel stand, from which the fibre tows are fed under the required tension from a set of reels. (b) A bath of resin, through which the fibre tows pass via a set of guides. (c) A delivery eye, through which the fibres emerge, the position of which is controlled by a mechanical system and (d) a rotating mandrel onto which the fibre tows are drawn. The key parameters are the fibre tension, the resin take-up efficiency and the winding geometry.

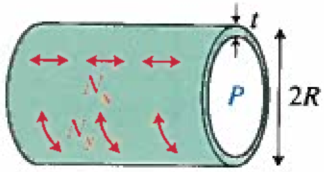
Q3/4c2

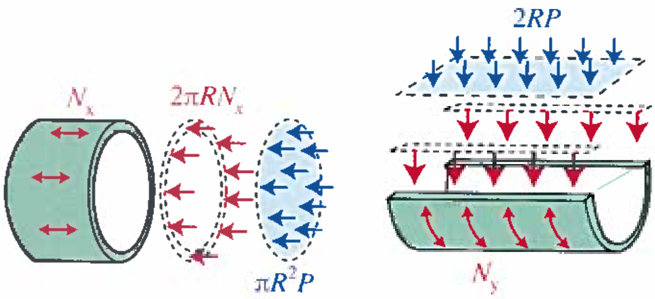


(ii) Applying a force balance between the force exerted by the pressure and the stresses in the wall gives

$$\pi R^2 P = 2\pi R N_x \Rightarrow N_x = \frac{PR}{2}$$

$$2N_y = 2RP \Rightarrow N_y = PR$$





$$\begin{pmatrix} N_x \\ N_y \\ N_{xy} \end{pmatrix} = [A]\begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{pmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix}\begin{pmatrix} \varepsilon_{\text{axial}} \\ \varepsilon_{\text{hoop}} \\ \gamma_{xy} \end{pmatrix}$$

$$\gamma_{xy} = 0$$

$$\therefore N_x = A_{11}\varepsilon_{\text{axial}} + A_{12}\varepsilon_{\text{hoop}}$$

$$N_y = A_{12}\varepsilon_{\text{axial}} + A_{22}\varepsilon_{\text{hoop}}$$

The transformed stiffness matrix for the $\pm 60^\circ$ plies is given by

$$\left(\overline{Q}_{11}\right)_{60^\circ} = \left(\overline{Q}_{11}\right)_{-60^\circ} = 39.76\ c^4 + 8.46\ s^4 + 2(2.54 + 2\times 4.1)s^2c^2 = 11.27\text{ GPa}$$

$$\left(\overline{Q}_{12}\right)_{60^\circ} = \left(\overline{Q}_{12}\right)_{-60^\circ} = (39.76 + 8.46 - 4\times 4.1)s^2c^2 + 2.54\left(c^4 + s^4\right) = 7.55\text{ GPa}$$

$$\left(\overline{Q}_{22}\right)_{60^\circ} = \left(\overline{Q}_{22}\right)_{-60^\circ} = 39.76\ s^4 + 8.46\ c^4 + 2(2.54 + 2\times 4.1)s^2c^2 = 26.92\text{ GPa}$$

where $c = \cos 60,\ s = \sin 60$

If t is the wall thickness

$$A_{11}=\frac{t}{2}\left(\overline{Q}_{11}\right)_{60}+\frac{t}{2}\left(\overline{Q}_{11}\right)_{-60}=11.27\ t$$

$$A_{12}=\frac{t}{2}\left(\overline{Q}_{12}\right)_{60}+\frac{t}{2}\left(\overline{Q}_{12}\right)_{-60}=7.55\ t$$

$$A_{22}=\frac{t}{2}\left(\overline{Q}_{22}\right)_{60}+\frac{t}{2}\left(\overline{Q}_{22}\right)_{-60}=26.92\ t$$

Therefore

$$N_x=\frac{PR}{2}=\left(11.27\varepsilon_{\text{axial}}+7.55\varepsilon_{\text{hoop}}\right)t \quad (1)$$

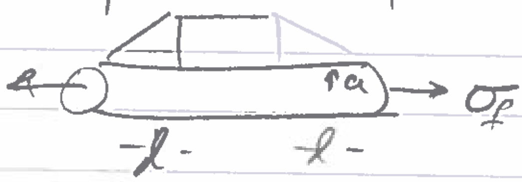
$$N_y=PR=\left(7.55\varepsilon_{\text{axial}}+26.92\varepsilon_{\text{hoop}}\right)t \quad (2)$$

$$\therefore \frac{(1)}{(2)} \qquad \frac{1}{2}=\frac{11.27\varepsilon_{\text{axial}}+7.55\varepsilon_{\text{hoop}}}{7.55\varepsilon_{\text{axial}}+26.92\varepsilon_{\text{hoop}}}=\frac{11.27\left(\varepsilon_{\text{axial}}/\varepsilon_{\text{hoop}}\right)+7.55}{7.55\left(\varepsilon_{\text{axial}}/\varepsilon_{\text{hoop}}\right)+26.92}$$

$$\frac{\varepsilon_{\text{axial}}}{\varepsilon_{\text{hoop}}}=\frac{11.82}{14.99}=0.79$$

4. (a) $\sigma_c \sim \frac{\tau_y}{\bar{\Phi}}$ for compressive failure by fibre microbuckling, where τ_y is the shear yield strength and $\bar{\Phi}$ is ~~the~~ the fibre misalignment angle.

The toughness of a lamina is due to fibre pull-out from the matrix.



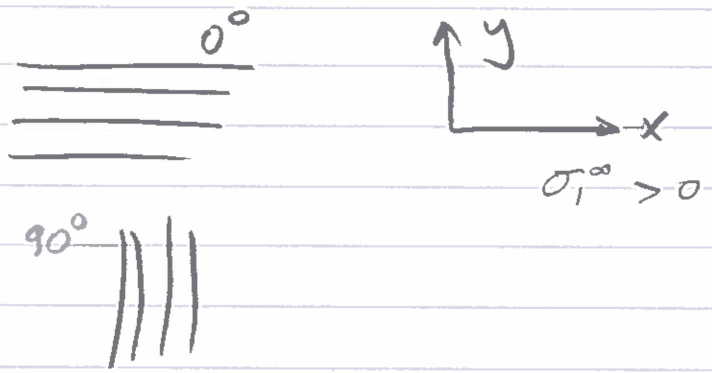
$$\pi a^2 \sigma_f = 2\pi a \tau_y \ell$$

$$\Rightarrow \ell = \frac{a}{2}\frac{\sigma_f}{\tau_y}$$

where a = fibre radius, and σ_f = fibre tensile strength, τ_y = matrix shear strength

Toughness scales with $\frac{1}{2}\sigma_f \cdot \ell$, so a high τ_y leads to a low toughness.

4. (b) (i)



$N_x = \sigma_1^\infty . t \qquad N_y = 0 \qquad N_{xy} = 0$

$M_x = M_y = M_{xy} = 0$

$$\begin{pmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \end{pmatrix} = \left[A^{-1} \right] \begin{pmatrix} N_x \\ N_y \\ N_{xy} \end{pmatrix}$$

Write $\sigma_1^\infty = \lambda \ \sigma_{ref}$ where $\sigma_{ref} = 1$ GPa

Then
$$\varepsilon_x^o = 0.0135 \ N_x = 0.0135 \ \lambda \ (\sigma_{ref} \ t)$$
$$\varepsilon_y^o = -0.0005 \ N_x = -0.0005 \ \lambda \ (\sigma_{ref} \ t)$$
$$\gamma_{xy}^o = 0$$

Strain allowables :

$$e_L^+ = \frac{1.448}{138} = 0.0105$$

$$e_L^- = -\frac{1.172}{138} = -0.0085$$

$$e_T^+ = \frac{48.3}{9000} = 0.00537$$

$$e_T^- = \frac{-248}{9000} = -0.0276$$

Q4. (b) (i) contd.

0° ply : $\varepsilon_L = 0.0135\lambda = 0.0105 \Rightarrow \lambda = 0.778$

$\varepsilon_T = -0.0005\lambda = -0.0276 \Rightarrow \lambda = 55.2$

90° ply $\varepsilon_L = -0.0005\lambda = -0.0085 \Rightarrow \lambda = 17$

$\varepsilon_T = 0.0135\lambda = 0.00537 \Rightarrow \lambda = 0.4$

So, first failure occurs in the transverse direction of the 90° plies at $\sigma_1^\infty = 400$ MPa

Q4. (b) (ii) $(\sigma) = [Q](\varepsilon^\circ)$

0° ply : $\sigma_1 = Q_{11}\varepsilon_x^\circ + Q_{12}\varepsilon_y^\circ$

$= (139 \times 0.0135 - 2.7 \times 0.0005)\lambda$

$= 1.88\lambda$

$\sigma_2 = Q_{21}\varepsilon_x^\circ + Q_{22}\varepsilon_y^\circ = 0.032\lambda$

90° ply : $\sigma_1 = Q_{11}\varepsilon_y^\circ + Q_{12}\varepsilon_x^\circ = -0.032\lambda$

$\sigma_2 = Q_{12}\varepsilon_y^\circ + Q_{22}\varepsilon_x^\circ = 0.12\lambda$

check : $\frac{1}{2}\sigma_1(0°\text{ ply}) + \frac{1}{2}\sigma_2(90°\text{ ply}) = \lambda$ ✓

$\frac{1}{2}\sigma_2(0°\text{ ply}) + \frac{1}{2}\sigma_1(90°\text{ ply}) = 0$ ✓

0° ply : $\frac{\sigma_1}{S_L^+} = \frac{1.88\lambda}{1.448} = 1.30\lambda$ $\frac{\sigma_2}{S_T^+} = 0.66\lambda$

90° ply : $\frac{\sigma_1}{S_L^-} = \frac{-0.032\lambda}{-1.172} = 0.027\lambda$ $\frac{\sigma_2}{S_T^+} = \underline{\underline{2.48\lambda}}$

So, first failure occurs in the transverse direction of the 90° plies at $\sigma_1^\infty = \underline{400 \text{ MPa}}$

(Same value as for max. strain criterion, such is the simple loading).

Q1. Extensional stiffness of laminates, and application to stress analysis.

Most candidates showed a good understanding of laminate plate theory and of the notion of a balanced laminate. Many could so the matrix manipulations without error. A surprisingly large number of candidates treated the spherical pressure vessel as a circular cylindrical pressure vessel.

Overall, the candidates found this a straightforward question, and obtained a high average mark.

Q2. Qualitative questions on properties of laminates.

Candidates showed a general appreciation of the main issues, but many attempts lacked sufficient detail. Few drew sketches or made quantitative references. Overall, the answers were somewhat vague.

Q3. Pressure vessel stress analysis for a filament wound pressure vessel.

Most candidates knew how to manipulate the compliance of a lamina due to rotation of axes. A surprisingly large number struggled to obtain the most compliant orientation of a lamina. There was an excellent understanding of the process of filament winding, but a mixed understanding of how to obtain the strain ratio in the axial and hoop directions.

Q4. Failure of laminates.

Most candidates understood the main ideas behind the various failure criteria. The first part on compressive failure by microbuckling and fibre pull-out were well done. Marks were lost in part (b) by candidates not setting out the various possibilities of which failure mode dominated.