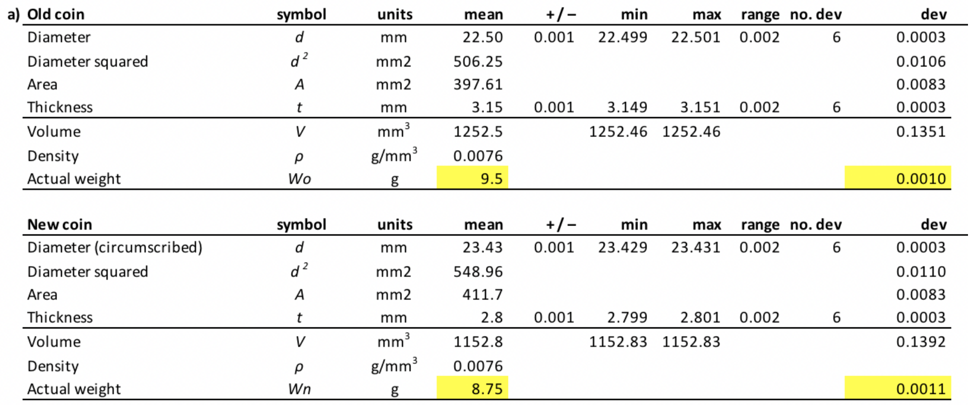
Question 1

a) Marks 10%

The diameter and thickness of the coins are both machined to ± 0.001 mm. The mean and standard deviation for the volume can be calculated using:

$$\begin{array}{ccc} ax & a\mu_x & a^2\sigma_x^2 \\ x_1x_2 & \mu_1\mu_2 & \mu_2^2\sigma_1^2 + \mu_1^2\sigma_2^2 \end{array}$$

The volume and weight are proportional (i.e. density) so the standard deviations are also proportional. The standard deviations are similar for the old coin = 0.0010 g and the new coin = 0.0011 g. Calculation details are given below:



a) **Old coin**	**symbol**	**units**	**mean**	**+/−**	**min**	**max**	**range**	**no. dev**	**dev**
Diameter	d	mm	22.50	0.001	22.499	22.501	0.002	6	0.0003
Diameter squared	d^2	mm2	506.25						0.0106
Area	A	mm2	397.61						0.0083
Thickness	t	mm	3.15	0.001	3.149	3.151	0.002	6	0.0003
Volume	V	mm^3	1252.5		1252.46	1252.46			0.1351
Density	ρ	g/mm^3	0.0076						
Actual weight	Wo	g	9.5						0.0010

New coin	**symbol**	**units**	**mean**	**+/−**	**min**	**max**	**range**	**no. dev**	**dev**
Diameter (circumscribed)	d	mm	23.43	0.001	23.429	23.431	0.002	6	0.0003
Diameter squared	d^2	mm2	548.96						0.0110
Area	A	mm2	411.7						0.0083
Thickness	t	mm	2.8	0.001	2.799	2.801	0.002	6	0.0003
Volume	V	mm^3	1152.8		1152.83	1152.83			0.1392
Density	ρ	g/mm^3	0.0076						
Actual weight	Wn	g	8.75						0.0011

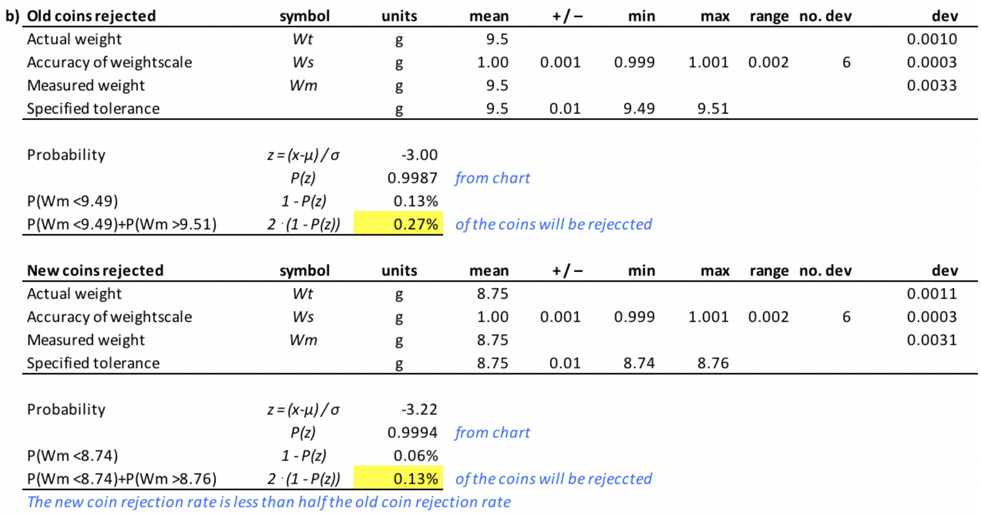
b) Marks 20%

The measured coin weight W_m is given by the actual coin weight W_t multiplied by the accuracy of the weigh-scale W_s. Note the weight-scale accuracy is expression as a percentage 0.1%, so the mean is taken as one. The mean and the standard deviation can be calculated using:

$$\begin{array}{ccc} x_1x_2 & \mu_1\mu_2 & \mu_2^2\sigma_1^2 + \mu_1^2\sigma_2^2 \end{array}$$

The probability is found by dividing the required coin tolerance ± 0.01 g by the measured coin weight standard deviation, then reading from the chart. The probability is doubled because coins are rejected for being both above and below the mean.

The fraction of new coins rejected is less than half the fraction of old coins rejected, because the new coin diameter is reduced but the reject tolerance remains the same. Calculation details are given below:



b) **Old coins rejected**	**symbol**	**units**	**mean**	**+/−**	**min**	**max**	**range**	**no. dev**	**dev**
Actual weight	*Wt*	g	9.5						0.0010
Accuracy of weightscale	*Ws*	g	1.00	0.001	0.999	1.001	0.002	6	0.0003
Measured weight	*Wm*	g	9.5						0.0033
Specified tolerance		g	9.5	0.01	9.49	9.51			

Probability	*z = (x-μ)/σ*	-3.00	
	P(z)	0.9987	*from chart*
P(Wm <9.49)	*1 - P(z)*	0.13%	
P(Wm <9.49)+P(Wm >9.51)	*2 · (1 - P(z))*	0.27%	*of the coins will be rejeccted*

New coins rejected	**symbol**	**units**	**mean**	**+/−**	**min**	**max**	**range**	**no. dev**	**dev**
Actual weight	*Wt*	g	8.75						0.0011
Accuracy of weightscale	*Ws*	g	1.00	0.001	0.999	1.001	0.002	6	0.0003
Measured weight	*Wm*	g	8.75						0.0031
Specified tolerance		g	8.75	0.01	8.74	8.76			

Probability	*z = (x-μ)/σ*	-3.22	
	P(z)	0.9994	*from chart*
P(Wm <8.74)	*1 - P(z)*	0.06%	
P(Wm <8.74)+P(Wm >8.76)	*2 · (1 - P(z))*	0.13%	*of the coins will be rejeccted*

The new coin rejection rate is less than half the old coin rejection rate

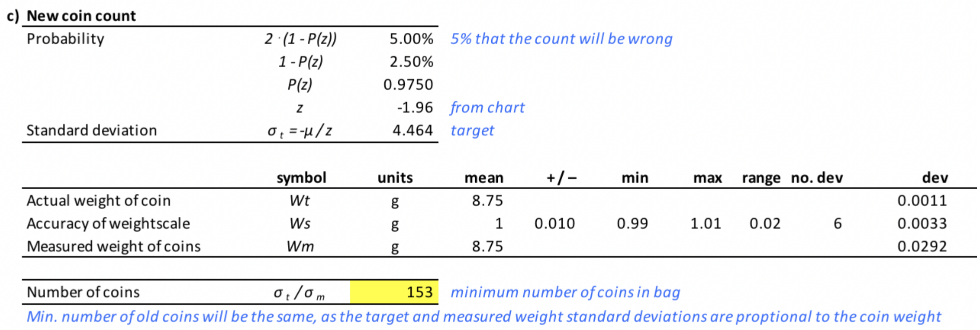
c) Marks 30%

The fraction of bags with an incorrect bag count is given as 5%. Therefore, we must work in reverse, starting with the fraction, halve it (as we could be over or under), find $P(z)$ and z from chart, and determine a target standard deviation for a bag of coins.

Then the actual weight calculated in (a) is multiplied with the weight-scale accuracy 1% to find a new measured weight of a coin, using:

$$x_1 x_2 \qquad \mu_1 \mu_2 \qquad \mu_2^2 \sigma_1^2 + \mu_1^2 \sigma_2^2$$

Finally, the target standard deviation for the bag of coins is divided by the measured coin weight, to find 153 coins in the bag. The minimum number of old coins will be the same, as both the standard deviations are proportional to the coin weight. Calculation details are given below:



c) **New coin count**			
Probability	*2 · (1 - P(z))*	5.00%	*5% that the count will be wrong*
	1 - P(z)	2.50%	
	P(z)	0.9750	
	z	-1.96	*from chart*
Standard deviation	*σ* $\sigma_t = -\mu / z$	4.464	*target*

	symbol	**units**	**mean**	**+/−**	**min**	**max**	**range**	**no. dev**	**dev**
Actual weight of coin	*Wt*	g	8.75						0.0011
Accuracy of weightscale	*Ws*	g	1	0.010	0.99	1.01	0.02	6	0.0033
Measured weight of coins	*Wm*	g	8.75						0.0292

Number of coins	σ_t / σ_m	153	*minimum number of coins in bag*

Min. number of old coins will be the same, as the target and measured weight standard deviations are proptional to the coin weight

d) Marks 30%

Candidates should outline at least 3 options which might affect the weight of the bag, which might include:

1. The measured weight of the bag of coins will be affected by the buildup of dirt and oils on the coins (increasing the weight of the coins) and any wear and tear on the coins (decreasing the weight of the coins).
2. The weigh-scale may need to be calibrated differently for new and old coins.
3. Bags containing a mixture of coins of different value will cause problems.
4. The weigh-scale needs to be calibrated regularly to avoid measurement drift.

Candidates should describe, with rough calculations, what happens if the bag contains both new and old coins. The possible permutations for overall weight of the bag increase greatly. For example, each new coin in an bag of olds coins, reduces the overall weight by 0.75g. If six new coins are included a bag of old coins, then the weight will be reduced by 4.5g, exactly half of an old coin, and the weigh-scale will not be able to determine an accurate count. So errors could start occurring as low as 7 coins (1 old, 6 new), compared to 153 coins if the coins are kept separate.

Weighing the bag of coins is only one way of assessing the coin count. Candidates should outline at least 3 other options, which might include:

1. Mechanically counting coins, individually, as they pass through a channel: slower, issues with blockages, expensive, noisy, larger in size, more moving parts so higher maintenance costs.
2. Optical/laser scanning of coins on belt: slower, more complex, expensive electronics, can sort between new and old but still needs mechanical separation, accuracy probably lower (reflections, misreads, calibration)
3. Stacking coins in tubes, using thickness to measure: accuracy improved as only one dimension, wear and tear, buildup of dirt, foreign bits of paper all affect accuracy, mechanical, noisy, more moving parts so higher maintenance costs. Good solution for small numbers of coins and manual counting.
4. Diameter measurement through a slot to distinguish old from new: 12-sided coin might jam, wear might allow new coins through old slot, mechanical, noisy, more moving parts so higher maintenance costs.

This was a popular question on probabilities related to counting new and old one pound coins. Many students made simple errors applying the mean and standard deviation equations to the problem of finding the coin weights (a), although found it easier to calculate the rejection rates (b). Very few candidates were able to reverse the calculations for part (c), but most gave good written answers about the other factors involved and alternative methods for counting bags of coins.

Question 2

Falls are a common and often serious cause of injury. Your company has been asked to reduce injuries due to falls in a hospital environment.

a) Marks 20%

The types of requirements will be business requirements, technical requirements, regulatory requirements and user-elicited requirements. After definition the problem and the objectives the next step is to define functions, gain a complete understanding of the environment and operating characteristics, and apply regulatory requirements.

Business requirements, technical and regulatory requirements can be elicited by a combination of interviews, focus groups and tender in an iterative process. User-elicited requirements are gathered by identifying target audiences, sampling representative participants from these target audiences, and then gather information for requirements. This process can be carried out via surveys, non-directed interviews, focus groups or observational studies. It is also possible to use probes (cultural and technological probes), although this is rarely carried out in practice.

b) Marks 30%

Requirements must be complete, testable and tracable. Key requirements, assuming a wearable solution, include:

1. The system is effective in detecting falls (e.g. $> 95\%$ precision). Source: For instance, medical/technical professional consensus.
2. The system has a false alarm rate less than 5%. Source: For instance, medical/technical professional consensus.
3. A small form factor that ensures the system is worn at all times (e.g. no larger than 20 mm (w) × 20 mm (h) × 10 mm (d)). Source: For instance, human factors engineers or via on-site observations and/or interviews with domain experts.
4. A low weight to ensure it is worn at all times (e.g. less than 100 g). Source: For instance, human factors engineers or via on-site observations and/or interviews with domain experts.
5. Long battery life (e.g. a device should work continuously for four weeks without charging). Source: For instance, procurement officer.
6. Cost-effective (e.g. maximum 100 unit cost per device). Source: For instance, procurement officer.
7. An operator response time less than 5 minutes. Source: For instance, medical/technical professional consensus.

c) Marks 30%

A solution-neutral problem statement can for instance be formulated as “Devise a system for alerting staff to patient falls.” Function structures can be elaborated via either FAST-diagrams or by explicitly drawing function structures and flows of energy, materials and signals. Regardless of method, a model answer elaborates on the function of fall detection (sensor, power on/off, and flow of energy and signals) and staff alert (actuator/display, and flow of energy and signals) and make it clear how these sub-functions are interlinked.

d) Marks 20%

Verification is the process of ensuring requirements are met. Requirements on effectiveness in diagnosis need to be verified in medical testing procedures. Requirements on weight, size, battery life and other similar characteristics can be measured or lab-tested. Unit cost can be verified by taking into account volume, manufacturer and bill of materials. Ergonomics, such as whether the system is securely fastened can be verified by calculation and lab-based stress testing. Requirements on precision and false alarm rates can be verified by collecting surrogate training data (by actors falling) to create benchmark datasets and then run classification algorithms via off-line experiments on data. Validation is ensuring the system works in practice in the target environment for the target audience. This can be done by on-site interviews with patients and staff, by monitoring injury rates and complications due to falls and by examining case-studies of falls and analysing desired versus actual outcomes.

This was a popular question which involved designing a solution for preventing falls in a hospital environment. Candidates were generally able to identify critical requirements and explain how to verify and validate the system. Some candidates had difficulties identifying the function structures and many candidates failed to give a coherent solution strategy for eliciting requirements for the problem.

Question 3

A system consists of n identical components configured for parallel redundancy.

a) Marks 20%

The definition of the hazard function is $h(t) = \frac{f(t)}{R(t)}$, where $f(t)$ and $R(t)$ and $f(t)$ is the probability density function and reliability function respectively. It is a conditional failure rate since it expresses a conditional statement: the probability of failing in the next instant *given* the system has not failed so far.

b) Marks 30%

As the hazard function reduces to the failure rate, the time t until a failure is exponentially distributed. Therefore the cumulative distribution function is $F(t) = 1 - e^{-\lambda t}$ and the reliability function is $R(t) = e^{-\lambda t}$.

Since the system has the components configured for parallel redundancy all components must fail for the system to fail. The probability of all n components failing is the product of the individual probabilities of each component failing. The expression for the probability of a system failure is thus $(1 - e^{-\lambda t})^n$.

c) Marks 20%

In reliability engineering, $R(t) = 1 - F(t) = \int_t^\infty f(t)dt$ and $h(t) = \frac{f(t)}{R(t)}$, where $R(t)$, $F(t)$ and $f(t)$ are the reliability function, cumulative density function and probability density function respectively.

By inspection, the derivative of $R(t)$ is therefore $-f(t)$ and we can rewrite $h(t)$ as $h(t) = -\frac{d}{dt} \ln R(t)$.

d) Marks 30%

Answer: A linear hazard function means the hazard function is now of the form $h(t) = kt$. We will need to derive an expression for $R(t)$ in terms of $h(t)$.

From c) we know $h(t) = -\frac{d}{dt} \ln R(t)$. Integrating from 0 to t, using the boundary condition $R(0) = 1$ (the probability of failure at $t = 0$ is zero), and solving for $R(t)$ yields $R(t) = \exp\left(-\int_0^t h(t)\, dt\right)$.

This results in $R(t) = \exp\left(-\int_0^t kt\, dt\right) = \exp\left(-\frac{kt^2}{2}\right)$. The probability of system failure in time t is therefore $\left(1 - e^{-kt^2/2}\right)^n$.

This was a less popular question which asked candidates to calculate the probabilities of system failure for a hypothetical system. Candidates were generally able to define the hazard function and use it to calculate the probability of system failure when the failure rate was constant. There was considerable more struggle when the hazard function was linear. On the positive side, quite a few *candidates were able to derive in-depth solutions, demonstrating an excellent understanding of the topic.*

Question 4

a) Marks 20%

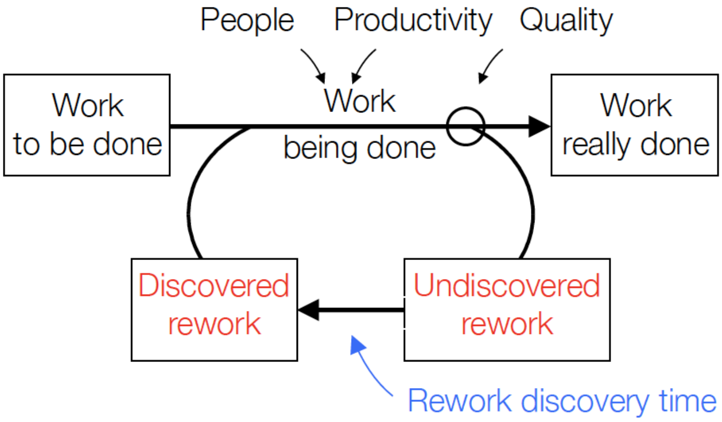
From the lecture notes: a *system* is a set of parts which, when combined, have qualities that are not present in any of the parts themselves, where those qualities are the emergent properties of the system.

In complex systems (such as this manufacturing site) individual parts interact with each other and with the outside world in many ways. The relationships between the parts determine how the system behaves. Thus a *systems thinking* approach would consider the interaction between the energy use and the material use across the site.

Understanding the interactions between energy and material use across the site is an example of *holistic thinking*, one of the six principles for systems given in the lecture notes. An integrated system can only be satisfactorily assessed by considering the system as a whole, looking across all of the parts.

b) Marks 30%

The standard rework model from the lecturer notes is:



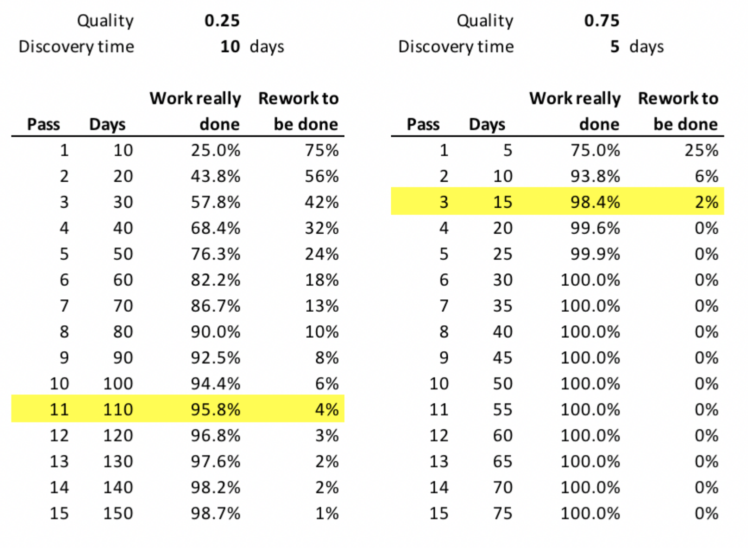
Considering the *material flows*, we have 1 tonne of product (work really done) requiring 4 tonnes of melted steel (work being done). If we ignore the 1% loss of material in the system, then we can calculate that 3 tonnes of steel must be being remelted in the rework loop.

Considering the *energy use*, 160 TJ is required to melt 5,000 tonne of product, giving 32 GJ/t (note melting of steel in an electric arc furnace normally uses 6.7 GJ/t). Each tonne of steel is melted 4 times, hence the energy use per melt is roughly 8 GJ/t.

Eliminating this remelt (rework) loop could save up to 24 GJ/t of product produced and deliver a 75% cut in energy use for melting on the site. This is significantly more than the 5% energy savings revealed by the previous energy review.

c) Marks 30%

The delay between casting and testing (rework discovery time) requires that a fraction of the products (1 minus the quality of work) must be return for remelting, creating an ongoing ever decreasing remelt loop. The quality of work = work really done / work being done = 1t/4t = 25%. A 95% complete rate is achieved after 11 passes and 110 days (for q=0.25, t=10 days), and 3 passes and 15 days (for q=0.75, t=5 days), as shown in the table below:



Quality **0.25**
Discovery time **10** days

Pass	Days	Work really done	Rework to be done
1	10	25.0%	75%
2	20	43.8%	56%
3	30	57.8%	42%
4	40	68.4%	32%
5	50	76.3%	24%
6	60	82.2%	18%
7	70	86.7%	13%
8	80	90.0%	10%
9	90	92.5%	8%
10	100	94.4%	6%
11	110	95.8%	4%
12	120	96.8%	3%
13	130	97.6%	2%
14	140	98.2%	2%
15	150	98.7%	1%

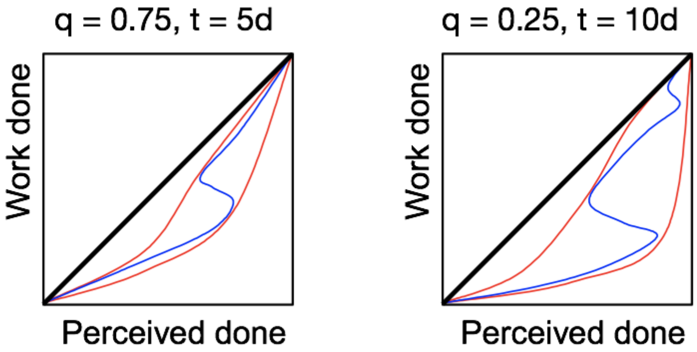
Quality **0.75**
Discovery time **5** days

Pass	Days	Work really done	Rework to be done
1	5	75.0%	25%
2	10	93.8%	6%
3	15	98.4%	2%
4	20	99.6%	0%
5	25	99.9%	0%
6	30	100.0%	0%
7	35	100.0%	0%
8	40	100.0%	0%
9	45	100.0%	0%
10	50	100.0%	0%
11	55	100.0%	0%
12	60	100.0%	0%
13	65	100.0%	0%
14	70	100.0%	0%
15	75	100.0%	0%

d) Marks 20%

Lower work quality and a longer rework discovery time, leads to a larger gap between perceived and real work done, and more uncertainty in estimating progress of the order.

The exact shape of the sketches is not important, but they should look something like:



This was a new question, and least popular, which asked candidates to apply the rework cycle to a manufacturing plant making forged steel parts. Candidates who attempted this question (and weren't running out of time) did well in the question. Sadly, many candidates couldn't remember the formal dentitions relating to systems and systems thinking (a). Those who could draw the rework cycle (b) found it simple to calculate the number of rework cycles and days (c). Few drew accurate sketches of work done versus perceived work done (d).