

407 Question 1

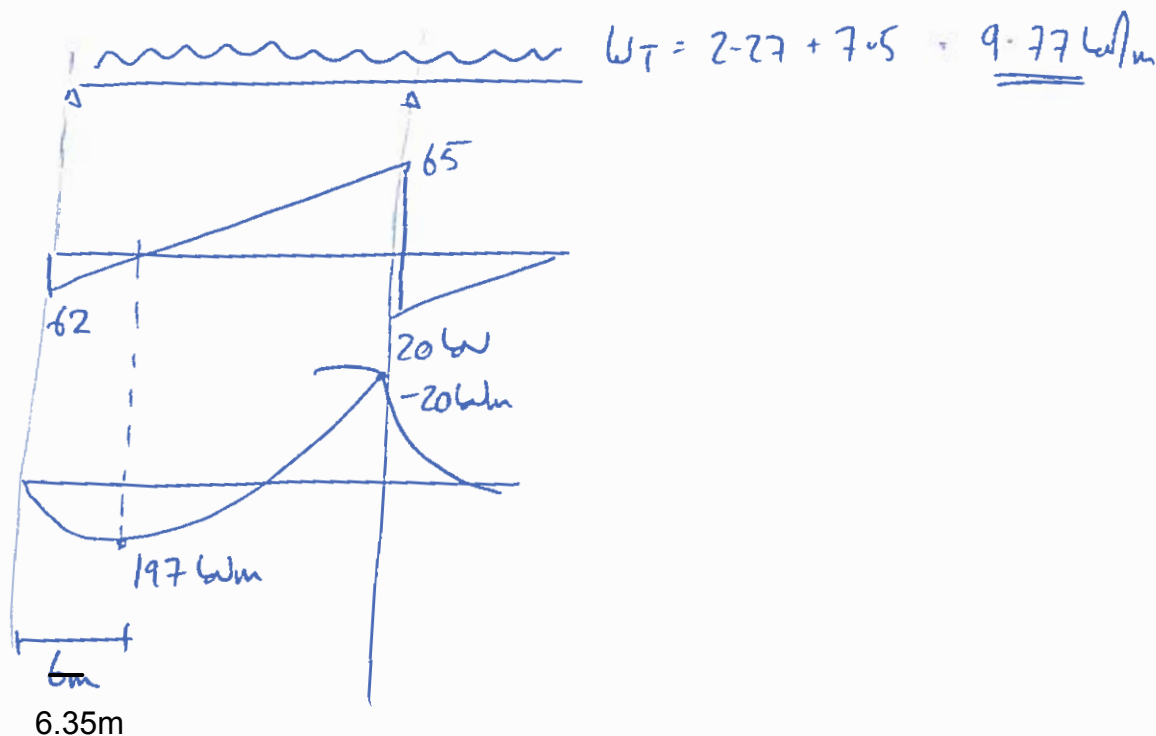
V2 JJ0

①

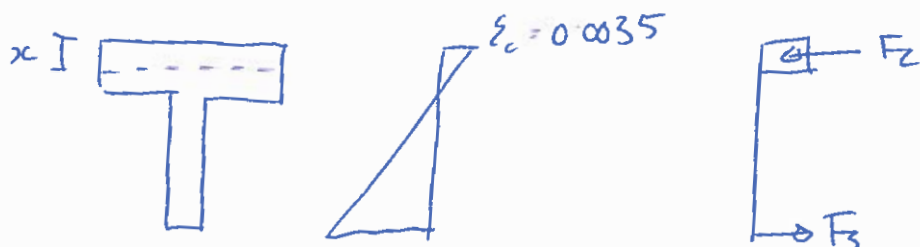
$$\delta_g = 1.35 \quad w_g = 1.68 \text{ kN/m} = 2.27 \text{ kN/m}$$

$$\delta_q = 1.50 \quad w_q = 5 \text{ kN/m} = 7.5 \text{ kN/m}$$

(a) (i)



(b) assume NA above web



$$F_s = A_s f_y d = \underline{\underline{546 \text{ kN}}}$$

$$F_c = \lambda x \rho f_c d (b)$$

$$\therefore x = \underline{\underline{38 \text{ mm}}} \quad (< 50 \text{ mm OK})$$

$$d = 550 - 50 = 500 \text{ mm}$$

$$z = d - x/2 = 481 \text{ mm}$$

$$M = F_c z = \underline{\underline{263 \text{ kNm}}}$$

(c)

(2)

→ check shear

$$\delta_c = 1.00$$

$$V_{R,c} = \frac{0.18}{1.00} \left(k (100 \rho f_{ct})^{1/3} \right) b_w d$$

$$k = 1.60 \quad f_{ct} = 45 \times 1.5 = 67.5 \text{ MPa}$$

$$\rho = 0.02 \quad b_w = 100 \text{ mm} \quad d = 500 \text{ mm}$$

$$V_{R,c} = 0.18 (1.6 (100 (0.02) (67.5)^{1/3}) 100 (500))$$

$$= \underline{\underline{81.4 \text{ kN}}}$$

$$w_{DL} = 1.68 \text{ kN/m}$$

applied load to cause shear failure

→ scale SFD from (a)

$$\text{Total } w_T = 12.2 \text{ kN/m}$$

$$w_q = \underline{\underline{10.55 \text{ kN/m}}} \quad (w_T - w_g)$$

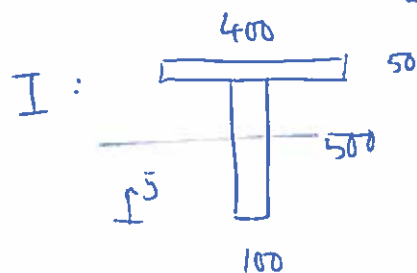
$$\text{at } w_q = 10.6 \text{ \& } w_a = 1.68$$

$$M_{\max} = \underline{\underline{246 \text{ kNm}}}, \text{ less than resistance (which is factored) } \underline{\underline{d_c}} \checkmark$$

$$\text{Note real } M_{\text{sagging}} = \underline{\underline{305 \text{ kNm}}}$$

check hogging

$$\text{simply } \sigma = \frac{M_y}{I}$$



$$A\bar{y} = \sum A_i y_i$$

$$A = 70,000$$

$$\bar{y} = \frac{400(50)(525) + 500(100)(250)}{70,000}$$

$$I = \frac{400 \cdot 50^3}{12} + 400(50)(525 - 329)^2 + \frac{100(500^3)}{12} + 500(100)(329 - 250)^2$$

$$= 2126 \times 10^9 \text{ mm}^4$$

$$M_{\text{hogging}} = 24 \text{ kNm} \quad (@ \text{ WT} = 12.2 \text{ kNm})$$

$$y = 550/2$$

$$\sigma = \frac{24(550/2)}{2126} = \underline{\underline{3.10 \text{ MPa}}}$$

→ for our beam $f_{ck} = 67.5 \text{ MPa}$, this is quite high
 ∴ shear or hogging (or both)

(1)(d)

(4)

Current EE = 2520 kg concrete + 148 kg steel
= 5480 MJ.

we have Shear capacity ✓ (81 vs 65 kN)
bending capacity ✓ (~~305~~ kNm vs 197 kNm)
hogging m.fts. (24 kNm vs 20 kNm)
263

→ hogging steel
↓ concrete volume

- eg. cut depth at support
↳ check shear capacity
- add bars to hogging zone
 - taper cantilever section

simple calc is sufficient.

eg for hogging

$M = 20 \text{ kNm}$

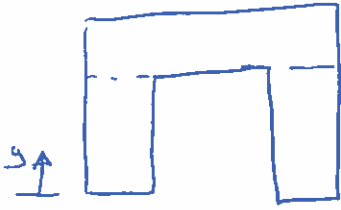
$z = 450 \text{ mm}$

$A_{s, req} = \frac{20 \times 10^6}{434(450)} = 100 \text{ mm}^2 = 1 \times \phi 12 \text{ mm bar}$
@ 5m = 4 legs

- add l. bars to web
- taper flange
- all = m³ concrete = MJ EE saving.

407 Question 2

V2 JJo ①



$$\bar{y} = \frac{\sum A_i y_i}{A}$$

$$= \frac{900(200)(400)(+) + (2)(200 \times 300)(150)}{900 \times 200 + 2 \times 200 \times 300}$$

$$= \underline{\underline{300 \text{ mm}}}$$

$$I = \sum I_{local} + A y^2$$

$$= \frac{900(200)^3}{12} + (900)(200)(100) + 2 \left[\frac{200(300)^3}{12} + (200)(300)(150^2) \right]$$

$$= \underline{\underline{6 \times 10^9 \text{ mm}^4}}$$

(a)(ii) $f_t = 4 \text{ MPa}$

$$\sigma_t = \frac{M(y)}{I} \quad M = \frac{4(6 \times 10^9)}{200} = \underline{\underline{120 \text{ kNm}}}$$

$$\omega = \frac{2M}{L^2} = \underline{\underline{9.6 \text{ k/m}}}$$

$$\omega_{DL} = 7.2 \text{ k/m}$$

\therefore first crack @ $\omega_{cr} = \underline{\underline{2.4 \text{ k/m}}}$

Question 2(b)

2

(i) Calc I_{cr}

$$m = 200/30 = 6.7$$

$$A_s = 342 \text{ mm}^2$$

$$2bx(x/2) = mA_s(d-x) \quad [b = 400 \text{ mm}]$$

$$x = \frac{mA_s}{b} \left(\sqrt{1 + \frac{2bu}{mA_s}} - 1 \right)$$

$$x = \underline{\underline{171 \text{ mm}}}$$

$$I_{cr} = \sum I_{load} + A_s^2$$
$$= 2.29 \times 10^9 \text{ mm}^4$$

At SLS $w = \text{self weight} + 8$
 $= 15.2 \text{ kN/m}$
 $M = \frac{wL^2}{2} = \underline{\underline{190 \text{ kNm}}}$

$$\sigma_c = \frac{M(x)}{I_{cr}} = \underline{\underline{14 \text{ MPa}}}$$

$$\epsilon_c = \sigma_c / E = 0.00047$$

$$\sigma_s = \epsilon_s E = \underline{\underline{154 \text{ MPa}}}$$

$$\epsilon_s = \frac{\epsilon_c}{x} \times (400 - 50 - x)$$
$$= 0.00077$$

Question 2(b)

(3)

(ii)

$$\delta_{\text{uncracked}} = \frac{wL^4}{8EI_0} = \underline{6.6 \text{ mm}} \quad [w = 15.2 \text{ k/m}]$$

$$\delta_{\text{cracked}} = \frac{wL^4}{8EI_c} = \underline{17.2 \text{ mm}}$$

$$(iii) \quad \xi = 1 - \beta \left(\frac{\sigma_{SR}}{\sigma_s} \right)^2$$

$$\beta = 1 \quad (\text{shaft term})$$

$$w_{\text{crack}} = \underline{9.6 \text{ k/m}}$$

At 1st crack, using $M = 120 \text{ kNm}$, $x = 171 \text{ mm}$, I_c from before

$$\sigma_c = \underline{8.9 \text{ MPa}} \quad (My/I)$$

$$\sigma_{SR} = \frac{500 - 50 - 171}{x} (\sigma_c)(m) = \underline{97 \text{ MPa}}$$

$$\sigma_s \text{ at full load (from before)} = \underline{154 \text{ MPa}}$$

$$\xi = 0.60$$

$$\delta = \xi \delta_{\text{uncracked}} + (1 - \xi) (\delta_{\text{cracked}})$$

$$= \underline{10.8 \text{ mm}}$$

Q2 (b) (iv)

(4)

actual will be less since not cracked on whole length

3		Marks
a	<p>Students should identify the key parameters:</p> <ul style="list-style-type: none"> • Vehicular traffic (salt) • Crossing a river (impact from debris, scour) • Very cold • Very hot • Quite rainy • Maintenance issues will be a key part of the answer • Specification of mix to be impermeable etc. To prevent: 4C's and w/c ratio: Adequate cover, cement content, compaction, curing, and impermeable (low w/c). • Some exploration of what whole life cost is (theory from books) and then an extension to support their arguments above is necessary • Excellent students will consider whole life cost beyond £s and talk about energy use, in the materials, and the construction process. Some comments on longevity are required • Students may reference the Turcot interchange as this was shown in Lectures but not essential. 	
(b) (i)	<ul style="list-style-type: none"> • Chloride ions present from road salts/sea spray, or possibly older buildings with less control over mix water or use of CaCl to accelerate setting • Ions diffuse towards the reinforcing steel. Can include equation • Once present at the bar surface, then can act as a catalyst in the electrochemical cell leading to corrosion • Typically initiated at $Cl^- > 0.4\%$ by weight cement • Carbonation from atmosphere diffuses through the concrete • Generates a front dividing areas of higher and lower pH • (Higher pH due to acidity of the carbonation process) • When this reaches the steel, the passivating alkaline environment around the bar is destroyed and corrosion can be initiated 	
(ii)	<p>Content covered in lecture notes. Links to parts listed above.</p> <ul style="list-style-type: none"> • Some description of the problem would be acceptable, and some estimation of the cause of the corrosion. • How is the problem identified: <ul style="list-style-type: none"> ○ Rust stains (visual inspection) ○ Cell potentials ○ Resistivity ○ Presence of spalling ○ Each method to be explained with pros and cons outlined (covered in notes in detail) • Options that could be explored: <ul style="list-style-type: none"> ○ Steel replacement ○ Stainless steel ○ Epoxy coating reinforcement ○ Surface treatments ○ Electrochemistry interference ○ Coatings ○ Silane ○ Patch repairs, and so forth 	

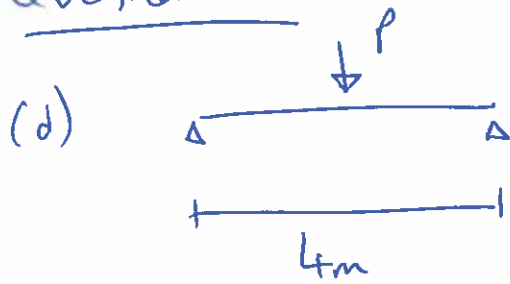
(c)(i)	<p>Choosing between the two options</p> <p>Option 1: £75k to install and £4.5k per year to operate Continuous method.</p> <ul style="list-style-type: none"> • 4% discount rate • $1.04 = \exp(r_c)$ • Therefore $r_c = \ln(1.04) = 0.0392$ • $NPV = NPV = \int_0^{125} \frac{4500}{\exp(r_c t)} dt$ • NPV = £113,883 • Capital cost = £75k • Total = £189k <p>Option 2: Major repairs every 25 years costing £300,000</p> <ul style="list-style-type: none"> • Cost = £300k • Life span 125 years • Costing at 24, 49, 74, 99 years • 4% discount rate • Total = $\frac{300,000}{(1+0.04)^{24}} + \frac{300,000}{(1+0.04)^{49}} + \frac{300,000}{(1+0.04)^{74}} + \frac{300,000}{(1+0.04)^{99}} = £184k$ <p>Close, but Option 2 is cheaper overall.</p>	
(ii)	<p>Even a small change in discount rates, or annual costs, would heavily affect the comparison. They are almost the same in this scenario. You might chose based on other considerations beyond cost.</p>	

4		Marks
(a)(i)	<p>ULS and SLS should be noted.</p>	
(ii)	<p>ULS is about safety, SLS is about serviceability.</p> <p>A full answer would consider some of the design situations (which relate to the limit states, and are either persistent, transient, or accidental design situations).</p>	
(iii)	<p>Key to reliability analysis. Students may talk also about safety.</p> <p>Answers should explore the different types of partial factors, e.g. uncertainty in values of actions, uncertainty in models of actions and action effects, uncertainty in resistance, and uncertainty in material properties.</p> <p>These four partial factors are usually reduced to just two partial factors – g_M for materials and g_F for actions.</p>	
(b)	<ul style="list-style-type: none"> • Lectures have explored the basis of Beta values, this should be explained by the student along with the role of partial factors • Answers should explore how calibration of a partial factor method against a non-partial factor method code (e.g. permissible stress) might lead to the same designs and the implications of this for 	

	<p>reliability.</p> <ul style="list-style-type: none"> • Can the numbers be trusted? Will you sign on the dotted line? 	
(c)	<p>Students should explain clearly the failure chosen, detail what went wrong and why, and identify implications. A number of examples were given in lectures, along with sources for further examples that the students may write about. The choice of failure is not limited to those in lecture notes, and good students will demonstrate their reading around this topic.</p>	

Question 4

①



load $P = \text{mean } 10\text{kN}$
 $\text{COV} = 0.20$

$$\gamma_{\text{req}} = 1.35$$

$$\gamma_{\text{M}} = 1.50$$

Load effect

$$M_s = M = \frac{PL}{4} = \frac{10(4)}{4} = \underline{\underline{10 \text{ kNm}}}$$

$$\text{COV} = 0.2 = \frac{\sigma_s}{\mu_s}$$

$$\sigma_s = 10(0.2) = \underline{\underline{2 \text{ kNm}}}$$

$$\gamma_g = 1.35$$

Strength

$$M_R = 40 \text{ kNm}$$

$$\text{COV} = \frac{\sigma_R}{M_R} = 0.18$$

$$\sigma_R = 0.18(40) = \underline{\underline{7.2 \text{ kNm}}}$$

$$\gamma_M = 1.5$$

(i) design load effect = $\gamma_r \cdot F_k$ \swarrow 95%

$$= 1.35(\mu_s + 1.645\sigma_s)$$
$$= 1.35(10 + 1.645(2)) = \underline{\underline{17.9 \text{ kNm}}}$$

~~13.29 kNm~~

design strength

$$X_d = \frac{X_k}{\gamma_m} = \frac{M_R - 1.645\sigma_R}{1.5} = \frac{298.2}{1.5} = \underline{\underline{198.8 \text{ kNm}}}$$

(ii) $\beta = \frac{M_R - M_s}{\sqrt{\sigma_R^2 + \sigma_s^2}} = \underline{\underline{4.015}}$

↓
 σ_Z function from notes.

Students could work out P_f also

$$P_f = \Phi(-\beta)$$

↳ standard normal CDF. (tables)

$\beta = 4.015$	
$\Phi = 0.946964$	0.947090
4.01	4.02

$$\Phi = 0.947027$$

$$P_f = 1 - 0.947027 = 29.7 \times 10^{-6} \approx \underline{\underline{3 \times 10^{-5}}}$$

- (iii) - the $X_d > F_d$ ✓
- $\beta > \text{ECO}$ requires it to be.
- ignored self weight.

4D7: Examiner's comments

Q1 Ultimate limit state design

Answered by 7 candidates.

Candidates often did not identify sign conventions for their shear force diagram, and these were often inverted compared to the data book convention. In part (b) most candidates were able to identify the moment capacity. In part (c) some students did not consider shear or hogging capacity of the beam. There were some minor errors with partial safety factors. Part (d) was generally well answered, the best scores achieved when statements were backed up with some outline calculation.

Q2 Serviceability limit state design

Answered by 24 candidates.

Candidates either scored rather high, or rather low, with this question. Part a(i) and a(ii) were generally answered well. Beyond this the submissions were divided between those that calculated the cracked moment and those that did not. Most candidates identified the correct equation for deflection to use in Part (b)(iii). The interpolation formula in Part b(iii) was generally outlined correctly. Some candidates calculated deflections that by inspection alone should be identified as incorrect (either very large, or very small), but rarely was this noted in the analysis.

Q3 Durability and whole life costing

Answered by 28 candidates.

The question was rather well answered over all. In part (a) some students simply regurgitated a list of facts, without stopping to consider the particular situation (location, climate, etc) of the bridge in the question. Part (b) was quite well considered. Those who attempted Part (c) tended to use the correct methodology to compare the two options. Commentary of results was rather patchy, and in some cases not attempted.

Q4 Reliability analysis

Answered by 25 candidates.

The majority of candidates scored highly in part (d) of this question, showing clear working and analysis of the results. Some commentary in Parts (a)-(c) was often less convincing, with some students repeating the same points multiple times. The issues of safety and reliability were key to these sections.