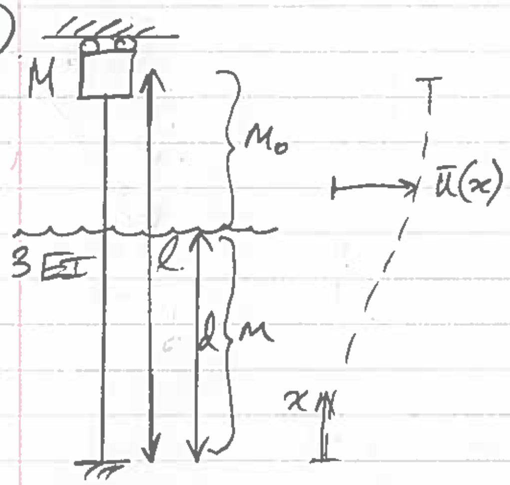
2017

Part IIB 4D6: Dynamics in Civil Engineering

(1)



a) $\bar{u} = 1 - \cos\left(\frac{\pi x}{\ell}\right) \Rightarrow \bar{u}(0) = 0$

$\bar{u}(\ell) = 2$

$\frac{d\bar{u}}{dx} = \frac{\pi}{\ell}\sin\left(\frac{\pi x}{\ell}\right) \Rightarrow \left.\frac{d\bar{u}}{dx}\right)_0 = 0$

$\left.\frac{d\bar{u}}{dx}\right)_\ell = 0$

So assumed mode captures the expected shape and boundary conditions. [2]

b). Neglecting the vertical motion of the deck, the total potential energy is due to strain in the legs.

$$\Rightarrow K_{eq} = 3 \times \int_0^\ell EI\left(\frac{d^2\bar{u}}{dx^2}\right)^2 dx$$

$$= 3EI\int_0^\ell \left(\frac{\pi^2}{\ell^2}\cos\left(\frac{\pi x}{\ell}\right)\right)^2 dx = 3EI\frac{\pi^4}{\ell^4}\int_0^\ell \frac{1}{2}\left(1+\cos\frac{2\pi x}{\ell}\right) dx$$

$$= \frac{3EI}{2}\frac{\pi^4}{\ell^4}\left[x + \frac{\ell}{2\pi}\sin\frac{2\pi x}{\ell}\right]_0^\ell = \frac{3EI}{2}\frac{\pi^4}{\ell^4}\cdot\ell$$

$$= \frac{3\pi^4 EI}{2\ell^3}$$

[4]

The kinetic energy comprises that of the submerged part of the legs, that of the legs above water and that of the deck

$$\Rightarrow M_{eq} = 3 \times \int_0^d m\left(1-\cos\left(\frac{\pi x}{\ell}\right)\right)^2 dx + 3 \times \int_d^\ell M_0\left(1-\cos\left(\frac{\pi x}{\ell}\right)\right)^2 dx$$

$$+ M(2)^2 \quad \leftarrow \bar{u}(\ell)$$

For $d \simeq \ell$: [As assumed in question]

$$= 3m\int_0^\ell 1 - 2\cos\frac{\pi x}{\ell} + \cos^2\frac{\pi x}{\ell}\, dx + 4M$$

$$= 3m\int_0^\ell 1 - 2\cos\frac{\pi x}{\ell} + \frac{1}{2}\left(1+\cos\frac{2\pi x}{\ell}\right) dx + 4M$$

① continued.

$$M_{eq} = 3m\left[\frac{3x}{2} - \frac{2l}{\pi}\sin\frac{\pi x}{l} + \frac{1}{2}\cdot\frac{l}{2\pi}\sin\frac{2\pi x}{l}\right]_0^l + 4M = 3m\cdot\frac{3l}{2} + 4M$$

$$= \frac{9ml}{2} + 4M$$ [4]

$$\therefore \omega_n^2 = K_{eq}/M_{eq} \Rightarrow \omega_n = \sqrt{\frac{3\pi^4 EI}{2l^3}\cdot\frac{2}{9ml+8M}}$$

$$\Rightarrow \omega_n = \sqrt{\frac{3\pi^4 EI}{l^3(9ml+8M)}}$$ [2]

c). $$\omega_n = \sqrt{\frac{3\times\pi^4\times 1.8\times10^{11}}{80^3(9\times25\times10^3\times80+8\times4.6\times10^6)}} = 1.37\ \text{rad/s} \equiv 0.218\ \text{Hz}$$

$$\Rightarrow T_n = 4.59\ \text{s} \quad \therefore \frac{t_d}{T_n} \simeq 0.2 \Rightarrow DAF \simeq 0.65$$ [3]

$$K_{eq} = \frac{3\times\pi^4\times1.8\times10^{11}}{2\times80^3} = 51.4\times10^6\ \text{N/m}$$

$$F_{eq} = F\bar{u}(x=L) = 70\times2 = 140\ \text{MN}$$

$$\therefore u_{max} \simeq 0.65\times\frac{140\times10^6}{51.4\times10^6}\times\bar{u}(x=L) = 3.54\ \text{m}$$ [3]

d). The lower sea level ~~reduces~~ exposes the upper portion of the legs, eliminating the entrained water mass and hence reducing the equivalent mass. This increases the natural frequency, decreasing the natural period and hence increasing the DAF. However, the wave loading is no longer at deck level (the antinode of the sway mode) and therefore the equivalent force is likely to be lower, which may cancel out the effect of the increased DAF.

Damping may also be reduced (viscous and radiation damping) but this is less significant in transient response.

[2]

② a) i) $k_1 = k_2 = 2\left(\frac{12EI}{L^3}\right) = 26.7 \times 10^6$ N/m

$k_3 = \frac{3EI_{tower}}{\left[\left(\frac{2}{3}\right)L\right]^3} = 2.78 \times 10^6$ N/m

$M_{eq} = (1)^2(10{,}000) + (2)^2\left(10{,}000 + \frac{1}{3}(6000)\right) + (5)^2\left(\frac{2}{3}(6000)\right) = 158{,}000$ kg

$K_{eq} = k_1(1)^2 + k_2(2-1)^2 + k_3(5-2)^2 = 78.3 \times 10^6$ N/m

$\omega_n = \sqrt{\frac{K_{eq}}{M_{eq}}} = \sqrt{\frac{78.3 \times 10^6}{158 \times 10^3}} = 22.3$ rad/s $= \underline{\underline{3.55 \text{ Hz}}}$

ii) Try $[1, 2, 6]$

$\Rightarrow M_{eq} = 202{,}000$; $K_{eq} = 9.78 \times 10^7 \rightarrow \omega_n = 22.0$ rad/s

$\underline{\underline{f_n = 3.50}}$ Hz < 3.55

∴ better

b) $T_n < 0.6$

$S_{a,des} = 2.5(9.81) = 24.5$ m/s² (plateau)

$\ddot{u}_{max} = \Gamma S_a \rightarrow S_a = \frac{\ddot{u}_{max}}{\Gamma}$

Bottom story! $F = \Sigma ma = [m_1 \ m_2 \ m_3] \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix} \ddot{u}_{max}$

∴ $S_a = \frac{F}{[m_1(1) + m_2(2) + m_3(5)]\,\Gamma}$ where $F = 100$ kN

$\Gamma = \frac{\Sigma m\bar{u}}{\Sigma m\bar{u}^2} = 0.287$

$S_a = 6.0$ m/s²

For elastic → $PGA = \frac{6.0}{24.5} = \underline{\underline{0.25\,g}}$

(2) b) ii) Equal energy rule for plateau portion of spectra.

$$\mu = \frac{\left(\frac{PGA_{design}}{PGA_{elastic}}\right)^2 + 1}{2} = \underline{\underline{2.5}}$$

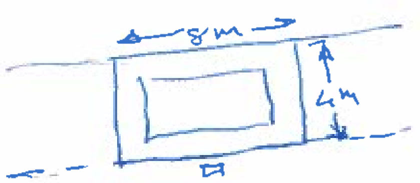
4D6

Q3) a) It possible to solve a dynamic problem by obtaining the response of the system at individual frequency components of the input motion and then superposing these to obtain the overall response. Such an analysis is called frequency domain analysis. It has the advantage of requiring less computational effort and can be carried out quickly. However it has the disadvantage of not being applicable for non-linear systems due to non-orthogonal modes.

Time domain analysis on the other hand attempts to solve directly the equations of motion. This is normally done by using an integration scheme such as Newmark-β method. However the computational effort can be quite large but non-linear systems can be analysed using this method. [15%]

b) It is well known that soil behaviour is highly non-linear especially at large strains. Earthquake loading normally involves this non-linear behaviour due to the large cyclic strains it induces in the soil. Hence to include soil-behaviour while analysing soil-structure systems, time domain analysis is required. Further it is also possible to carry out analysis of coupled solid and fluid phases to capture the behaviour of both solid and fluid phases present in saturated soils [10%]

c) Calculate the small-strain shear modulus G_{max}



Vertical stress below the tunnel = $\sigma_v = \sigma_v'$

Weight = $(8\times4 - 7.5\times3.5)\times24\times1 = 138$ kN/m

$\therefore \sigma_v = \sigma_v' = \frac{138}{8\times1} = 17.25$ kPa

Poisson's ratio for sand = 0.3

$\therefore K_0 = \frac{\nu}{1-\nu} = \frac{0.3}{0.7} = 0.428$

$p' = \sigma_v'\left(\frac{1+2K_0}{3}\right) = 17.25\times\left(\frac{1+2\times0.428}{3}\right) = 0.619\times17.25$

$= 10.6786$ kPa

$\therefore G_{max} = 100\,\frac{[3-e]^2}{1+e}\sqrt{p'} = 100\,\frac{(3-0.67)^2}{1.67}\left(\frac{10.67}{1000}\right)^{0.5}$

$= \underline{\underline{33.59 \text{ MPa}}}$

3 c) Contd.

Now consider the dimensions of the tunnel. I use Wolf's formulae.

$2l = 8\text{ m} \quad 2b = 1\text{m} \quad e = 4\text{m}$

$\therefore l/b = 8 \quad e/b = \frac{4}{0.5} = 8$

From data sheets :- Horizontal stiffness.

$$K_{hx} = G\frac{b}{1-\nu}\left(6.8\,(l/b)^{0.65} + 2.4\right)\left[1 + \left(0.33 + \frac{1.34}{(1+l/b)}\right)\left(\frac{e}{b}\right)^{0.8}\right]$$

$$= G\frac{0.5}{(1-0.3)}\left(6.8 \times 8^{0.65} + 2.4\right)\left[1 + \left(0.33 + \frac{1.34}{9}\right)8^{0.8}\right]$$

$$G \times 0.714 \times 28.6735 \times 3.5275$$

$$= G \times 72.2469$$

$$\therefore K_{hx} = \underline{2426.776}\ \text{MN/m}$$

$$\text{Vertical stiffness } K_v = \frac{Gb}{2-\nu}\left(3.1\,(l/b)^{0.75} + 1.6\right)\left[1 + \left(0.25 + 0.25\frac{b}{l}\right)\left(\frac{e}{b}\right)^{0.8}\right]$$

$$= G\frac{0.5}{1.7} \times \left(3.1 \times 8^{0.75} + 1.6\right)\left(1 + \left(0.25 + \frac{0.25}{8}\right)8^{0.8}\right)$$

$$= G \times 0.2941 \times 16.3462 \times 2.4844$$

$$= G \times 11.9435$$

$$\therefore K_v = \underline{401.1838}\ \text{MN/m}$$

$$\text{Rotational stiffness } K_{ry} = \frac{Gb^3}{1-\nu}\left(3.73\left(\frac{l}{b}\right)^{2.4} + 0.27\right)\left(1 + \frac{e}{b} + \frac{1.6}{0.35 + (l/b)^4}\left(\frac{e}{b}\right)^2\right)$$

$$= G\frac{0.5^3}{0.7}\left(3.73 \times 8^{2.4} + 0.27\right)\left(1 + 8 + \frac{1.6}{0.35 + 8^4} \times 8^2\right)$$

$$= G \times 0.17857 \times 548.7045 \times 9.025$$

$$= G \times 884.289$$

$$K_{ry} = \underline{29703.2681}\ \text{MN-m/m}$$

3 d) Mass of the tunnel per m width = 138 kN ≃ 14067.278 kg.

For Horizontal mode, Soil participating will $30 \times 14067 = 422018.3486$ kg

Horizontal stiffness $= K_{hx} = 2426.776$ MN/m.

$$\therefore f_h = \frac{1}{2\pi}\sqrt{\frac{K_{hx}}{M}} = \frac{1}{2\pi}\sqrt{\frac{2426.776 \times 10^6}{422018.3486}} = \underline{\underline{12.069\ \text{Hz}}}$$

3 d) Contd.

Vertical vibration mode: Soil participating $\approx 10 \times 14067.278$
$= 140672.78$ kg.

$K_v = 401.1838$ MN/m.

$\therefore f_v = \frac{1}{2\pi}\sqrt{\frac{K_v}{M_v}} = \frac{1}{2\pi}\sqrt{\frac{401.1838 \times 10^6}{140672.78}} = 8.5$ Hz

Rocking mode :- Mass moment of inertia = 984,600 kg-m².
$K_{ry} = 29703.2681$ MN m/m.

$\therefore f_{rocking} = \frac{1}{2\pi}\sqrt{\frac{K_{ry}}{I}} = \frac{1}{2\pi}\sqrt{\frac{29703.2681 \times 10^6}{984600}} = 27.64$ Hz [20%]

3 e) Shear wave velocity = 20 m/s. after liquefaction & cyclic strains.

$v_s = \sqrt{\frac{G}{\rho}} \Rightarrow G = \rho v_s^2$

Soil unit weight = 15 kN/m³ ⇒ Dry density $\rho = 1529.05$ kg/m³.

$\therefore G = 0.6116$ MPa

$\therefore K_{hx} = 72.2489\, G = 44.186$ MN/m
$K_v = 11.9435\, G = 7.3046$ MN/m
$K_{ry} = 884.289\, G = 540.8311$ MN-m/m.

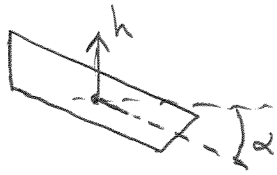
$\therefore f_h = \frac{1}{2\pi}\sqrt{\frac{44.186 \times 10^6}{422018.35}} = 1.6285$ Hz

$f_v = \frac{1}{2\pi}\sqrt{\frac{7.3046 \times 10^6}{140672.78}} = 1.1468$ Hz

$f_{rocking} = \frac{1}{2\pi}\sqrt{\frac{540.8311 \times 10^6}{984600}} = 3.73$ Hz [15%]

3 f) Clearly the natural frequencies in all the modes considered have significantly dropped during the large earthquake event. So the tunnel can undergo resonant vibrations in all of these modes, if the earthquake motion has components with significant energy at these frequencies ie 1 to 5 Hz.
Further the tunnel can also suffer floatation failure, if the soil fully liquefies. [15%]

4) a)



$$m\ddot{h} + c_h\dot{h} + k_h h = c_1 h + c_2\dot{h} + c_3\alpha + c_3\dot{\alpha}$$

$$I\ddot{\alpha} + c_\alpha\dot{\alpha} + k_\alpha \alpha = c_5\alpha + c_6\dot{\alpha} + c_7 h + c_8\dot{h}$$

- Instability that occurs when the motion-induced fluctuating lift force gets into phase with structural velocities, adding energy to system @ each cycle.

(See notes for sketches & further explanation)

b) SDOF flutter (torsion) - no vertical movement.

$$I\ddot{\alpha} + c_\alpha\dot{\alpha} + k_\alpha \alpha = c_5\alpha + c_6\dot{\alpha}$$

If $c_\alpha - c_6$ becomes negative, then motion is negatively damped, so torsional oscillations grow.

c) Restoring force $= k_\alpha \alpha$; Moment $= \frac{1}{2} C_m \rho V^2 b^2 L$

Angle increases by $d\alpha$: $k_\alpha\, d\alpha = \frac{dC_m}{d\alpha}\, d\alpha\, \frac{1}{2}\rho V^2 b^2 L$

Assume half sine wave = mode shape $\phi = \sin\frac{\pi x}{L}$, $\alpha = \alpha_0 \phi$

$$I_{eq} = \int_0^L I \phi^2 dx = I\frac{L}{2}$$

$$I = Mr^2 = 16.0\times10^3\,(9.5)^2 = 1.44\times10^6 \text{ kg-m}^2/\text{m}$$

$$I_{eq} = 8.64\times10^8 \text{ kg-m}^2$$

$$k_{eq} = I_{eq}(\omega^2) = 8.64 \times 10^8 (1.70^2) = 25.0 \times 10^8 \ \text{N·m}$$

Mode-generalised torque $= \int_0^L T \phi \, dx = T \int_0^L \sin\frac{\pi x}{L} dx = \frac{2}{\pi} L T$

$$k_{eq} = \frac{2}{\pi} L T = \frac{\cancel{2}}{\pi} L \left(\frac{dC_m}{d\alpha} \frac{1}{\cancel{2}} \rho V^2 b^2 \right)$$

$$V^2 = \frac{k_{eq} \pi}{\rho b^2 L \frac{dC_m}{d\alpha}} = \frac{(25.0 \times 10^8)\pi}{1.23 (30^2) 1200 (1.1)}$$

$$V = 73.3 \ \text{m/s} = 164 \ \text{mph}$$

Q1

All candidates attempted Q1, and generally produced good solutions. The vast majority of candidates correctly assessed the assumed mode shape as satisfying the required boundary conditions. Rayleigh's method was correctly applied, in general, although marks were lost for failing to account correctly for the three legs and/or the mass of the deck. A significant number of candidates lost marks on the forced response by failing to calculate correctly the equivalent force (by scaling by the mode shape) and/or the maximum displacement (again, by mode scaling).

Q2

31 candidates attempted Q2, and most did quite well. Most students used mass lumping methods in part (a), although the appropriateness of assumptions varied significantly. No student observed that a mass lumping approach would add mass to the second storey. In part a(i), about half of students quickly checked another mode shape to see if the natural frequency was lower; the other half attempted to solve for the "real" eigenmodes, which no one did successfully. In part (b), the majority of students used an appropriate method, with a minority forgetting the modal participation factor. About 2/3 of students correctly calculated the ductility, though many overcomplicated the solution.

Q3

35 candidates attempted Q3, and most did quite well. The initial parts on differences between frequency and time domain analyses were well recognised and most candidates identified soil non-linearity as the main reason for requiring time domain analyses. The quantitative parts of the question required the students to calculate the horizontal, vertical and rotational stiffness of the tunnel. Almost all candidates had the right approach but a few made arithmetic errors which promulgated into calculation of natural frequencies. With the strong earthquake event and liquefaction, this time round many candidates could see floatation as a mode of failure apart from tunnel wall resonance.

Q4

This question was attempted by only 6 candidates. Most candidates performed very well on this question, explaining flutter in parts (a) and (b) in a clear fashion, and calculating the critical wind speed for static torsional divergence using an appropriate method. The errors that did occur were primarily related to calculating the mode-generalised torque.

JPT/MSPG/MJD/FAM