DDM

4D6 – Dynamics in Civil Engineering.
April 2019 – Gibs

1 a) Grand floor columns stiffness $k_1 = \frac{3 \times 12\ 6000}{3^3} = 8000$ kN/m

I Floor Column Stiffness $k_2 = \frac{2 \times 12 \times 4000}{3^3} = 3555.56$ kN/m

m_1 = Mass of the slab $= (2.5+5)\,200 = 1500$ kg

m_2 = Mass of the 2nd floor slab $= 5 \times 200 = 1000$ kg.

Try mode shape $\begin{bmatrix} 1 \\ 0.25 \end{bmatrix}$ first.

$$M_{eq} = 1 \times 1000 + 0.25^2 \times 1500 = 1093.75 \text{ kg}$$

$$K_{eq} = k_1 \times 0.25^2 + k_2 (1-0.25)^2$$
$$= 8000 \times 0.25^2 + 3555.56 \times 0.75^2 = 2500 \text{ kN/m}.$$

$$\therefore f_n = \frac{1}{2\pi}\sqrt{\frac{K_{eq}}{M_{eq}}} = 7.61 \text{ Hz} \text{ or } T_n = 0.13 \text{ sec}$$

Try mode shape $\begin{bmatrix} 1 \\ 0.42 \end{bmatrix}$ now.

$$M_{eq} = 1 \times 1000 + 0.42^2 \times 1500 = 1264.6 \text{ kg}.$$

$$K_{eq} = k_1 \times 0.42^2 + k_2 (1-0.42)^2$$
$$= 8000 \times 0.42^2 + 3555.56\,(0.58)^2 = 2607.29 \text{ kN/m}$$

$$\therefore f_n = \frac{1}{2\pi}\sqrt{\frac{K_{eq}}{M_{eq}}} = 7.23 \text{ Hz} \text{ or } T_n = 0.1384 \text{ sec.}$$

By Rayleigh's principle, the mode shape $\begin{bmatrix} 1 \\ 0.42 \end{bmatrix}$ gives a lower natural frequency, therefore this should be closer to the fundamental natural frequency of the structure.

[30%]

1 b) $\frac{t_d}{T_n} = \frac{100 \times 10^{-3}}{0.1384} = 0.723$.

Using the data sheets, DAF $\simeq 1.4$.

$F_{eq} = 10 \text{ kN} \times 0.42 \leftarrow (\bar{v})$

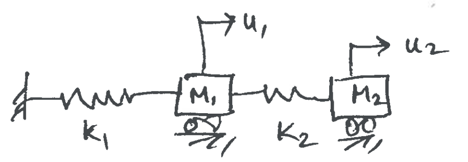
$= 4.2 \text{ kN}$.

static deflection $\times$DAF $= \frac{F_{eq}}{k_{eq}} \times$ DAF

$\therefore$ Dynamic deflection (max) $= \frac{4.2 \times 10^3}{2607.29 \times 10^3} \times 1.4$

[30%]

$= \underline{\underline{2.25}}$ mm

1 c) 

$$\begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix} \begin{bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \end{bmatrix} + \begin{bmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\therefore$ Det of characteristic equation $= \begin{vmatrix} k_1+k_2-M_1\omega^2 & -k_2 \\ -k_2 & k_2 - M_2\omega^2 \end{vmatrix}$

Substituting $\begin{vmatrix} 11555.56 \times 10^3 - 1500\omega^2 & -3555.56 \times 10^3 \\ -3555.56 \times 10^3 & 3555.56 - 1000\omega^2 \end{vmatrix} = 0$.

Solving $\omega^2 = 2061.69$ or 9197.56.

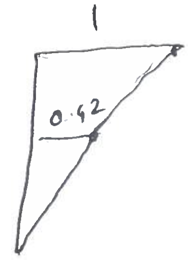
$\omega_1 = 45.4$ rad/s $\Rightarrow f_1 = 7.23$ Hz

$\omega_2 = 95.9$ rad/s $\quad f_2 = 15.26$ Hz

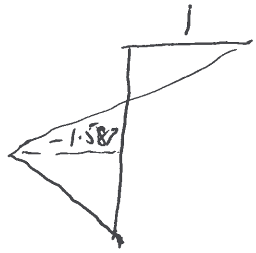
f_1 is the same natural frequency we obtained from mode shape $\begin{bmatrix} 1 \\ 0.42 \end{bmatrix}$ in part (a). Therefore this is true fundamental natural frequency. or

substituting ω^2 and normalising first column gives the eigen vectors as $\begin{bmatrix} 1 \\ 0.42 \end{bmatrix}$ & $\begin{bmatrix} 1 \\ -1.5867 \end{bmatrix}$ for 1st & 2nd modes.

1c) Contd :-



1st mode



2nd mode.

We already have the 1st mode deflection as = 2.25 mm from part b)

2nd mode natural frequency = 15.26 Hz.

$\therefore T_n = 0.0655$ sec.

$$\frac{t_d}{T} = \frac{100 \times 10^{-3}}{0.0655} = 1.526$$

From data sheets the DAF = 1.7

$$F_{eq} = -1.587 \times 10 \text{ kN} = -15.87 \text{ kN}.$$

$$K_{eq} = 8000 \times \cancel{1000} (-1.587)^2 + 3555.56 (1-(-1.587))^2$$
$$= 43944.38 \text{ kN/m}.$$

$$\therefore \delta_{\substack{dynamic \\ Max}} = \frac{F_{eq}}{K_{eq}} = \frac{-15.87 \times 1000}{43944.38 \cancel{\times 10^3}} = \underline{0.361 \text{ mm}}$$

Using SRSS method :-

$$\delta_{dynamic} = \sqrt{2.25^2 + 0.361^2} = \underline{\underline{2.28}} \text{ mm}.$$

Very small improvement ! No need to consider any higher modes of vibration.

[40%]

4D6 2019, crib

2 a) The response spectrum is a plot of the highest magnitude response of an infinite array of oscillators of differing natural frequencies. There are three main "pure" types of response spectrum, where the "response" is the relative displacement, the relative velocity or the absolute acceleration. If one takes the relative displacement response spectrum and multiplies it by ω or ω^2 one obtains the pseudo-velocity and the pseudo-acceleration spectra respectively. These are very similar to the relative velocity and the absolute acceleration spectra, but they are not strictly equal to them. So, they provide useful estimates, and the reason for using the pseudo- rather than the real "pure" versions is that you can then plot all three (relative displacement, pseudo-velocity and pseudo-acceleration) on one graph, by first plotting the relative displacement spectrum on log-log axes. The log of the peak magnitude of relative displacements is plotted on the "y-axis", but this "y-axis" is tilted at 45 degrees to the right. The x-axis is log ω and it follows that the pseudo-velocity and pseudo-accelaration results can then be read directly from axes that are angled at successively further 45 degree directions to these (i.e log S_v vertical and log S_a at 45 degrees to the left)..

Marks will be awarded for sensible explanations that state that the pseudo- variants are similar to, but not exactly equal to, the pure variants, with the advantage that the tripartite graph then gives three sets of information from one diagram. Students are provided with an example of a tripartite response spectrum, so this should not be difficult.

4D6

2b) i) $d = 8$ in, $a \approx 0.32g$.

ii) $f_1 = 0.34$ Hz, $w_1 = 2\pi f = 2.14$ rad/s $T_1 = 1/f_1 = 2.94$ s $S_a = 0.13g$

$f_2 = 1.46$ Hz $w_2 = = 9.18$ rad/s $T_2 = 1/f_2 = 0.7$ s. $S_a = 0.7g$

$$\Gamma_1 = \frac{1+0.883}{1+(0.883)^2} = 1.058$$

$$\Gamma_2 = \frac{1-1.13}{1+(1.13)^2} = 0.058$$

Mode 1: $S_{d_1} = \frac{S_a}{w_1^2} = \frac{0.13(9.81)}{(2.14)^2} = 0.279$ m (~ 11 inches)

Mode 2: $S_{d_2} = \frac{S_a}{w_2^2} = \frac{0.7(9.81)}{(9.18)^2} = 0.08$ m (~ 3 inches)

$$u_1 = \Gamma_1 S_{d_1} \Phi_1 = 1.058(0.279)\begin{bmatrix}1\\0.883\end{bmatrix} = \begin{bmatrix}0.295\\0.260\end{bmatrix} \text{ metres}$$

$$u_2 = \Gamma_2 S_{d_2} \Phi_2 = 0.058(0.08)\begin{bmatrix}1\\-1.13\end{bmatrix} = \begin{bmatrix}0.004\\-0.005\end{bmatrix}$$

Bottom floor, SRSS $\rightarrow$ $u_2 - u_1 = \sqrt{(0.26)^2+(0.005)^2} = \underline{0.26}$

Column shear $= \frac{3EI\delta}{L^3} = \frac{3(1.50\times10^3)(0.26)}{3^3}$

$= \underline{\underline{4.33 \text{ kN}}}$ per column

No need to check top floor, as interstorey drift is clearly much smaller.

4D6 Q2 cont'd.

c) i) $T_1 = 2.94 \approx 3$ seconds.

$$\delta_{\text{Allowable}} = 0.26\,\text{m}\left(\frac{2}{4.33}\right) = 0.12\,\text{m}.$$

$$\delta_1 = \Gamma_1 S_{d_1} \Phi_{1(\text{top floor})} = \Gamma_1\left(\frac{S_a}{\omega_1^2}\right)(0.883) = 1.058\,\frac{S_a}{\underbrace{\omega_1^2}_{(2.14)}}(0.883) = 0.12\,\text{m}$$

$$\therefore\ \cancel{0.12} \Rightarrow S_a = \frac{0.12\,(2.14)^2}{1.058(0.883)} = 0.5887\ \text{m/s}^2 = \underline{0.06\,g}$$

PGA = 0.4g | S_{a_1} on plot $= \dfrac{0.06g}{0.4} = 0.15g.$

From plot, $T_1 = 2.94 \approx 3$ secs. , $S_a = 0.15g$ $\Big\} \rightarrow \underline{\mu \approx 4}$ req'd

3 a) When loose saturated sands are subjected to cyclic loading they will undergo a reduction in volume as the soil grains rearrange into a closer packing. This tendency to undergo volumetric contraction is impeded by the presence of pore water in saturated soils. As a result the tendency to undergo volumetric contraction is expressed as an increase in pore water pressure, which are over and above hydrostatic water pressures. Hence they are called excess pore water pressure. [10%]

b) The stiffness of soil is proportional to the effective stress in the soil by Terzaghi's principle.

$$\sigma' = \sigma - u$$

effective stress = Total stress - pore water pressure.

In case of excess pore pressure generation

$$\sigma' = \sigma - \{u_{hydrostatic} + u_{excess}\}$$

If the excess pore pressures are so large as to match the total stress when added to the hydrostatic pore pressure, then

$$\sigma' \simeq 0$$

This is called full liquefaction. If this happens then any structure founded on such soil will undergo severe settlements/rotations, as the soil has no stiffness left.

If the excess pore pressures increase, but only to a level where there is still some effective stress is present, this is called partial liquefaction. In this case, the degradation in soil stiffness may cause the natural frequency of stiff soil-structure system to degrade, bringing it closer to resonance. The structure may still see damage due to excessive vibrations from resonance. [10%]

3 c) Shear wave velocity $= 150 \, m/s$

$\gamma = 15 \, kN/m^3 \Rightarrow \rho = 1529.05 \, kg/m^3$.

$G = \rho v_s^2 = \underline{36.4} \, MPa$.

Consider the block foundation.

$2l = 6m$ $\quad l/b = 1$ $\quad$ Poisson's ratio $\nu = 0.3$

$2b = 6m$ $\quad e/b = 1.33$

$e = 4m$

Using the data sheets

$$K_{hx} = \frac{G \times 3}{2-0.3} \left(6.8+2.4\right)\left(1+\left(0.33+\frac{1.34}{1+1}\right)(1.33)^{0.8}\right)$$

$$= G \times \frac{3}{1.7} \times 9.2 \times 2.2562 = \underline{\underline{36.63 \, G}}$$

$\therefore K_{hx} = 1260.07 \, MN/m$.

$$K_{v} = \frac{Gb}{2-\nu}\left(3.1+1.6\right)\left(1+(0.75+0.25)(1.33)^{0.8}\right)$$

$$= \underline{\underline{13.5039 \, G}} = \underline{464.53} \, MN/m.$$

$$K_{ry} = \frac{G \times 3^3}{0.7}\left(3.73+0.27\right)\left(1+1.33+\frac{1.6}{1.35} \times 1.33^2\right)$$

$$\fallingdotseq 682.94 \, G = \underline{23493.195} \, MN\text{-}m/rad. \qquad [30\%]$$

c) $f_h = \frac{1}{2\pi}\sqrt{\frac{K_h}{M_h}} = \underline{\underline{6.23 \, Hz}}$.

$f_v = \frac{1}{2\pi}\sqrt{\frac{K_v}{M_t}} = 3.78 \, Hz$

$f_{rocking} = \frac{1}{2\pi}\sqrt{\frac{K_{ry}}{I}} = \frac{1}{2\pi}\sqrt{\frac{23493.195 \times 10^6}{4 \times 10^6}} = \underline{\underline{12.19 \, Hz}} \qquad [20\%]$

3 d) Due to large earthquake.

$G = 3\ MPa$

$K_h = 36.63\ G = 109.89\ MN/m$.

$\therefore f_h = \frac{1}{2\pi}\sqrt{\frac{109.89 \times 10^6}{820000}} = \underline{1.84\ Hz}$

$K_v = 13.5039\ G = 40.51\ MN/m$

$f_v = \frac{1}{2\pi}\sqrt{\frac{40.51 \times 10^6}{820000}} = \underline{1.118\ Hz}$

$K_{rock} = 682.94 \times 3 = 2048.82\ MNm/rad$

$f_{rock} = \frac{1}{2\pi}\sqrt{\frac{2048.82 \times 10^6}{4 \times 10^6}} = \underline{3.6\ Hz}$

Could get into either horizontal, vertical or rocking modes of resonance depending on where the driving frequencies in the earthquake.

More significantly, the block foundation can suffer significant settlement as the vertical stiffness has degraded. Also as it supports the crane structure, it can suffer rotational failure aswell.

[30%]

4D6 2019 Crib

4. a) Marks will be awarded for explaining how in EQ analysis, the excitation is via ground excitation and is thus coherent over the whole structure (very long span structures excepted), whereas in wind engineering the wind pressures vary spatially over the structure. So EQ is purely temporal, whereas wind is spatio-temporal, so need to take account of spatial decorrelations. Further marks will be awarded for explaining how the spatial decorrelations are handled by the aerodynamic admittance function applied to the spectral density of wind velocities. Spectral density of structural response is then found via the mechanical admittance, and the integral of the spectral density of the response is the response variance. Taking the square root and multiplying by a gust factor (typically a factor around 3) leads to a design value for the peak dynamic response. This must then be added to the static component due to the mean wind.

b) Marks will be awarded for correct descriptions that include references to – in order of increasing sophistication - Selberg's formula for avoiding coincidence of vertical and torsional frequencies, Theodorsen's flat plate theory, Scanlan's method of measuring flutter derivatives for section models of bluff bodies (assuming along-span coherence) and finally fully-transient CFD in its various forms, including discrete vortex particle methods to perform Scanlan-like analysis computationally on numerical section models.

4D6: Examiner's comments:

Q1 (Attempts 19)
All candidates attempted Q1, and generally produced good solutions. There were quite a few candidates who tried to solve the Eigen value problem rather than try the two suggested mode shapes and use the Rayleigh's principle. Marks were given for both methods. There were a few candidates who made arithmetic mistakes in working out the deflection of the top storey use mode superposition method.

Q2 (Attempts 17)
17 candidates attempted Q2, and most did quite well. The standard of responses was generally good. A surprising number failed to pick up the first two easy marks in part b) by reading from the left and right of the tripartite spectra. There were only a few correct solutions to the final inelastic part, with quite a few students attempting to recall formulae rather than simply using the graph provided.

Q3 (Attempts 19)
All candidates attempted Q3 and most did quite well. The initial parts of the question on liquefaction and excess pore pressure generation was answered well. However surprising number of candidates did not link partial liquefaction to degradation in soil stiffness and hence the natural frequency of the soil-structure system. Most candidates answered the calculation of the soil stiffness in horizontal, vertical and rocking modes well and could calculate the drop in natural frequencies following the degradation in shear modulus.

Q4 (Attempts 2)
This question was attempted by only 2 candidates (and 2 graduate students). There were only a few attempts, but these were of a pleasingly high standard. This part of the course had been covered in less than a single lecture this year, and it was clear that these students had read and absorbed the more extensive handouts on random buffeting excitation and on flutter instabilities.