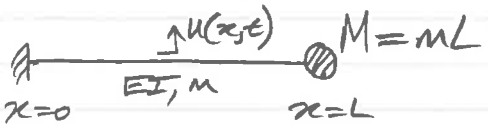
Part IIB 4D6 : Dynamics in Civil Engineering

1.



a) Assume $\bar{u}(x) = a\left(\frac{x}{L}\right)^3 + b\left(\frac{x}{L}\right)^2 + c\left(\frac{x}{L}\right) + d$

$$= \frac{ax^3}{L^3} + \frac{bx^2}{L^2} + \frac{cx}{L} + d$$

BCs : $\bar{u}(0) = 0$ (1) ; $\bar{u}'(0) = 0$ (2), no slope ; $\bar{u}''(L) = 0$ (3), no tip BM. [2]

(1) $\Rightarrow d = 0$

$\bar{u}' = \frac{3ax^2}{L^3} + \frac{2bx}{L^2} + \frac{c}{L}$ ∴ (2) $\Rightarrow c = 0$

$\bar{u}'' = \frac{6ax}{L^3} + \frac{2b}{L^2}$ ∴ (3) $\Rightarrow \frac{6a}{L^2} + \frac{2b}{L^2} = 0$, $b = -3a$

∴ $\bar{u} = \frac{ax^3}{L^3} - \frac{3ax^2}{L^2}$ or $\frac{x^3}{L^3} - \frac{3x^2}{L^2}$ (arbitrarily setting $a = 1$)

$\bar{u}' = \frac{3x^2}{L^3} - \frac{6x}{L^2}$; $\bar{u}'' = \frac{6x}{L^3} - \frac{6}{L^2}$ [1]

$$M_{eq} = \int_0^L m\bar{u}^2\,dx = m\int \frac{x^6}{L^6} - \frac{6x^5}{L^5} + \frac{9x^4}{L^4}\,dx + M\left(\bar{u}(x=L)\right)^2$$

$$= m\left[\frac{x^7}{7L^6} - \frac{6x^6}{6L^5} + \frac{9x^5}{5L^4}\right]_0^L + mL(1-3)^2 = \frac{33mL}{35} + 4mL = \frac{173}{35}mL$$ [2]

$$K_{eq} = \int_0^L EI\left(\frac{d^2\bar{u}}{dx^2}\right)^2 dx = EI\int_0^L \frac{36x^2}{L^6} - \frac{72x}{L^5} + \frac{36}{L^4}\,dx = EI\left[\frac{36x^3}{3L^6} - \frac{72x^2}{2L^5} + \frac{36x}{L^4}\right]_0^L$$

$$= \frac{EI}{L^3}(12 - 36 + 36) = \frac{12EI}{L^3}$$ [2]

$$\Rightarrow \omega_1 = \sqrt{\frac{K_{eq}}{M_{eq}}} = \sqrt{\frac{12EI}{L^3}\cdot\frac{35}{173mL}} = \sqrt{\frac{420EI}{173L^4}} = 1.56\sqrt{\frac{EI}{mL^4}}$$ [1]

$$\equiv 0.248\sqrt{\frac{EI}{mL^4}}\ \text{Hz}$$

b) $EI = 2\times10^{11}$ Nm², $L = 60$m, $m = 3000$ kg/m

$$\Rightarrow f_1 = \frac{1}{2\pi}\times 1.56\sqrt{\frac{2\times10^{11}}{3000\times60^4}} = 0.56\ \text{Hz}.$$

∴ $T_1 = 1.78$s [2]

1 Cont.

$$F_{eq} = \int_0^L f\bar{u}\,dx = \int_0^L \frac{Fx}{L}\left(\frac{x^3}{L^3} - \frac{3x^2}{L^2}\right) dx = F\int_0^L \frac{x^4}{L^4} - \frac{3x^3}{L^3}\,dx$$

$$= F\left[\frac{x^5}{5L^4} - \frac{3x^4}{4L^4}\right]_0^L = FL\left(\frac{1}{5} - \frac{3}{4}\right) = -\frac{11FL}{20} \quad [2]$$

N.B. $\bar{u}(L) = -2$

$$\therefore |u_{stat}| = \frac{F_{eq}\bar{u}}{K_{eq}} = \frac{11FL}{20} \cdot \frac{L^3}{12EI} \cdot 2 \quad \frac{11FL^4}{120EI} = \frac{11 \times 60\times10^3 \times 60^4}{120 \times 2\times10^{11}} = 0.356\ m$$

[2]

$$\frac{t_d}{T} = \frac{3}{1.78} = 1.69 \therefore DAF = 2 \text{ (datasheet)}$$

$$\therefore |u_{max}| = 2 \times 0.356 = 0.713\ m \quad [2]$$

c) $K_{eq} \propto \frac{1}{L^3}$, $\omega_1 \propto \frac{1}{L^2}$

$L \times 2 \Rightarrow K_{eq} \times \frac{1}{8} \therefore u_{stat} \times 8$

$\omega_1 \times \frac{1}{4} \therefore T \times 4 \Rightarrow \frac{t_d}{T} = \frac{1.7}{4} = 0.4 \therefore DAF = 1.6$

$\Rightarrow |u_{max}| = 1.6 \times 8 \times 0.356 = 4.56\ m$ [3]

NB: the DAF used here should be 1.9, not 1.6, with the final Umax corrected accordingly.

This is now $\frac{4.56}{120} \approx 4\%$ of the cantilever length and may be becoming unacceptable. Furthermore, the simple scaling analysis assumes the load remains distributed over the entire length (monopile + tower) when it will be more concentrated towards the top, and there is the extra concern of wave loading.

[1]

Entrained water mass and extra damping may mitigate the response.

Alternative form

(1) Cont.

$u = ax^3 + bx^2 + cx + d$

$u(0) = 0 \Rightarrow d = 0$

$u' = 3ax^2 + 2bx + c$

$u'(0) = 0 \therefore c = 0$

$u'' = 6ax + 2b$

$u''(L) = 0 \therefore 0 = 6aL + 2b \Rightarrow b = -3aL$

Set $u(L) = 1 \Rightarrow 1 = aL^3 + bL^2$, $1 = aL^3 - 3aL^3 = -2aL^3$, $a = -\frac{1}{2L^3}$

$b = \frac{3}{2L^2}$

$\therefore u = -\frac{1}{2L^3}x^3 + \frac{3}{2L^2}x^2$

$$M_{eq} = \int_0^L m u^2 \, dx = \int_0^L \frac{m}{L^6}\left(-\frac{1}{2}x^3 + \frac{3L}{2}x^2\right)^2 dx \quad + mL(1)^2$$

$$= \frac{m}{4L^6}\int_0^L x^6 - 6Lx^5 + 9L^2x^4 \, dx \quad + mL$$

$$= \frac{m}{4L^6}\left[\frac{1}{7}x^7 - Lx^6 + \frac{9}{5}L^2x^5\right]_0^L + mL \quad = \frac{m}{4L^6}\left(\frac{L^7}{7} - L^7 + \frac{9}{5}L^7\right) + mL$$

$$= \frac{173\,mL}{140}$$

$$K_{eq} = \int_0^L EI\left(\frac{d^2u}{dx^2}\right)^2 dx. \qquad u' = -\frac{3}{2L^3}x^2 + \frac{3x}{L^2} \quad ; \quad u'' = -\frac{3x}{L^3} + \frac{3}{L^2}$$

$$= \int_0^L \frac{EI}{L^6}\left(9x^2 - 18xL + 9L^2\right) dx \quad = \frac{EI}{L^6}\left[3x^3 - 9Lx^2 + 9L^2x\right]_0^L$$

$$= \frac{3EI}{L^3}$$

$$F_q = \int_0^L f u \, dx = \int_0^L \frac{Fx}{L}\left(-\frac{1}{2L^3}x^3 + \frac{3}{2L^2}x^2\right) dx \quad = \frac{F}{2L^4}\int_0^L -x^4 + 3Lx^3 \, dx$$

$$= \frac{F}{2L^4}\left[-\frac{x^5}{5} + \frac{3L}{4}x^4\right]_0^L$$

$$= \frac{11FL}{40}$$

② a) $\ddot{u}_{g_{max}} = 1g$

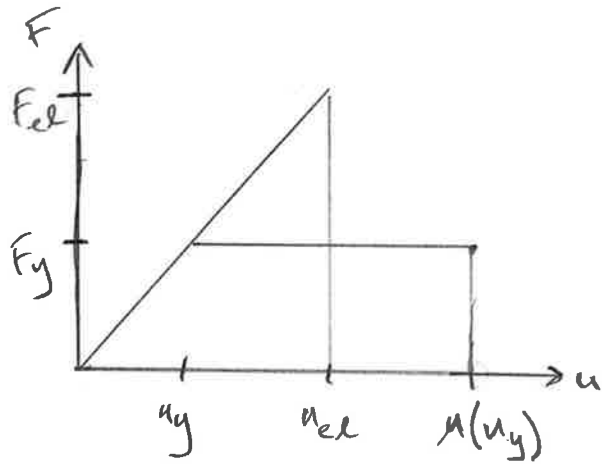
$$S_d = \frac{S_a}{\omega^2} = \frac{2.5\left(\frac{1.8}{T_n^2}\right)}{\frac{4\pi^2}{T_n^2}}(9.81) = 1.08\,m$$

$\therefore\; S_{d_{max}} = 1.12\,m$

$u_{g,max} = 1.12\,m$

[15%]

b)



$$\frac{F_{el}u_{el}}{2} = \frac{F_y u_y}{2} + (\mu-1)\,u_y F_y$$

$$\frac{F_{el}}{F_y}\frac{u_{el}}{u_y} = 1 + 2(\mu-1) = 2\mu-1 \rightarrow \left(\frac{u_{el}}{u_y}\right)^2 = 2\mu-1$$

$$\therefore\; u_y = \frac{u_e}{\sqrt{2\mu-1}} \Rightarrow u_e = S_d = \frac{S_a}{\omega^2} = \frac{2.5\,T_n^2}{4\pi^2}$$

$$u_y = \frac{2.5\,T_n^2}{4\pi^2\sqrt{2\mu-1}}$$

[35%]

② c) $[u]_{max} = [\bar{u}]\,\Gamma\, S_{d,design}$

$u_{max,1} = (1)\,\Gamma\, S_{d,design}$

$$\Gamma = \frac{m_1\bar{u}_1 + m_2\bar{u}_2}{m_1\bar{u}_1^2 + m_2\bar{u}_2^2} = 0.72 \rightarrow S_{d,design} = \frac{u_{max,1}}{\Gamma} = \frac{0.5}{0.72}$$

$$\underline{S_{d_{design}} = 0.69 \text{ m}}$$

→ T_n must be less than 3 s. because $S_d < 1.08$ m

→ Try $0.6 < T_n < 3$

$$S_a = \frac{2.5(0.6)}{\cancel{T_n}}(9.81) = \frac{4\pi^2 S_d}{T_n^{\cancel{2}}} \rightarrow \underline{\underline{T_n = 1.85 s}}$$

$$\rightarrow \omega_n = 3.4 \text{ rad/s}$$

Find ω_1: $M_{eq} = 1000(1)^2 + 1000(1.62)^2 = 3{,}624$ kg

$K_{eq} = 2k(1)^2 + 2k(0.62)^2 = 2.77\,k$

$$\omega_1 = \sqrt{\frac{K_{eq}}{M_{eq}}} \rightarrow 3.4 = \sqrt{\frac{2.77\,k}{3624}}$$

$$\underline{\underline{k = 1.5\times 10^4 \text{ N/m}}}$$

$\boxed{50\%}$

Q3)

a) Civil Engineering dynamics is an important area and covers both interesting and important problems. From traffic induced vibrations in a bridge structure to earthquake loading on large buildings, bridges etc. dynamic response must be considered in designs.

Response of structural elements like beams can be determined with respect to the frequency of excitation. Traditionally the response of multi degree freedom structures is obtained by superposing the response of several modes. This is allowed due to orthogonal nature of Eigen modes. Such analyses are termed as frequency domain analyses as the response in terms frequency is superposed.

Another way of analysing the dynamic problems is time domain analyses. In these analyses the equations of motion are considered and solutions are obtained by using numerical integration schemes such as 'generalised Newmark method'. Solutions are obtained at every time step and therefore the response of the structure is determined as a time-varying function.

When dealing with non-linear materials such as soils, it is imperative that the dynamic problems are analysed in the time domain. The material non-linearity i.e. elasto-plastic stress-strain behaviour of soils can only be captured in such time domain analyses.

[15%]

3 b) Degradation of soil stiffness can occur in a fully saturated sand bed due to the following reasons;

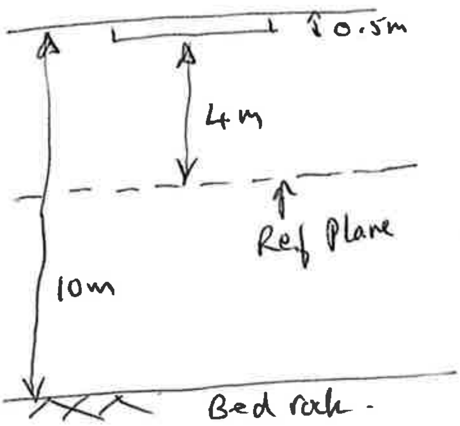
i) Imposed cyclic strains in the soil
ii) Generation of excess pore pressures

It is well known that the soil stiffness reduces with increasing amplitude of strains. During an earthquake loading the soil is subjected to cyclic strains. The larger the magnitude of the earthquake the larger will be the magnitude of cyclic strains. This causes a reduction in the shear modulus of the soil and hence a reduction in its stiffness.

Loose sands beds are liable to suffer volumetric contraction during earthquake loading. If the sand bed is fully saturated this tendency to suffer volumetric contraction is manifested as an increase in pore water pressure. With the generation of such excess pore water pressures the effective stress in the soil decreases leading to a degradation in the soil stiffness. If the excess pore pressures cause the effective stress to reduce to near zero values, we call such an event 'full liquefaction'. If on the other hand excess pore water pressures cause a reduction in the effective stress, this is called 'partial liquefaction'.

[15%]

Q3 c)



Consider Hardin & Drnevich equation

$$G_{max} = 100 \frac{[3-e]^2}{1+e} (p')^{0.5}$$

Void ratio $e = 0.7$ $G_s = 2.65$

Dry Den: $\rho_d = \frac{G_s \rho_w}{1+e} = \frac{2.65 \times 1000}{1.7} = 1558.82 \ kg/m^3$

Unit weight $= \rho_d g = 15.29 \ kN/m^3$.

Vertical stress on Reference plane

$$\sigma_v = \sigma_v' = 4 \times 15.29 + 30 = 91.16 \ kPa.$$

Poisson's ratio $= 0.3$ $K_o = \frac{\nu}{1-\nu} = \frac{0.3}{1-0.3} = 0.4285$.

$$\therefore p' = \left(\frac{1+2K_o}{3}\right)\sigma_v' = \underline{56.43} \ kPa$$

$$\therefore G_{max} = 100 \ \frac{[3-0.7]^2}{1.7} \times \left(\frac{56.43}{1000}\right)^{0.5} = \underline{\underline{73.92}} \ MPa$$

$$v_s = \sqrt{\frac{G_{max}}{\rho_d}} = \sqrt{\frac{73.92 \times 10^6}{1558.82}} = \underline{\underline{217.76}} \ m/s$$

[20%]

3 d) Actual shear wave velocity $v_s = 120 \ m/s$.

$$\Rightarrow G = \rho v_s^2 = 1558.82 \times 120^2 = 22.447 \ MPa$$

This reduction has occurred due to the cyclic shear strains induced by earthquake loading.

$$\frac{G}{G_{max}} = \frac{22.447}{73.92} = 0.303666 = \frac{1}{1+\gamma_h}.$$

$$\gamma_h = \frac{1}{0.303666} - 1 = 2.2931$$

Using Datasheets $\gamma_h = \frac{\gamma}{\gamma_r}\left[1 + a e^{-b \gamma/\gamma_r}\right]$

For Sands

$a = -0.2 \ln N = -0.2 \ln 5$ (5 cycles in the earthquake)

$a = -0.3219$

$b = 0.16$

Say $\frac{\gamma}{\gamma_r} = \alpha$

$\therefore \gamma_h = \alpha \left[1 - 0.3219\, e^{-0.16\alpha}\right] = 2.2931$

Solving by Trial & Error; $\alpha = 2$ LHS $= 1.5325$
$\alpha = 3$ LHS $= 2.4024$
$\alpha = 2.9$ LHS $= 2.313$. ✓ ok.

$\therefore \frac{\gamma}{\gamma_r} = 2.9$

$\gamma_r = \frac{\tau_{max}}{G_{max}}$ ≡ $\tau_{max} = \sqrt{\left[\left(\frac{1+2K_0}{2}\right)\sigma_v' \sin\phi\right]^2 - \left[\left(\frac{1-K_0}{2}\right)\sigma_v'\right]^2}$

$= \left\{\left[\left(\frac{1+0.4285}{2}\right) 91.16 \times \sin 33\right]^2 - \left[\left[\frac{1-0.4285}{2}\right] 91.16\right]^2\right\}^{0.5}$

$\tau_{max} = 24.06$ kPa ; $G_{max} = 73.92$ MPa

$\therefore \gamma_r = \frac{24.06 \times 10^3}{73.92 \times 10^6} = 3.25487 \times 10^{-4}$

$\therefore \gamma = 2.9\, \gamma_r = 2.9 \times 3.25487 \times 10^{-4} = \underline{9.44 \times 10^{-4}}$ [25%]

$= \sim \underline{0.1\%}$

3 e) Saturated soil $\rho_s = \frac{(G + e S_r)\rho_w}{1+e} = \frac{2.65 + 0.7}{1.7} \times 1000 = 1970.59$ kg/m³.

Sat unit weight $= 19.33$ kN/m³.

$\sigma_v' = 4 \times 19.33 + 30 - 4 \times 10 - 65 = 2.32$ kPa

$p' = \left(\frac{1+2K_0}{3}\right)\sigma_v' = 1.436$ kPa ; $G_{max} = 100 \frac{[3-0.7]^2}{1.7} \sqrt{\frac{1.436}{1000}}$

$= 11.79$ MPa.

$G = \frac{G_{max}}{1+\gamma_h} = \frac{11.79}{1+2.2931} = \underline{3.58 \text{ MPa}}$ (Same γ_h as same earthquake event as part d)

$v_s = \sqrt{\frac{G}{\rho_{sat}}} = \sqrt{\frac{3.58 \times 10^6}{1970.59}} = \underline{42.62 \text{ m/s}}$. [25%]

Q4)

a) Marks will be awarded for description of Strouhal number as a dimensionless measure of the frequency, non-dimensionalised with respect to flow velocity and object dimension. Knowing the Strouhal number, one can thus work out the windspeeds at which vortex frequencies will coincide with structural frequencies to cause vortex-induced resonance.
Similarly the Scruton number is a (dimensionless) mass-damping parameter, so it gives a measure of the likely magnitude of the vortex induced response. Large Scruton numbers are good (lots of mass and lots of damping).
For the human-induced case, similar quantities can be defined - a Strouhal-like parameter concerning footfall frequencies, with pedestrian excitation likely when footfall frequencies coincide with modal frequencies, and a Scruton-like mass-damping parameter , but the mas part being (mode-generalised) mass of the bridge relative to the mass of the people. [20%]

b) Basic marks will be awarded for a description of the eight flutter derivatives as the (dimensionless) coefficients relating lift forces and moments to the angle of attack and the heave displacement and their first derivatives. Extra marks will be awarded for noting that these are only defined for periodic solutions due to fluid memory effects. Descriptions of wind tunnel testing should focus on the need for prescribed harmonic motions of deck section models. [20%]

c) Marks will be awarded for a description of how buffetting analysis – unlike earthquake Response Spectrum Analysis - takes account of **spatial** decorrelations of forces. [20%]

d) Marks will be obtained for descriptions of principles of structural robustness and redundancy, increased stand-off distances, bollards etc as per lecture notes, and with a description of glazing options, particularly laminated glass. [20%]

e) Marks will be awarded for descriptions of rivulets affecting the aerodynamic cross-section and why this can lead to large-amplitude galloping response. [20%]

Part IIB 4D6 – Assessor comments

1) Answered well, in general, with all candidates adopting the correct approach. Mistakes were made in choosing the appropriate cubic for the mode shape that would meet the boundary conditions, and one or two candidates incorrectly assumed the shear force at x=L is zero. Equivalent force, stiffness and mass were generally calculated correctly, although some forgot the point-mass contribution to the equivalent mass. Another common mistake in calculating the static displacement was failing to account for the mode shape scaling when the chosen mode shape did not have unit amplitude at x=L.

2) About two-thirds of students answered this question. In part (a), relatively few students correctly distinguished between the ground movement and the structural response. In part (b), most students correctly equated the deformation energies, but fewer students derived the complete equation. In part (c), many students followed the correct procedure, but fewer students correctly checked the natural frequency of the structure to ensure it agreed with their assumption for Sa.

3) Nearly all students answered this question. In part (b) almost all students clearly explained stiffness degradation due to excess pore pressure, though significantly fewer people identified that cyclic strains also directly cause stiffness degradation. In parts (c) through (e), the correct procedure was often followed, but many students incorrectly calculated the dry density, the saturated density, and the vertical effective stress on the reference plane. The use of G or G_max for calculation of shear wave velocity was often confused.

4) The wind/blast engineering question was taken by many more students than usual this year, with almost all showing a good understanding across topics that ranged from galloping, vortex induced vibration, buffeting, pedestrian-induced vibration and blast resistant design.