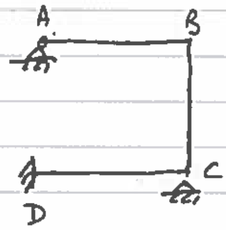
Q1 a)



The diagonal braces remove the effect of rollers at A.

Beams UB 457 × 191 × 98
Column UC 305 × 305 × 283

Steel grades irrelevant. Young's Moduli all the same (but affects compactness).

Check compactness.

COLUMN: flange $\frac{322.2 - 26.8}{2(44.1)}\sqrt{\frac{460}{355}} = 3.8$ compact

web $\frac{365.3 - 44.1 \times 2}{26.8}\sqrt{\frac{460}{355}} = 11.8$ compact in compression and bending.

BEAMS: flange $\frac{(192.8 - 11.4)}{2(19.6)}\sqrt{\frac{235}{355}} = 3.8$ compact

web $\frac{467.2 - 19.6 \times 2}{11.4}\sqrt{\frac{235}{355}} = 30.5$ compact only in bending.

.. but for this calculation the beams act only in bending, therefore everything is compact and can use full section properties.

	I_{maj}	L	I/L
Column	78 870	350	225.3
Beams	45 730	600	76.2
	cm⁴	cm	cm3

$$k_{TOP} = \frac{(EI/L)_{col}}{(EI/L)_{col} + 3/4\,(EI/L)_{beam}} = \frac{225.3}{225.3 + 3/4(76.2)} = 0.8$$

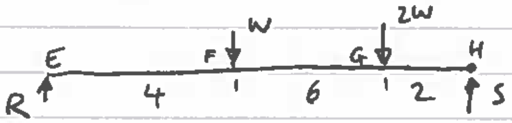
$$k_{BOT} = \frac{(EI/L)_{col}}{(EI/L)_{col} + (EI/L)_{beam}} = \frac{225.3}{225.3 + 76.2} = 0.75$$

Use graph a) $(0.8, 0.75) \rightarrow l_{eff} = 0.83L$
$= 0.83(3.5\text{m})$
$= \underline{\underline{2.905\text{m}}}$

$$P_{cr} = \frac{\pi^2 E I_{maj}}{L^2_{eff}} = \frac{\pi^2 (210 \times 10^9)\,[\text{N/m}^2]\,[78\,870 \times 10^{-8}\text{m}^4]}{(2.905)^2\,[\text{m}^2]}$$

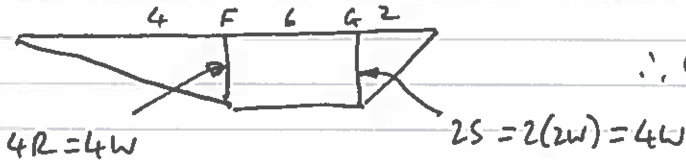
$= \underline{\underline{193.7 \text{ MN}}}$ Elastic critical load for major axis buckling. (Elasto-plastic not asked for).

Q1 b)



Moment about E: $4(W) + 10(2W) = 12S$

$\therefore 24W = 12S \quad \therefore S = 2W \quad \therefore R = W$



(Shear = 0 along FG)

$\therefore$ Critical section = FG (longest, and also equal + opposite).

S275 Steel : UB 356 × 127 × 39

$I_{maj} = 10170 \times 10^{-8}$ mm4

$I_{min} = 358 \times 10^{-8}$ mm4

$J = 15.1 \times 10^{-8}$ mm4

$Z_{maj,plastic} = 659 \times 10^{-6}\ m^3$

Check compactness.

Flange : $\lambda = \dfrac{126.0 - 6.6}{2} \dfrac{1}{10.7} \sqrt{\dfrac{275}{355}} = 4.91 < 8$ compact ✓

Web Bending : $\lambda = \dfrac{353.4 - 2(10.7)}{6.6} \sqrt{\dfrac{275}{355}} = 44.3 < 56$ compact ✓

Calculate the Plastic :

$$M_{pL} = \sigma_y Z_{pl,major} = \underset{N/m^2}{(275 \times 10^6)} \underset{m^3}{(659 \times 10^{-6})} = \underline{\underline{181.2\ kNm}}$$

Calculate Elastic

Basic $M_{cr,LTB} = \dfrac{\pi}{L}\sqrt{GJEI_{minor}} = \dfrac{\pi E}{L}\sqrt{\dfrac{JI_{minor}}{2.6}}$

$$= \frac{\pi}{6}(210 \times 10^9)\sqrt{\frac{15.1(358)}{2.6}} \times 10^{-8} = \underline{\underline{50.1\ kNm}}$$

Warping correction: $\sqrt{1 + \dfrac{\pi^2 E\Gamma}{L^2 GJ}}$; $\Gamma = \dfrac{I_{minor} D^2}{4}$

$$= \frac{(358 \times 10^{-8})(353.4 - 10.7)^2 \times 10^{-6}}{4}$$

$$= 10.5 \times 10^{-8}\ m^6$$

$$\sqrt{1 + \frac{\pi^2 E\Gamma}{L^2 GJ}} = \left(1 + \frac{\pi^2}{6^2}(2.6)\frac{10.5 \times \cancel{10^{-8}}}{15.1 \times \cancel{10^{-8}}}\right)^{1/2} = \sqrt{1.496} = 1.223$$

$$M_{LTB,\ EQ+TOP} = 1.223(50.1) = \underline{\underline{61.27\ kNm}}$$

Q1 b) cont'd.

$C_{unequal} = 1$ since equal and opposite on FG

$$\lambda = \sqrt{\frac{\text{Plastic}}{\text{Elastic}}} = \sqrt{\frac{181.2}{61.27}} = \underline{1.72}$$

Curve? Rolled I-section, $\frac{h}{b} = \frac{353.4}{126.0} = 2.8$

which is greater than 2 $\Rightarrow$ curve (b), DS3 (for LTB).

$\rightarrow \chi = 0.28$ (DS1)

$$M_{max} = \chi M_{pL} = 0.28(181.2) = \underline{\underline{50.7 \text{ kNm}}}$$

BMD:

$4W = 50.7$ kNm

$\therefore$ $\underline{\underline{W = 12.6 \text{ kN}}}$

Check Shear:

Max. shear $= 2W = 25.2$ kN

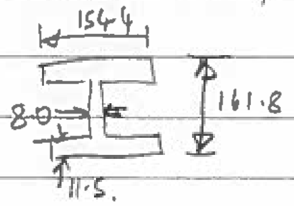
$$A_{web} = 6.6(353.4 - 2(10.7))$$
$$= 2191 \text{ mm}^2$$

$$\tau_{web} = \frac{25.2 \times 10^3}{2191} = 11.5 \text{ MPa}.$$

$$< \frac{275}{\sqrt{3}} = 159 \text{ MPa}$$

$\therefore$ shear OK.

Q2. 5m beam, UC 152×152×37. S275.



$I_{maj} = 2210 \times 10^{-8}\,m^4$

$I_{minor} = 706 \times 10^{-8}\,m^4$

$J = 19.2 \times 10^{-8}\,m^4$

$Z_{pl,maj} = 309 \times 10^{-6}\,m^3$

$A = 47.1 \times 10^{-4}\,m^2 = 4710\,mm^2$

Check compactness: $\lambda_{flange} = \left(\frac{154.4 - 8}{2}\right)\frac{1}{11.5}\sqrt{\frac{275}{355}} = 5.6 < 8$ ∴ OK
COMPACT DS4.

$\lambda_{web} = \frac{161.8 - 2(11.5)}{8.0}\sqrt{\frac{275}{355}} = 15 < 24$ ∴ OK compression
< 56 ∴ OK bending.

AXIAL: 3

Calc. plastic. $N_{pl} = (275)(4710) = \underline{1295\ kN}$

Calc. elastic $N_{el,maj} = \frac{\pi^2 (210 \times 10^{9})(2210 \times 10^{-8})}{5^2} = \underline{1832\ kN}$

$N_{el,min} = \frac{\pi^2 (210 \times 10^{9})(706 \times 10^{-8})}{(2.5)^2} = \underline{2341\ kN}$

$\lambda_{maj} = \sqrt{\frac{N_{pl}}{N_{el}}} = \sqrt{\frac{1295}{1832}} = 0.84$ | Curve. $y\text{-}y \rightarrow b$ | $\Rightarrow \chi = 0.7$

$\lambda_{min} = \sqrt{\frac{N_{pl}}{N_{el}}} = \sqrt{\frac{1295}{2341}} = 0.74$ | $z\text{-}z \rightarrow c$ | $\chi = 0.7$ also

$\frac{h}{b} = \frac{161.8}{154.4} = 1.05 < 1.2$, $t_f < 100\,mm$ 5

∴ $N_{allow,maj} = N_{allow,min} = \chi N_{pl} = 0.7(1295) = \underline{906.5\ kN}$.

FLEXURE. Calc. plastic $M_{pl} = Z_{pl,maj}\,\sigma_y = (275 \times 10^{6})(309 \times 10^{-6}) = \underline{85\ kNm}$

Calc. Elastic: $M_{LTB\,basic} = \frac{\pi}{L}\sqrt{GJEI_{min}} = \frac{\pi}{L}E\sqrt{\frac{JI_{min}}{2.6}}$

$= \frac{\pi}{2.5}(210 \times 10^{9})\sqrt{\frac{(19.2)(706)}{2.6}} \times 10^{-8} = \underline{190.5\ kNm}$

Warping correction: $\Gamma = \frac{ID^2}{4} = \frac{(706\times10^{-8})(161.8-11.5)^2\times10^{-6}}{4} = \underline{\underline{4.0\times10^{-8}\ m^6}}$

$\sqrt{1+\frac{\pi^2 E\Gamma}{L^2 GJ}} = \sqrt{1+\left(\frac{\pi}{2.5}\right)^2 2.6\left(\frac{4.0}{19.2}\right)} = \sqrt{1.855} = 1.36$

$\therefore M_{LTB,\ \cancel{ER\ tot}} = 1.36(190.5) = \underline{\underline{259.5\ kNm}}$

For BMD, $C_{unequal} = 0.6 \Rightarrow M_{LTB} = \frac{259.5}{0.6} = \underline{\underline{432.5\ kNm}}$

$\lambda = \sqrt{\frac{\cancel{\sigma}}{\lambda\lambda}} = \sqrt{\frac{85}{432.5}} = \sqrt{0.196} = \underline{\underline{0.44}}$

Which curve? Rolled I, $\frac{h}{b} = \frac{161.8}{154.4} = 1.05 < 2 \Rightarrow$ curve a). (DS3).

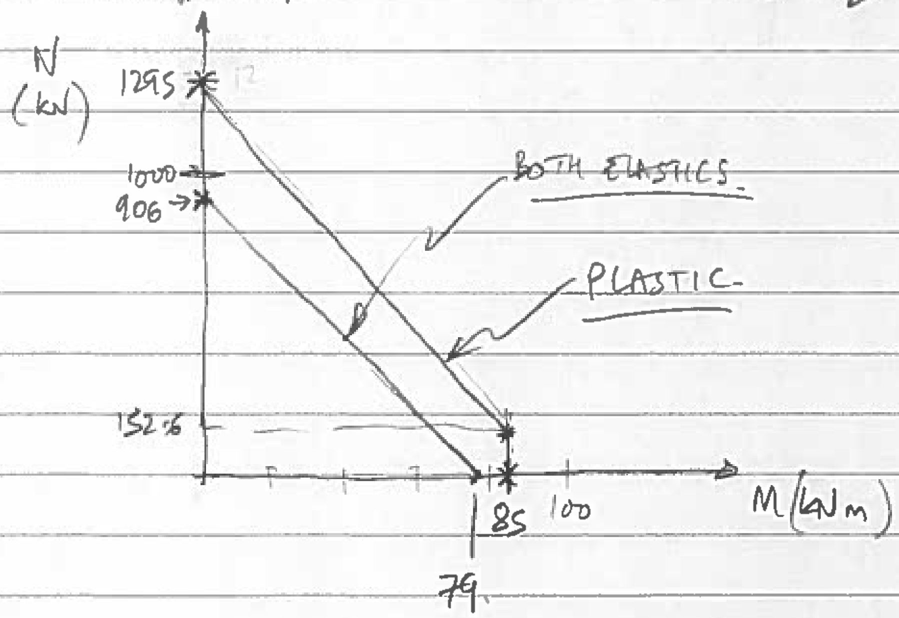
$\Rightarrow \chi = 0.93$ (DS1)

$\therefore M_{allow} = \chi M_{pl} = 0.93(85) = \underline{\underline{79\ kNm}}$ 8

Also need half web fraction.

Web fraction $= \frac{8(161.8-2(11.5))}{4710} = 0.236$

Half Web fraction = 0.118. $N_{pl}\times\frac{a}{2} = 1295\times0.118 = \underline{\underline{152.6\ kN}}$



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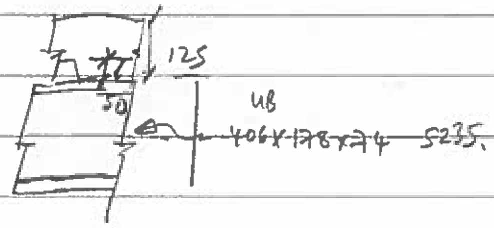
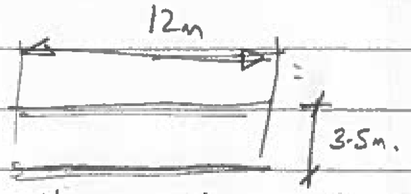
Q3 a)

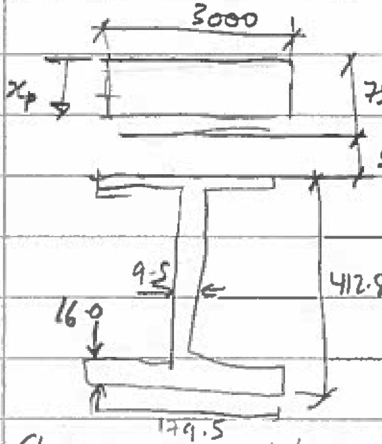


PLAN.



Effective width $= \min\left(\frac{span}{4}, w\right)$

$= \min(3, 3.5) = \underline{3m}$



Assume N/A in concrete.

$A_s \sigma_y = 0.6 f_{cd} b_c x_p$.

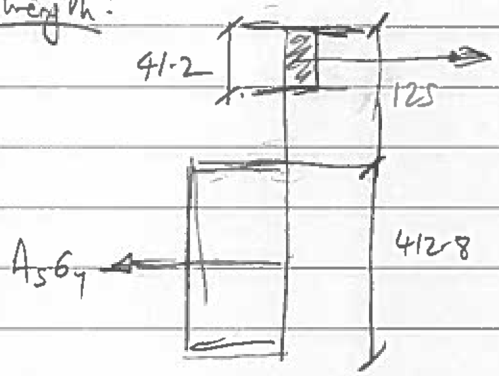
412.8 $N/mm^2 \rightarrow \frac{(9450)(235)}{(0.6)(30)(3000)} = \underline{41.2\ mm}$

Check compactness of beam:

flange $\frac{b}{t}\sqrt{\frac{235}{355}} = \left(\frac{179.5-9.5}{2}\right)\frac{1}{16.0}\sqrt{\frac{235}{355}} = 4.3 < 8$

web $\frac{b}{t}\sqrt{\frac{235}{355}} = \left(\frac{412.8-2(16)}{9.5}\right)\sqrt{\frac{235}{355}} = 32.6 < 56$ OK bending.

Strength:



$M = A_s\sigma_y \underbrace{\left[\frac{412.8}{2} + 125 - \frac{41.2}{2}\right]}_{310.8\ mm}$

$= (9450\ mm^2)(235\ N/mm^2)(310.8\ mm)$

$= \underline{690.2\ kNm}$ strength

Loads: DL $= (3.5m)(0.1m)(2400\ kg/m^3) = 840\ kg/m$

steel beam $= 74.2\ kg/m$

$914.2\ kg/m = \underline{8.97\ kN/m}$

Permanent services $= 1\ kN/m^2 \times 3.5m = \underline{3.5\ kN/m}$

Live load $= (3.5m \times w) = 3.5w$ kN/m

TOTAL $= 1.35(8.97 + 3.5) + 1.5(3.5w) =$

$= \underline{(16.83 + 5.25w)}$

Q3a) contd

$$\text{Load} = 1{\cdot}35\left(8{\cdot}97 + 3{\cdot}5\right) + 1{\cdot}5\left(3{\cdot}5\right)4$$

$$= 16{\cdot}83 + 5{\cdot}25(4) = 37{\cdot}83 \text{ kN/m.}$$

$$M = \frac{(37{\cdot}83)(12)^2}{8} = \underbrace{680{\cdot}9 \text{ kNm}}_{\text{req'd}} < \underbrace{690{\cdot}2 \text{ kNm}}_{\text{strength}}$$

$\therefore$ OK.

Check shear:

$$A_{web} = 9{\cdot}5\left(412{\cdot}8 - 2(16)\right) = 3618 \text{ mm}^2$$

$$S_{max} = A_{web}\,\tau_y = 3618 \times \frac{235}{\sqrt{3}} = 491 \text{ kN}$$

$$\text{Actual shear} = \underset{\text{kN/m}}{37{\cdot}83} \times 6\text{m} = 227 \text{ kN} \quad \therefore \text{OK.}$$

Q3 b). Shear studs: Total force at centre $= A_{st}\,\sigma_y$

$$= \left(9450 \text{ mm}^2\right)\left(235 \text{ N/mm}^2\right) = 2221 \text{ kN}$$

Shear studs in each half span

eg. 25mm dia, 100 mm $\longrightarrow$ 154 kN each $\Rightarrow$ need $\frac{2221}{154}$

Cover is OK ✓

$= 14{\cdot}4$ req'd

so 30 req'd in total.

We have $\frac{12\text{m}}{0{\cdot}2\text{m}} = 60$ troughs,

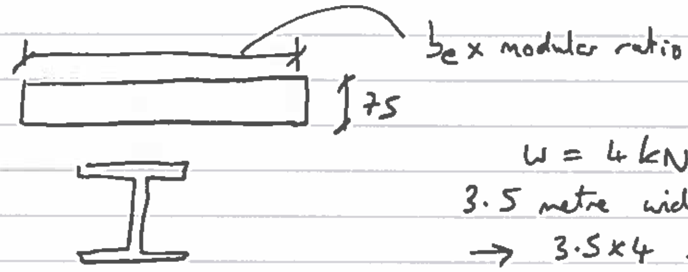
$\therefore$ Need one stud in every second trough.

(Alternative designs are possible:

eg. one 19 x 100 stud in each trough

or one pair of 13 x 65 studs in each trough.)

Q3 c) Deflection.



$w = 4\ kN/m^2$

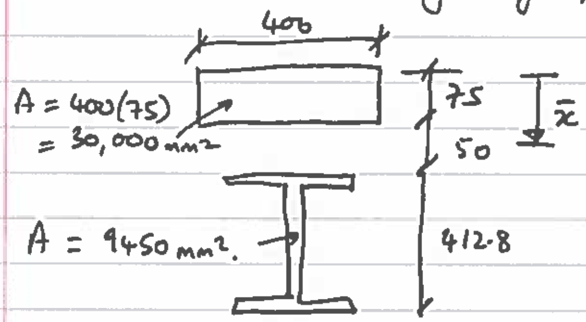
3.5 metre width

$\rightarrow\ 3.5 \times 4 = 14\ kN/m$

unfactored LL.

$$b_e \times \text{modular ratio} = 3.0\left(\frac{28\ GPa}{210\ GPa}\right) = 0.4\,m.$$

Assume N/A in trough region, and calculate elastic I.



$$A_{TOT}\,\bar{x} = \int x\,dA$$

$$39450\,\bar{x} = 30{,}000\left(\frac{75}{2}\right) + 9450 \times \left(\frac{412.8}{2} + 125\right)$$

$$\bar{x} = \frac{1.125\times10^6 + 3.132\times10^6}{3945\times10^3}$$

$$= \underline{107.9\ mm}$$

(which is in trough region ✓).

$$I = \frac{400(75)^3}{12} + 400(75)\left(107.9 - \frac{75}{2}\right)^2 + 27310\times10^4 + 9450\left[\frac{412.8}{2} + 125 - 107.9\right]^2$$

$$= 14.06\times10^6 + 148.7\times10^6 + 273.1\times10^6 + 472.0\times10^6$$

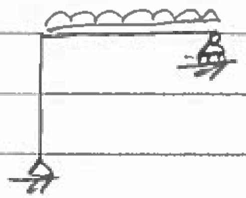
$$= \underline{907.9} \times 10^6\ mm^4$$

$$\Delta = \frac{5wL^4}{384\,EI} = \frac{5(14\times10^3)(12)^4}{384\,(210\times10^9)(907.9\times10^{-6})} \qquad \frac{[N/m]\,[m^4]}{[N/m^2]\,[m^4]}$$

$$= 0.0198\ m = \underline{20mm}$$

$$\frac{0.02m}{12\ m} = 0.0017 = \frac{1}{605} \rightsquigarrow \frac{\text{Span}}{600}$$

∴ OK for deflections.

Q4  Can treat as simply-supported.

$$\therefore M_{centre} = \frac{wl^2}{8} + \frac{WL}{4}$$

$$= \frac{(\cancel{120}\ 80)(24)^2}{8} + \frac{(200)(24)}{4}$$

$$= \cancel{8640}\ 5760 + 1200 = \cancel{9840}\ 6960 \text{ kNm}$$

(M_{centre})

~~a) Compactness.~~

Shear at centre = 100kN.

Shear at end $= (\cancel{120}\ 80)(12) + 100$

$= \cancel{1540 \text{ kN}}\ \ 1060 \text{ kN}$

a) Compactness. $\lambda = \frac{b}{t}\sqrt{\frac{\sigma_y}{355}}$ DS4, $\sqrt{\frac{235}{355}} = 0.8136$.

Flange. $b = \frac{900 - 2(10) - 2(20)}{3} = 280\text{mm}$

$t = 10\text{ mm} \rightarrow \frac{b}{t} = 28 \quad \frac{b}{t}\sqrt{\frac{235}{355}} = 0.8136(28) = 22.8$

< 24

$\therefore$ Compact in compression.

Web: $b = \frac{1800 - 2(10) - 20}{2} = 880\text{mm}$

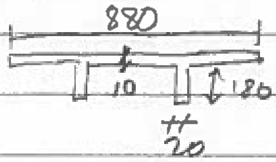
$t = 10\text{mm} \rightarrow \frac{b}{t} = 88 \quad \frac{b}{t}\sqrt{\frac{235}{355}} = 71.6$

which is > 54 so not compact in bending

> 24 not compact in compression

Stiffeners: $\frac{b}{t}\sqrt{\frac{235}{355}} = \frac{180}{20}(0.8136) = 7.3 < 8$ $\therefore$ Compact in compression.

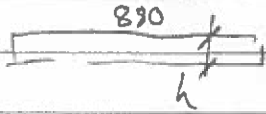
Q4 b) Smeared Section



Area $= 880(10) + 2(180)(20)$

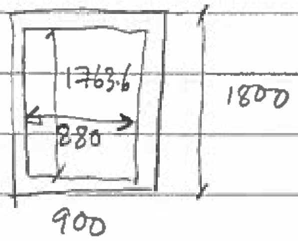
$= \underline{\underline{16000\ mm^2}}$

$=$



$\Rightarrow h = \dfrac{16000}{880} = \underline{\underline{18.18\ mm}}$

$I_{xx} =$



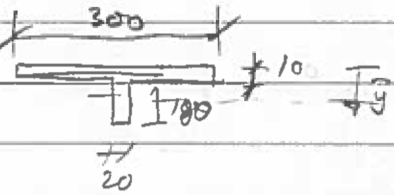
$= \dfrac{(900)(1800)^3}{12} - \dfrac{(880)(1763.6)^3}{12}$

$= \underline{\underline{35.35 \times 10^9\ mm^4}}$

4

NOTE: Compression ~~flange compact in~~ compression from pt a) so $b_e = b$

c) Top flange



$A = (300)(10) + (180)(20)$

$= \underline{6600\ mm^2}$

$A\bar{y} = (3000)(5) + (3600)(10+90) \rightarrow \bar{y} = \dfrac{15,000 + 360,000}{6600} = \underline{\underline{56.8\ mm}}$

$I_{xx} = \dfrac{(300)(10)^3}{12} + (300)(10)(56.8-5)^2 + \dfrac{(20)(180)^3}{12} + 20(180)(100-56.8)^2$

$= 25 \times 10^3 + 8.049 \times 10^6 + 9.72 \times 10^6 + 6.718 \times 10^6$

$= \underline{24.51 \times 10^6\ mm^4}$

Calc. plastic. $N_{pl} = A_{st}\sigma_y = (6.6 \times 10^3\ mm^2)(235\ N/mm^2) = \underline{\underline{1551\ kN}}$

Calc. Elastic. $N_{el} = \dfrac{\pi^2 EI}{L^2} = \dfrac{\pi^2 (210 \times 10^3)(24.51 \times 10^6)}{(3000)^2}\ \dfrac{N/mm^2\ mm^4}{mm^2}$

$= \underline{\underline{5644\ kN}}$

$\lambda = \sqrt{\dfrac{N_{pl}}{N_{el}}} = \sqrt{\dfrac{1551}{5644}} = 0.52$ Use curve c) say (welded)

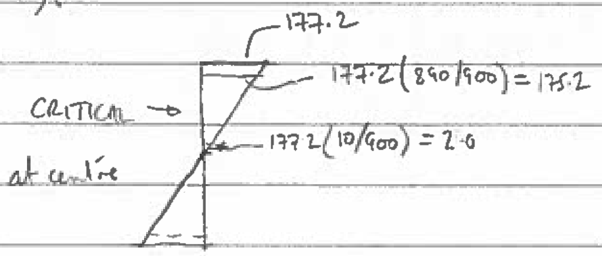
DS1 $\rightarrow \chi = 0.82$ $N_{design} = 0.82(1551) = 1272\ kN.$

Req'd: $\sigma = \dfrac{My}{I} = \dfrac{(\cancel{9840}\ 6960 \times 10^3\ Nm)(0.9\ m)}{(35.35 \times 10^{-3}\ m^4)} = \underline{177.2\ MPa}$ 4

$\therefore N_{req'd} = 177.2 \times 6600 = 1169\ kN$

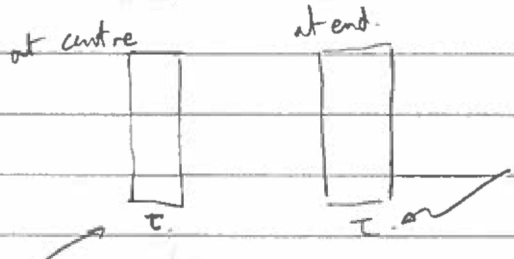
$\therefore$ OK req'd

d).



$\sigma_c = \dfrac{175.2+2}{2} = 88.6$ MPa

$\sigma_b = 175.2 - 88.6 = 86.6$ MPa



$\tau = \dfrac{(100+80(12))10^3}{20(1800)} = \underline{\underline{29.4 \text{ MPa}}}$

$\tau = \dfrac{100\times10^3 \text{ N}}{(20)(1800) \text{ mm}^2} = 2.8$ MPa

i) STRENGTH: Centre: — $\sigma^2 \leq \sigma_y^2 - 3\tau^2$

actual $177.2 \leq (235)^2 - 3(2.8)^2 = (234.9)^2$ OK

End $0 \leq (235)^2 - 3(29.4)^2$ ✓ OK

ii) Stability: Centre $\lambda = \dfrac{b}{t}\sqrt{\dfrac{235}{355}} = 71.6$ from part a)

DS4.

$\sigma_c = 88.6 \quad \lambda = 71.6 \rightarrow K_c = 0.43$

$\sigma_b = 86.6 \quad \lambda = 71.6 \rightarrow K_b = 1.1$

$\tau = 2.8 \quad \lambda = 71.6 \rightarrow K_q = 0.74$

$(\phi = \frac{3}{0.9} > 3)$

$$\frac{88.6}{(0.43)(235)} + \left(\frac{86.6}{1.1(235)}\right)^2 + \left(\frac{2.8}{(0.74)(235/\sqrt{3})}\right)^2$$

$$\frac{88.6}{101.1} + \left(\frac{86.6}{258.5}\right)^2 + \left(\frac{2.8}{100.4}\right)^2$$

$= 0.876 + 0.112 + \sim 0 = \underline{\underline{0.99}}$ ∴ OK, just.

End.

$\sigma_c = 0$

$\sigma_b = 0$

$\tau = 29.$

$K_q = 0.74$

$\left(\dfrac{29}{0.74(235/\sqrt{3})}\right)^2 = \left(\dfrac{29}{100.4}\right)^2 = 0.09 < 1$ ∴ OK.

8

e) No improvements req'd.

2.

ENGINEERING TRIPOS PART IIB 2019

ASSESSOR'S COMMENTS 4D10 Structural Steelwork

Q1 Subframe analysis, column buckling and Lateral-Torsional Buckling of a beam
A popular question, well-answered by most candidates. The question tested three separate topics. A common place where marks were lost was via the incorrect use of the 0.75 factor for "far-end-pinned" beams in the part a) stiffness factors.

Q2 Interaction diagrams for beam-column stability
A popular question and many answers were almost fully correct, despite the numerous calculations required. Many of the errors were arithmetical. Marks were lost for failing tocheck compactness.

Q3 Composite floor design
This was a popular question attempted by all candidates. The methodology is now well-established in the lecture notes, examples papers and past papers, and the students have demonstrated that they can reliably undertake all the numerous computations required, even under examinations conditions.

Q4 Thin-walled box girder using stiffened plates
There were numerous very good answers submitted. Marks were mostly lost for arithmetic errors, and when students ran out of time.

Allan McRobie (Principal Assessor)