

4D8 Prestressed Concrete Examination 2016

Solutions

Section A Long Questions

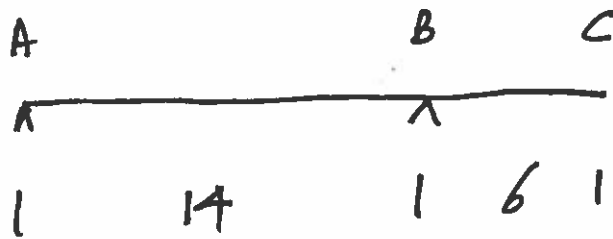
1. "Some engineers and codes of practice regard prestressed concrete as being the same as reinforced concrete, ..."

This is a fairly open-ended question. As the course is taught, prestressed and reinforced concrete are different, but Eurocode 2, for example, treats them as a continuum. I am prepared to accept either conclusion, provided it is argued. I would expect them to make reference to the fact that PSC needs to satisfy conditions at both the working load and the ultimate state, whereas RC typically only has to satisfy strength conditions at ULS and serviceability conditions such as deflections and cracking at the working load. I would expect them to argue that there is no continuum - you don't get structures with intermediate amounts of prestress, and it is argued in the course that mixing tensioned and untensioned steel leads to the prestress in the concrete being lost due to creep, the effect being that the prestress ends up going into the untensioned reinforcement. So in reality you have either I would expect them to argue that the reactant moments induced in indeterminate PSC beams have no parallel in RC. As for beams with advanced composite tendons/reinforcement the differences between the RC and PSC are even more pronounced and I would expect them to reflect on the strain diagrams that you use for design.

The counter argument to this is that once a beam has been built, the sequence of operations that you went through to get to its current state are unknown, and to be satisfactory the working load stresses and ULS capacity most all be adequate, so one should consider the two materials as a continuum, but this implies an ability to allow for creep, redistribution and so on.

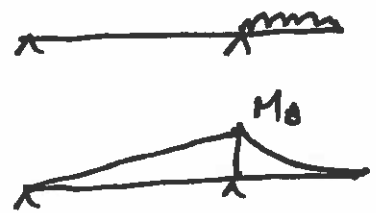
I would expect the good candidates to be able to argue their case, while the weaker ones will merely assert their answer.

2(a)



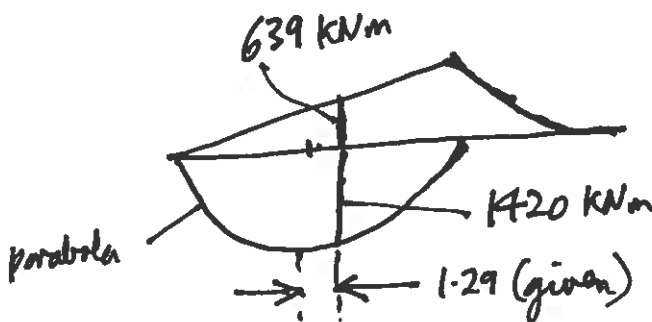
Max hogging - load BC only

$$M_B = 60 \times 6 \times 3 = 1080 \text{ KNm}$$



Max sagging - load AB only

$$M = \frac{60 \times 14^2}{8} = 1470 \text{ KNm}$$



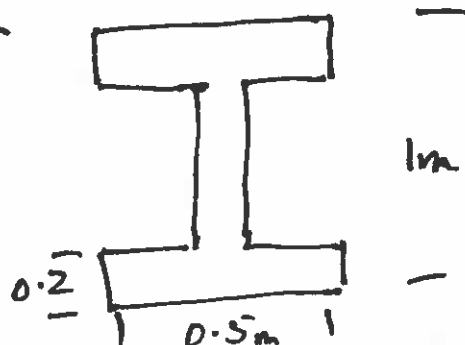
Moment range = 2059 KNm

$$\therefore \text{Required } Z_{\min} = \frac{2.06}{22} = \underline{\underline{0.0936 \text{ m}^3}}$$

Many candidates took several ways of algebra to get here. Few drew the B.M. diagram so many put the critical section on the wrong side of the C of G of the 14m span. This is IA statics!!

Possible section

Many others possible



4-D/2016/2/2

(b) Section given $A = 0.4 \text{ m}^2$ $Z_t = -0.17 \text{ m}^3$
 $Z_b = +0.14 \text{ m}^3$

Dead load = $0.4 \times 24 = 9.6 \text{ kN/m}$

Moment at critical section - either go back to bit principles or use result from (a)

$$M_D = \frac{9.6}{60} (1420 - 639) = 125 \text{ kNm}$$

Max moment = $1420 + 125 = 1545 \text{ kNm}$

Min moment = $-639 + 125 = -514 \text{ kNm}$

$$-\frac{Z_b}{A} = -0.35 \text{ m}$$

$$\frac{Z_t}{A} = +0.425 \text{ m}$$

Stress equations all of form

$$f \geq \frac{P}{A} + \frac{Pe}{Z} - \frac{M}{Z}$$

$$\Rightarrow e \geq -\frac{Z}{A} + \frac{fZ}{P} + \frac{M}{P}$$

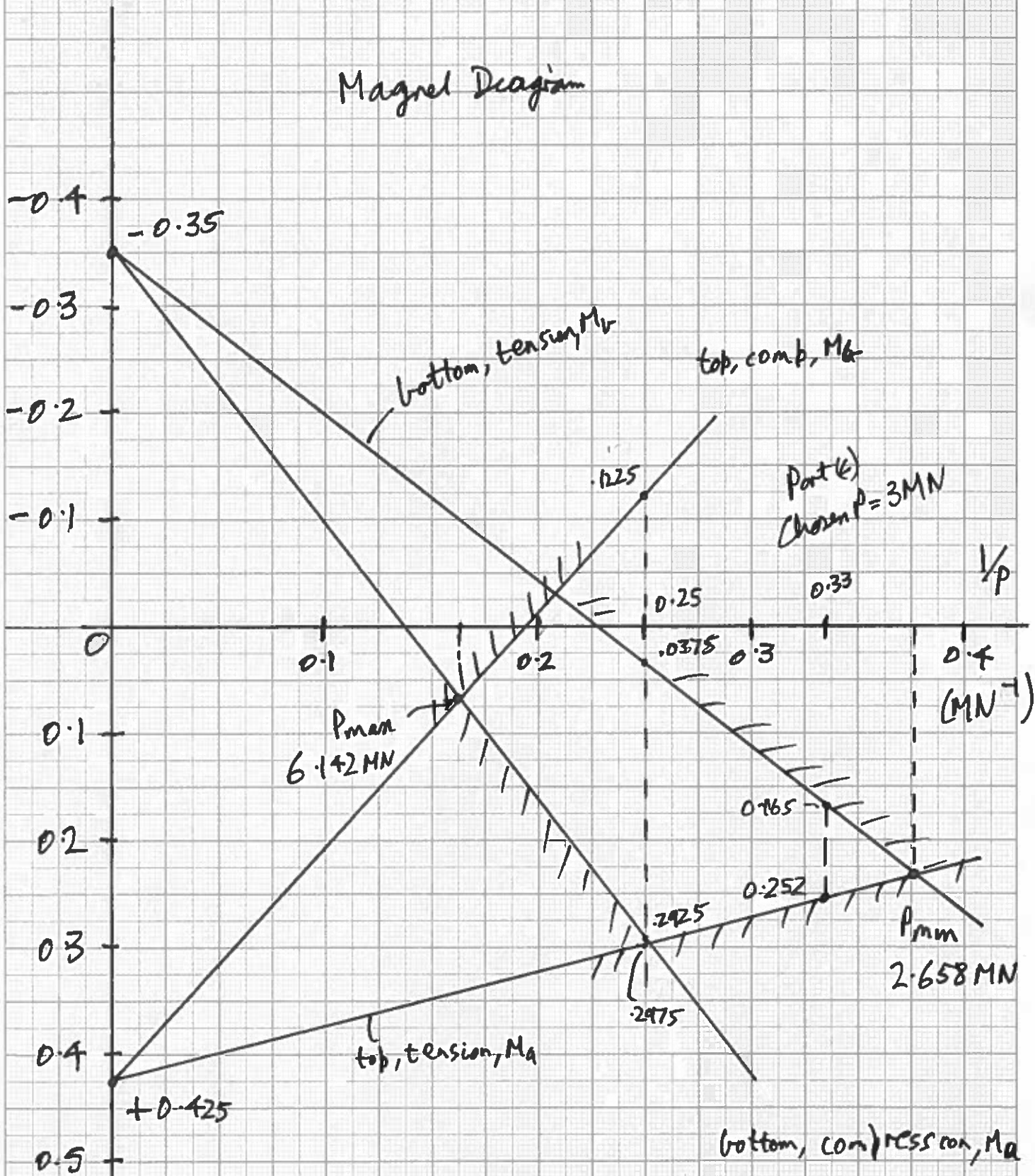
Direction of inequality depends on which fibre and which stress limit being considered

To draw Magerl diagram choose arbitrary value of P

$$\frac{f_c}{2} \cdot A = \frac{22}{2} \cdot 0.4 = 4.4 \text{ MN}$$

(Choose $P = 4 \text{ MN} \quad \therefore \frac{1}{P} = 0.25 \text{ MN}^{-1}$)

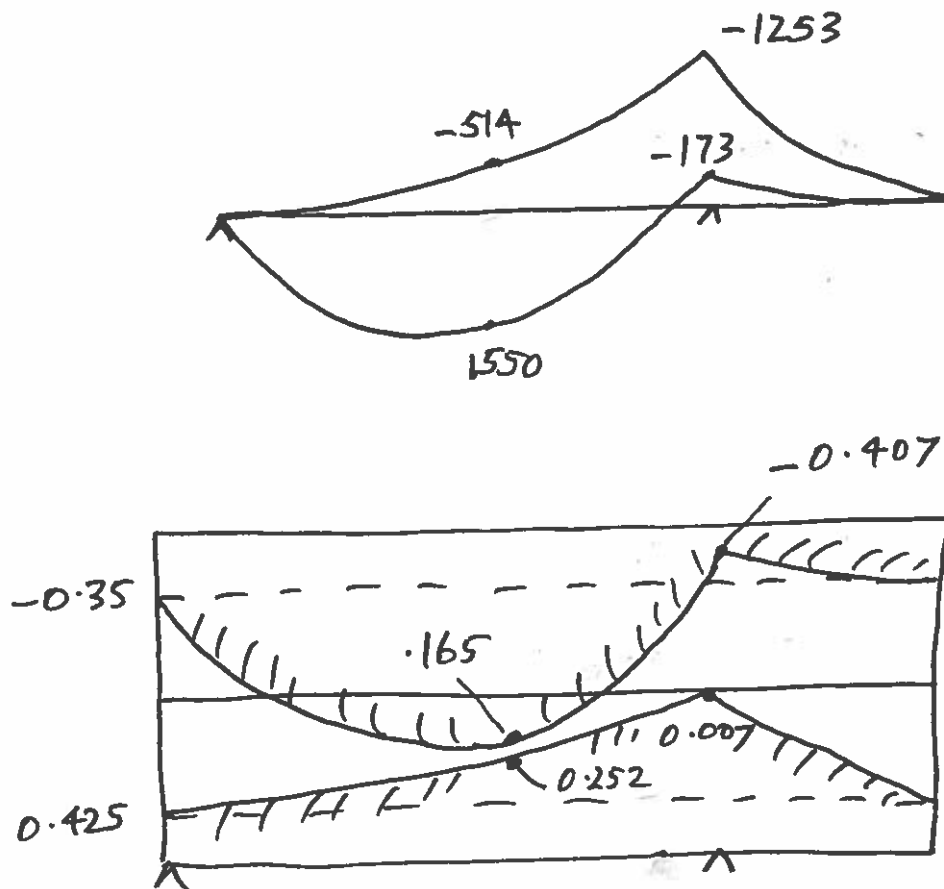
Magnel Diagram



P_{min} and P_{max} either from diagram
or from calculation

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(c) Moment range in whole beam (incl dead weight)



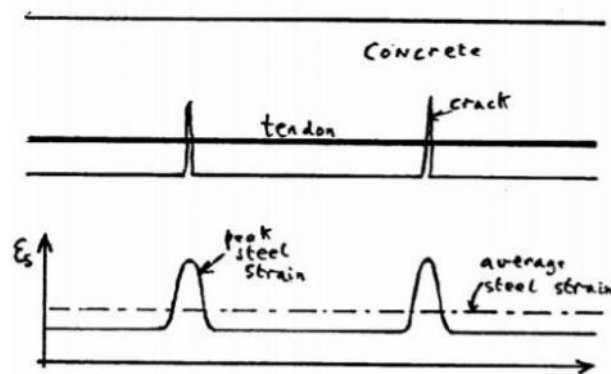
Not possible to pretension because no suitable straight profile exists, even if it is allowed to be inclined

Many candidates were very vague about this, and almost no one took account of the dead weight in the cantilever

Section B Short Questions

3. "When considering the ultimate load capacity of prestressed concrete beams, a limit is often placed on the maximum strain on the steel..."

The logic behind this restriction on the steel strain is based on the assumptions that underly the analysis of prestressed beams at the ultimate load. In order to give a reasonable relationship between the strains in the concrete and the strains in the steel, it is assumed that the steel strains are averaged across the cracks. The actual steel strain at the crack location can be significantly higher than this average because in reality the strain is concentrated at the crack locations. If the steel has very high strain capacity this does not matter - the real steel strain will be higher than the average value assumed, and if the steel work hardens the steel stress might be a bit higher too so the moment capacity is higher than assumed and everything is conservative.



But prestressing steels are cold-drawn and often have more limited strain capacity, of the order of 3% - 5%, so to avoid the possibility of the steel snapping the maximum strain at the ultimate load is limited to about 1.5%. Given that the initial prestrain is typically between 0.4% and 0.5%, a limit on the additional strain of about 1% is often applied. (The candidates will not get credit if they simply say that this is a safety factor without reference to the strain localisation argument given above.)

From their notes - to avoid this type of failure

$$\text{So:- } \beta_1 \epsilon_c + \beta_2 \frac{(d-x)}{x} \epsilon_{cu} \leq 0.01$$

But $\beta_1 \epsilon_c$ usually negligible, so

$$\frac{x}{d} \geq \frac{1}{\frac{0.01}{\beta_2 \epsilon_{cu}} + 1}$$

$$\text{If } \epsilon_{cu} = 0.0035 \text{ and } \beta_1 = 1.0 \quad \frac{x}{d} \geq 0.26$$

This gives an upper limit on the allowable ductility of a section, which is unusual. But tendon failure would be catastrophic

So this form of failure would occur in lightly prestressed sections where only a small compression zone is needed to balance the force in the tendon. So this type of failure would occur in lightly prestressed sections. To keep the neutral axis deeper in the beam ($x/d > 0.26$), the compression zone must be bigger so the tendon needs to be larger (A_{st}). If the prestrain is 0.4%, and the allowable additional strain is 1%, then we need the stress at 1.4% strain ($\sigma_{1.4}$) in the tendon. If the average stress in concrete in compression is $k_1 f_{cu}$ then the compressive force is $k_1 f_{cu} x b$, which must be balanced by the force in the tendon ($A_{st} \sigma_{1.4}$).

To ensure this type of failure does not occur, $A_{st} \sigma_{1.4} > k_1 f_{cu} x b = 0.26 k_1 f_{cu} b d$, so

$$A_{st} \geq \frac{0.26 k_1 f_{cu} b d}{\sigma_{1.4}}$$

Examiner's comments. This covered a fairly narrow but important detail that had been covered in their notes. Only attempted by a small number of weaker candidates (judging by their answers to other questions). There were no correct answers.

4. Difference in behaviour of bonded and unbonded beams.

(a) For both beams the behaviour up to first cracking will be very similar. There will normally be some negative curvature when prestressed due to the eccentricity of the tendons. The beams will behave linearly elastic until the tensile stress capacity is first exceeded.

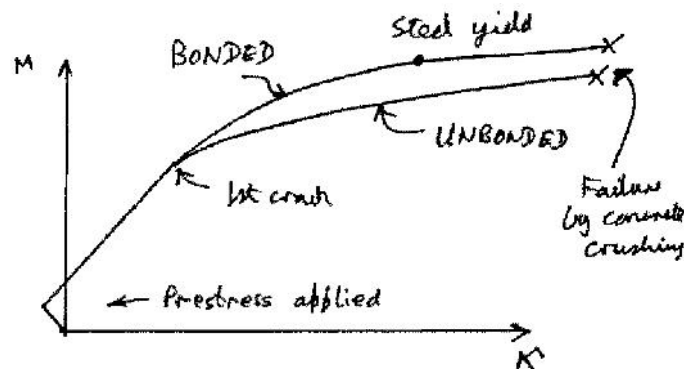
After cracking, the unbonded beam loses stiffness more quickly than the bonded beam, because the tendon does not pick up stress so quickly at crack locations. Unbonded beams only have to satisfy compatibility in global terms, not locally.

The bonded beam, if properly designed, will probably reach a point where the tendon yields, although the moment capacity may continue to increase due to changes in lever arm between the tension and compression forces.

The unbonded beam will continue to increase in moment capacity, but usually without yielding of the steel.

Final failure in both beams will occur when concrete crushes, at similar strains.

(The students have done a lab on this, so although this answer is long it should be familiar to them.)



(b) A complete analysis of the bonded beam can be carried out by satisfying compatibility locally, but for an unbonded beam this is not possible. The designer must calculate the strains in the tendon and in the concrete adjacent to the tendon all along the beam, and then integrate both separately. The two should be equal, satisfying the global compatibility condition.

What is normally of interest is to determine the peak moment capacity in the beam. This can be achieved by assuming that the change in strain in the tendon at the peak moment position is a certain fraction of the change in strain in the adjacent concrete. A factor of 0.25 is normally used for this but the exact value depends on the shape of the bending moment diagram and is higher for constant moments but lower for peaked moment diagrams.

(c) Need to make an initial guess for position of neutral axis. This would normally be between 0.25 and 0.5 of the effective depth, which in this case is at 600 mm, so a value of 200 mm seems reasonable.

The strain factor β is here defined as 0.25.

	1 st Guess	2 nd Guess	3 rd Guess
Neutral axis x (mm)	200	180	182
Additional steel strain ϵ_{s_s}	$0.25 \cdot 0.0035 \cdot \frac{400}{200} = .00175$.0024	.00201
Prestrain	0.004	0.004	0.004
Total Steel strain	0.00575	0.00604	0.00601
Steel stress (N/mm ²) from stress-strain curve	1150	1206	1200
Tension (kN) T	2070	2171	2160
Compression (kN) C	$30 \cdot 200 \cdot 400 = 2400$	2160	2184
	$C > T$	$C < T$	$C > T$
	increase x	decrease x	

Close enough. x must be between 180 and 182, say 181 mm

Take $T = C = 2165$ kN

Lever arm = $600 - 181/2 = 509$ mm

So ultimate moment = 1102 kNm

Examiner's comments

Mirrored the behaviour of the two beams they tested in the labs and they clearly had learnt from that.

5. True/False questions about losses

(a) Flexibility of the ducts should not affect friction losses, but the duct may tend to move more when concrete is cast, so wobble losses can be expected to be higher. So False.

(b) There is no relationship between secondary moments and friction, so False.

(c) False. There is no link between them.

(d) False. Friction occurs before grout is introduced ∴ no effect.

(e) False. The untensioned rebar does not creep so picks up much of the prestress.
∴ The loss of prestress in the concrete is higher but the loss in the prestressing steel is lower.

(f) False. The opposite is true. Wedge slip affects area near the anchorage only.
∴ Important in short tendons.

(g) False. Force transfer affects any tendon that is already bonded. ∴ It affects all tendons in pretensioned beams.

(h) False. The Relaxation of steel is independent of whether you are pre or post tensioning.

Examiner's comments

Most candidates did well with 6 of the questions, but two caused problems. Most recognised that the presence of untensioned reinforcement causes a loss of prestress in the *concrete* but the question asked for the effect of the loss of force in the *tendon* where the loss of force is reduced. This had been emphasised in lectures. The other question to cause problems was the last question about relaxation of steel, which should have no difference between pre- and post-tensioned beams. The effect of maturity of concrete, which many identified, will at best be a second-order effect.

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14th May 2016