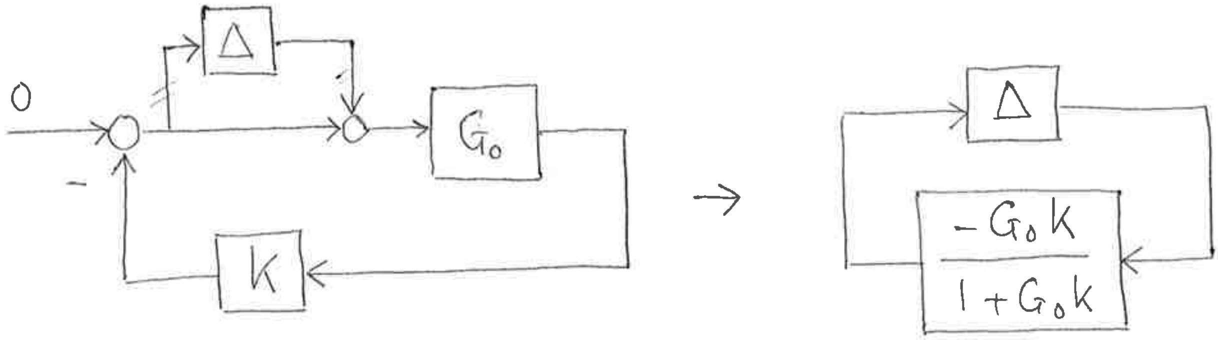
4F1 solutions 2015

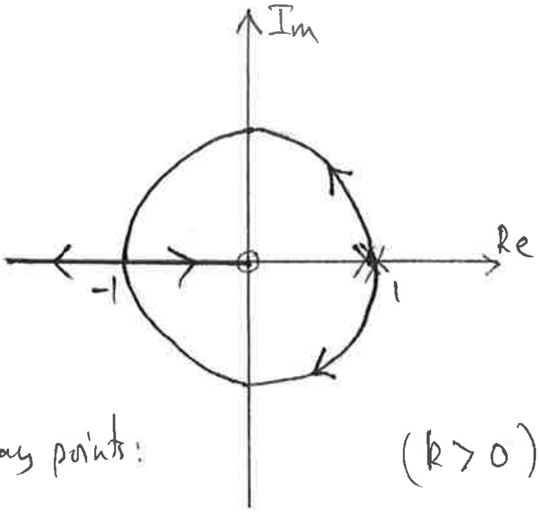
Prof M Smith

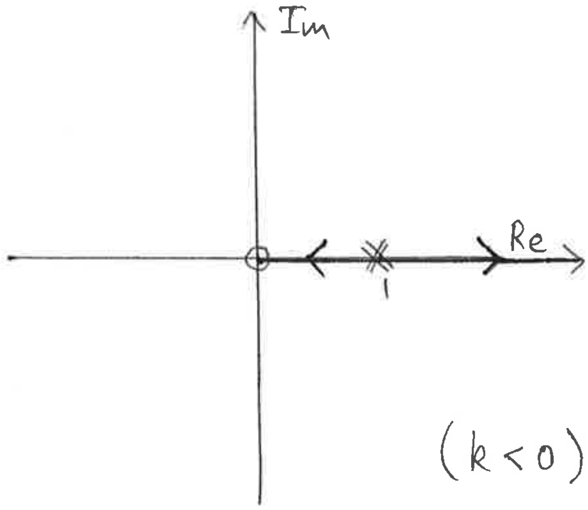
1(a)



Bookwork. Result follows from Small Gain Theorem.

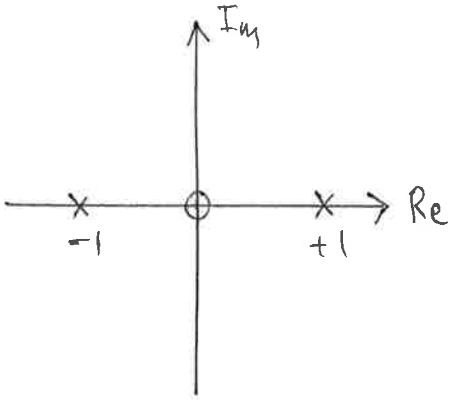
(b)





Breakaway points:
$s = \pm 1$

(c)(i)



Assume a stable compensator $k(s)$. Because of the zero at $s=0$ and deg. num. < deg. den. poles can only cross the imag. axis in pairs. Hence there will always be an odd number of RHP poles.

(ii) $$kL_o = \frac{k\,s}{(s-1)^2} = \frac{s}{(s-1)(s+1)} \cdot \frac{k(s+1)}{s-1} = G_o k$$

There are no RHP pole-zero cancellations between G_o and k, so internal stability of closed loop is possible. Part (b) shows that k can be chosen so that $k(s)$ is stabilising. Closed loop poles: $(s-1)^2 + ks \equiv (s+1)^2$ for $k=4$.

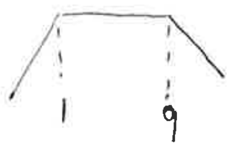
(iii) $$\frac{G_0 k}{1+G_0 k} = \frac{4s}{(s+1)^2}$$

From Part (a) need

$$\left|\frac{4j\omega}{(j\omega+1)^2}\right| \leqslant \left|\frac{j\omega+9}{\delta(j\omega+1)}\right|$$

$$\Leftrightarrow \left|\frac{4j\omega}{(j\omega+1)(j\omega+9)}\right| \leqslant \frac{1}{|\delta|} \qquad \text{for all } \omega \qquad (*)$$

From Bode mag. plot of LHS



peak mag. occurs at geometric mean of 1 and 9, i.e. $\omega = 3$

Hence

$$(*) \Leftrightarrow \frac{4.3}{3.(1+3^2)} \leqslant \frac{1}{\delta} \Leftrightarrow |\delta| \leqslant \frac{5}{2}$$

[Alter: $\delta^2 \leqslant \frac{(9-\omega^2)^2+100\omega^2}{16\omega^2} = \frac{81}{16\omega^2} + \frac{82}{16} + \frac{\omega^2}{16}$. By differentiation min of RHS occurs at $\omega = 3$.]

[Closed looppoles are at $s=-1$ and roots of $(s+1)(s+9)+4\delta s$ $= s^2+(10+4\delta)s+9$ which has LHP roots providing $10+4\delta > 0 \Leftrightarrow \delta > -5/2$

For a bound on $|\delta|$, above result is not conservative. (Not required)]

A very popular question which was attempted by all candidates. Most candidates gave good or excellent solutions. Not many candidates got a completely correct solution to the final part.

2(a) Bookwork: effect of disturbances, notion of sensitivity, effect of nonlinearities. $S(s) = \frac{1}{1+G(s)k(s)}$

(b) (i) $$G(s) = \frac{s}{(s+1)(ms^2+cs+k)}$$

$S(0) = 1$ since internal stability requires no pole-zero cancellations between $G(s)$ and $k(s)$.

(ii) RHP zero (at $s=1$) $\Rightarrow$ $S(1) = 1$

$$\Rightarrow \quad 0 = \log S(1) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{1}{1+w^2} \log|S(jw)|\, dw$$

$$\Rightarrow \quad 0 = \int_0^{\infty} \frac{1}{1+w^2} \log|S(jw)|\, dw$$

$$\leq \log\varepsilon \int_0^1 \frac{1}{1+w^2} dw + \int_1^{\infty} \frac{1}{1+w^2} dw \log|1+\delta|$$

$$= \log\varepsilon \underbrace{\left[\tan^{-1} w\right]_0^1}_{\frac{\pi}{4}} + \log|1+\delta| \underbrace{\left[\tan^{-1} w\right]_1^{\infty}}_{\pi/2 - \pi/4}$$

$$\Rightarrow \quad 0 \leq \log\varepsilon + \log 1+\delta$$

$$\Rightarrow \quad 0 \leq \log\varepsilon(1+\delta) \quad \Rightarrow \quad 1 \leq \varepsilon(1+\delta)$$

$$\Rightarrow \quad \varepsilon \geq \frac{1}{1+\delta} = \frac{2}{3}$$

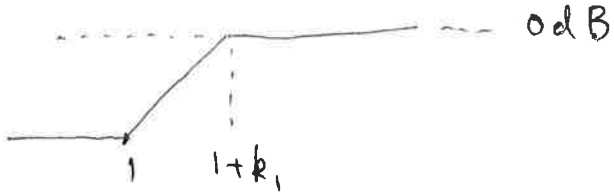
(iii) $G(s) = \dfrac{Ts+1}{ms^2+cs+k}$. Hence a controller to achieve internal stability can be found to give a return ratio equal to

$$L(s) = \frac{k_1}{s+1}$$

for any $k_1 > 0$. This gives

$$S(s) = \frac{s+1}{s+1+k_1}$$

Bode mag.



To achieve the spec. need $\frac{2}{1+(1+k_1)^2} \leq \varepsilon^2$

which is always possible for large enough k_1.

(iv) Return ratio will have at least 2^{nd} order roll-off at high frequency, hence

$$\int_0^\infty \ln|S(j\omega)|\, d\omega = 0$$

(or >0 if RHP poles are included in the controller - unlikely).
Hence the specs are not achievable for any $\varepsilon < 1$ and $\delta = 0$ since that would make $\ln|S(j\omega)| \leq 0$ for all ω.

A very unpopular question with 5 out of 9 attempts scoring 25% or less and only two first class solutions.

3 (a) (ii) Excess phases increases from 0 to 360° suggesting 2 RHP poles. Sharp rise in excess phase sugests a complex conjugate pair of poles with real part around $\omega = 3$ rad/sec. (This is consistent with peak in magnitude plot, indicating a damping ratio ~ 0.2). Possible poles at:

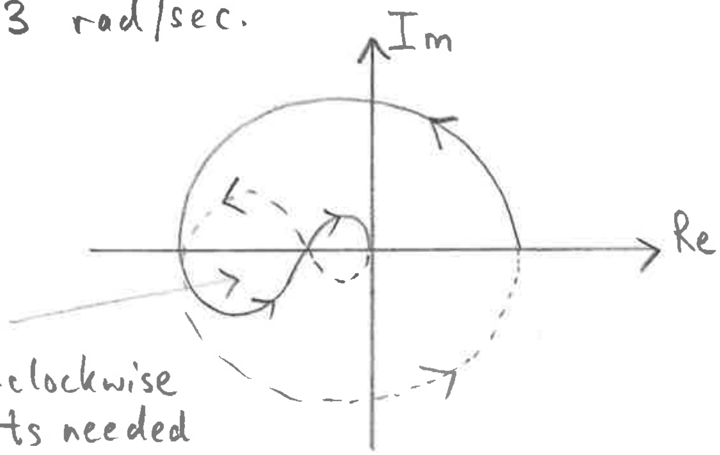
$$0.6 \pm j3$$

[Actual transfer function used

$$\frac{15000(s+5)}{(s^2-s+9)(s^2+40s+800)}$$
]

(iii) Difficult to achieve a cross-over frequency much below 3 rad/sec.

(b)



2 counter-clockwise encirclements needed for closed-loop stability

180° phase crossing points occur at ~ 23 dB and -4 dB

Implies closed-loop stable for $0.07 < k < 1.6$

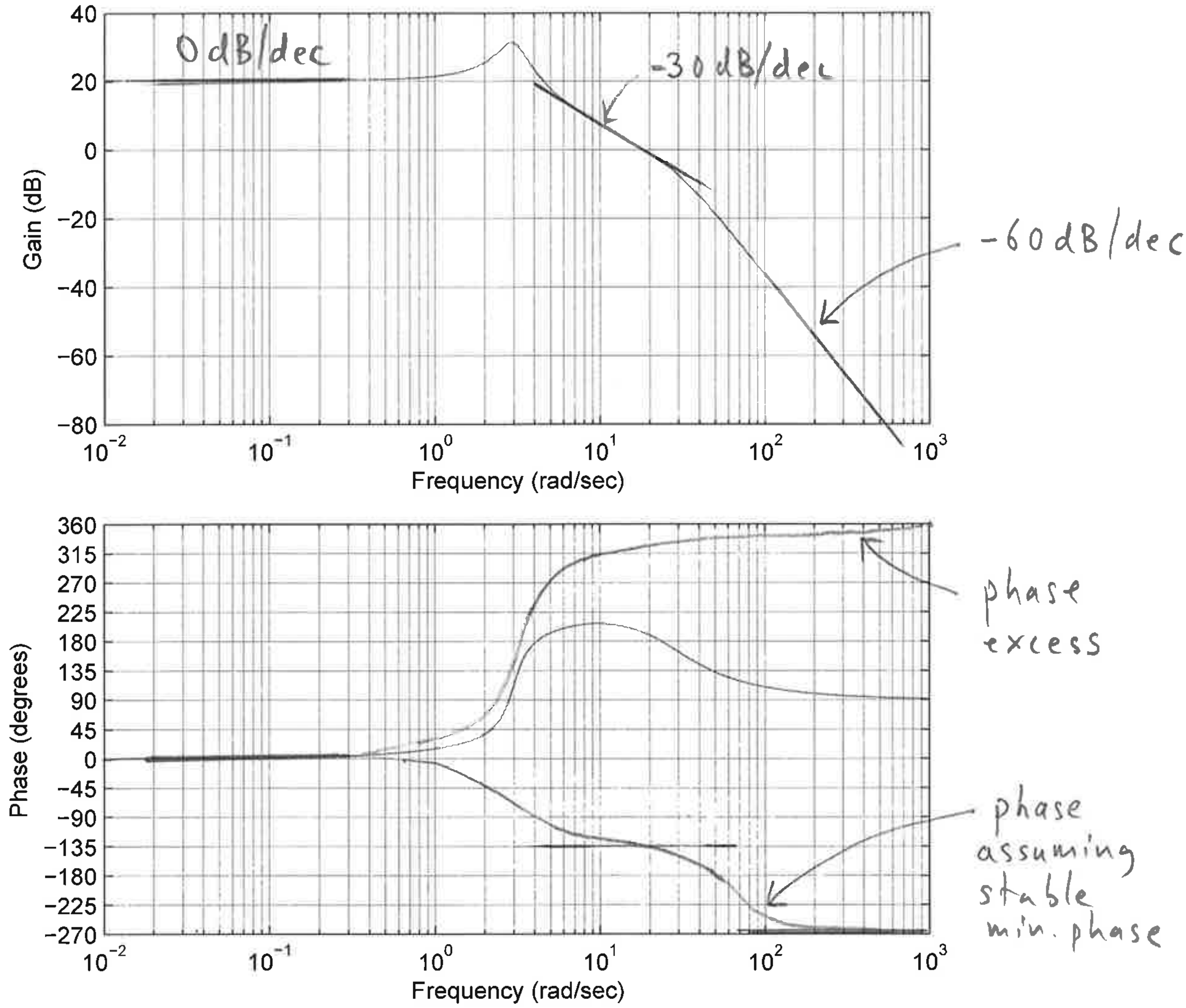
(c) (i) Lag. comp. can't satisfy D. Lead increases gain at high frequency relative to low frequency. B + C means not enough phase lead can be added to satisfy D.

[Close observation of the Bode plot shows about 1 dB in reserve for both B and C, but this is not enough for the (at least) 17° of phase lead needed to achieve D. This level of detail was not required.]

Version MCS/1 Candidate Number:

EGT3
ENGINEERING TRIPOS PART IIB
2015, Module 4F1, Question 3.

3(a)(i)



Extra copy of Fig. 1: Bode diagram for Question 3.

Version MCS/1

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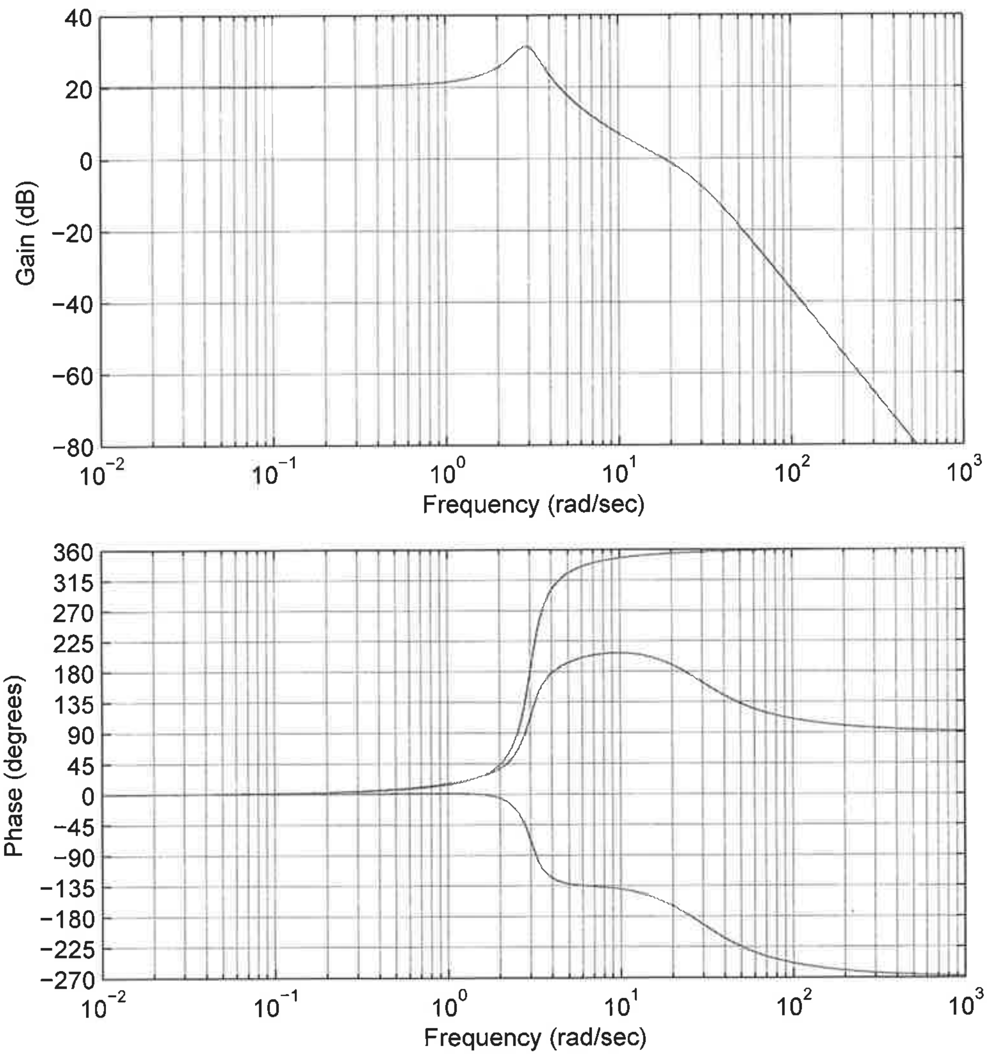
2015, Module 4F1, Question 3.

3(a)(i) Accurate computer plot.



Extra copy of Fig. 1: Bode diagram for Question 3.

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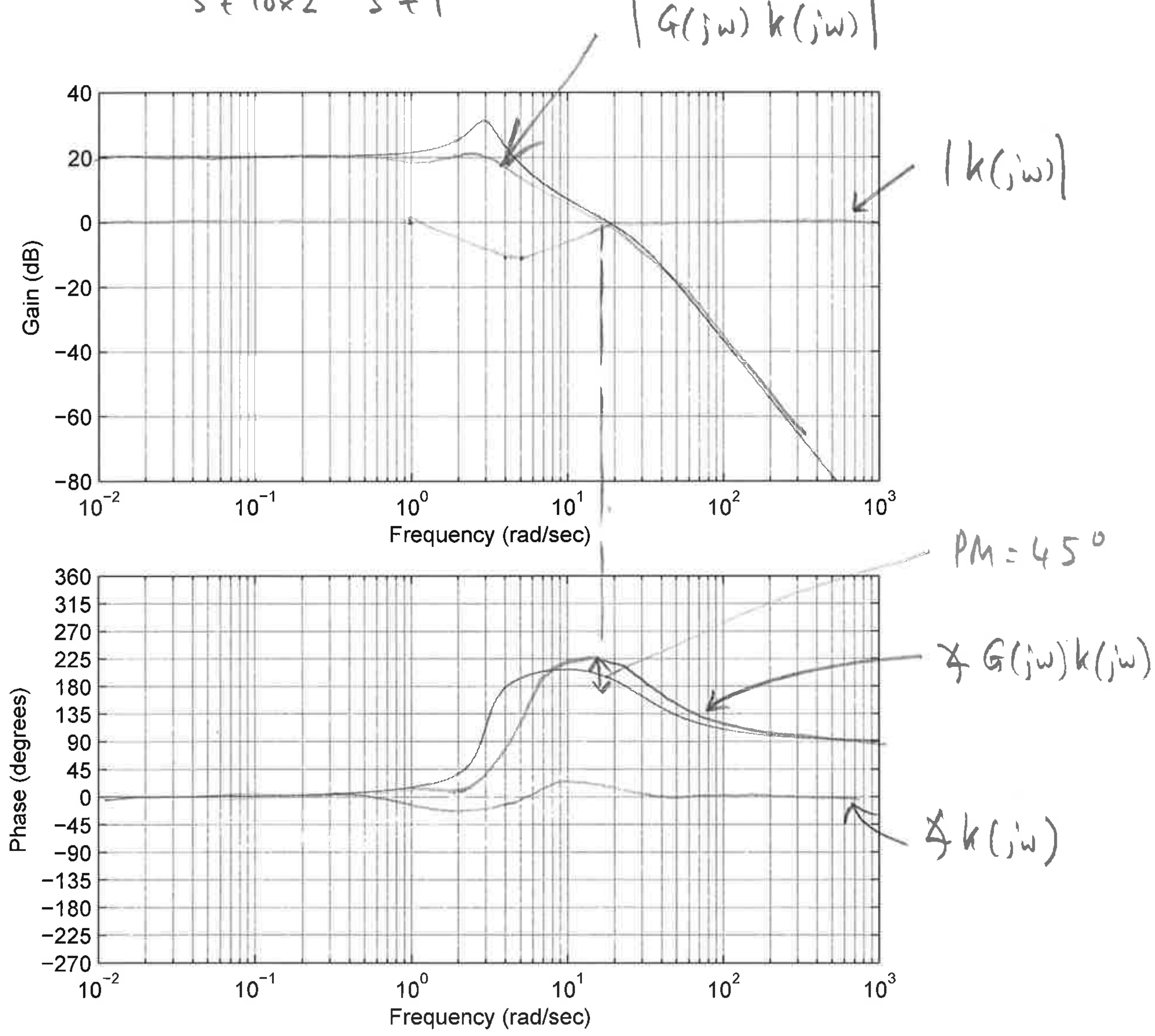
3(c)(ii) Needs a lead comp. around 10 rad/sec to get phase margin and a lag comp. for spec. B

e.g. $k(s) = \frac{s + 10/2}{s + 10 \times 2} \cdot \frac{s+4}{s+1}$



Extra copy of Fig. 1: Bode diagram for Question 3.

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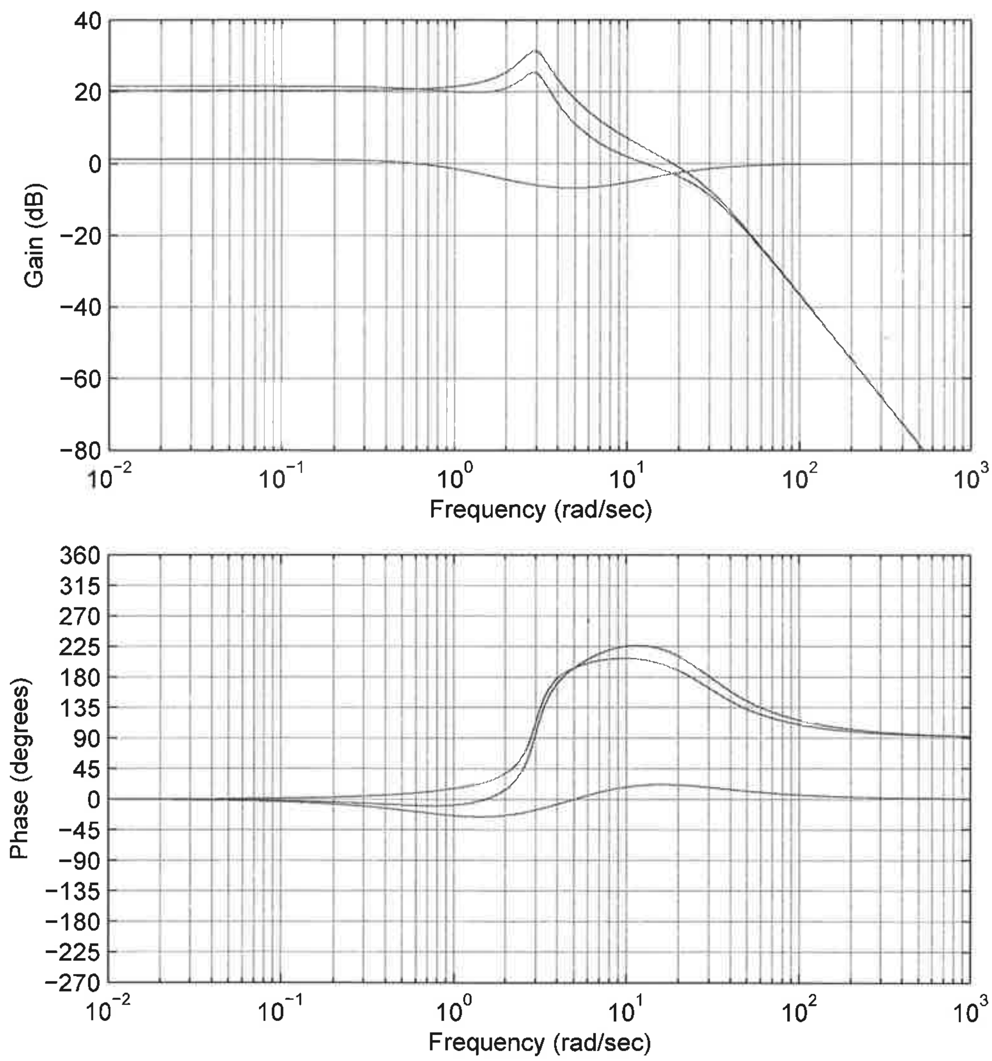
3 (c) Accurate computer plot

$$k(s) = \frac{(s+5)(s+4.6)}{(s+20)(s+1)}$$



Extra copy of Fig. 1: Bode diagram for Question 3.

Most candidates who attempted this question showed good understanding of the Bode gain/phase relationships and presented good solutions to the compensator design.