Prof M C Smith

4F1 Control System Design 2016

1(a) Plant zeros at $s = \pm\sigma = \pm\sqrt{mgr/J}$

RHP zero $\Rightarrow$ $S(\sigma) = 1$

Hence

$$0 = \log S(\sigma) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{\sigma}{\sigma^2+\omega^2} \log S(j\omega)\, d\omega$$

$$\Rightarrow \quad 0 = \int_{-\infty}^{\infty} \frac{\sigma}{\sigma^2+\omega^2} \log|S(j\omega)|\, d\omega$$

$$\Rightarrow \quad 0 = \int_{0}^{\infty} \frac{\sigma}{\sigma^2+\omega^2} \log|S(j\omega)|\, d\omega$$

(b)

$$0 \leq \int_0^{\omega_1} \frac{\sigma}{\sigma^2+\omega^2} \log\varepsilon\, d\omega + \int_{\omega_1}^{\infty} \frac{\sigma}{\sigma^2+\omega^2} \log(1+\delta)\, d\omega$$

$$= \left[\tan^{-1}\frac{\omega}{\sigma}\right]_0^{\omega_1} \log\varepsilon + \left[\tan^{-1}\frac{\omega}{\sigma}\right]_{\omega_1}^{\infty} \log(1+\delta)$$

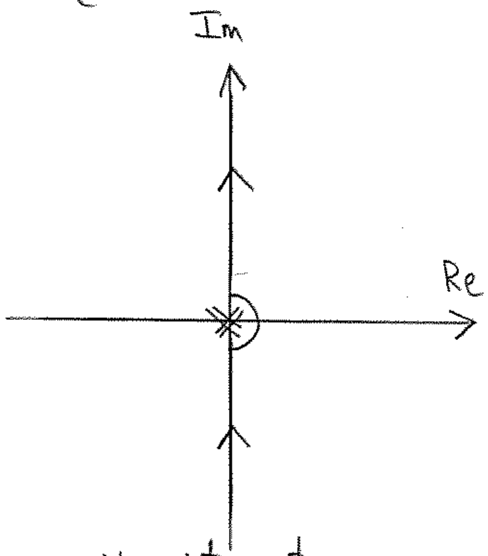
$$= \tan^{-1}\frac{\omega_1}{\sigma} \log\varepsilon + \left(\frac{\pi}{2} - \tan^{-1}\frac{\omega_1}{\sigma}\right)\log(1+\delta)$$

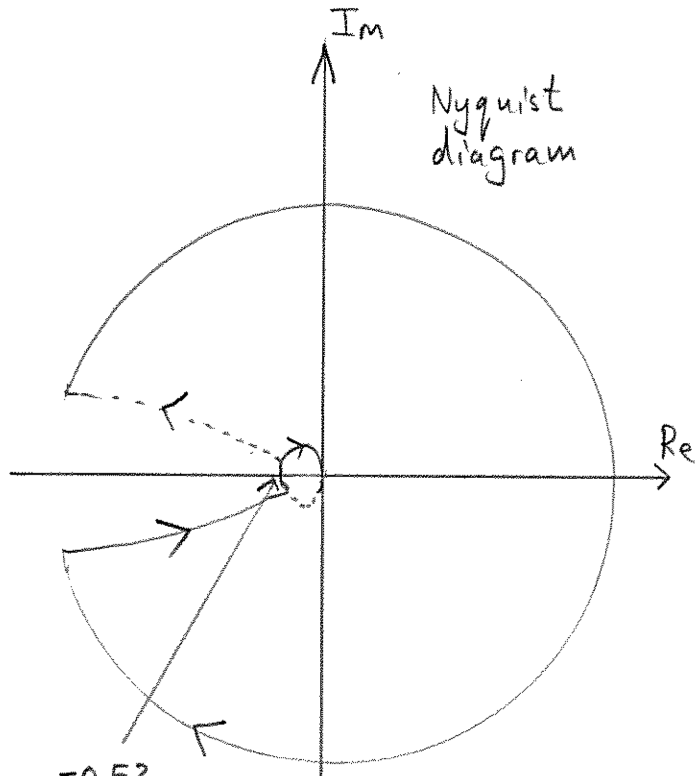
$$\Rightarrow \quad \left(-\log\varepsilon + \log(1+\delta)\right)\tan^{-1}\frac{\omega_1}{\sigma} \leq \frac{\pi}{2}\log(1+\delta)$$

$$\Rightarrow \quad \omega_1 \leq \sigma \tan\left(\frac{\pi}{2}\,\frac{\log(1+\delta)}{\log((1+\delta)\varepsilon^{-1})}\right)$$

(c) For good feedback properties it is desirable to achieve sensitivity reduction over as wide a bandwidth as possible. The larger σ is the less restrictive is the bound on ω_1 in Part (b). Hence r should be as large as possible.

(d)





2 poles at $s=0$ give large clockwise circular arc

$-\infty < -\frac{1}{k} < -0.53$

$\Updownarrow$

$0 < k < 1.9$ (approx.)

$\Updownarrow$

no encirclements

$\updownarrow$

closed-loop stable

$-0.53 < -\frac{1}{k} < 0$

$\Updownarrow$

$1.9 < k < \infty$

$\Updownarrow$

2 clockwise encirclements

$\updownarrow$

2 RHP poles

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(e)

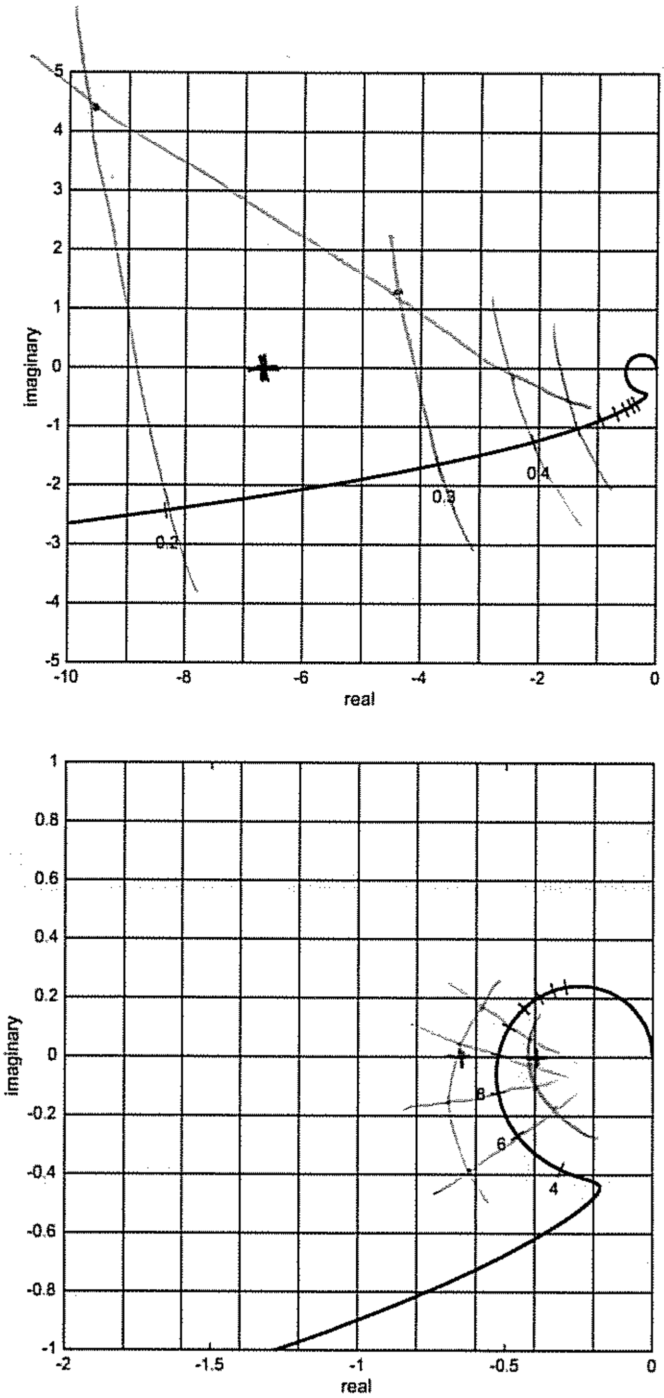

Fig. 1

$-\frac{1}{0.15} = -6.67$

roots:

$-0.04 \pm j0.23$

$-\frac{1}{1.5} = -0.67$

roots:

$-1.9 \pm j9.7$

$-\frac{1}{2.5} = -0.4$

roots:

$2.2 \pm j10.8$

Actual roots

k	s
0.15	$-0.036 \pm j0.22$
1.5	$-1.89 \pm j9.37$
2.5	$2.78 \pm j10.05$

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2(a)(i)

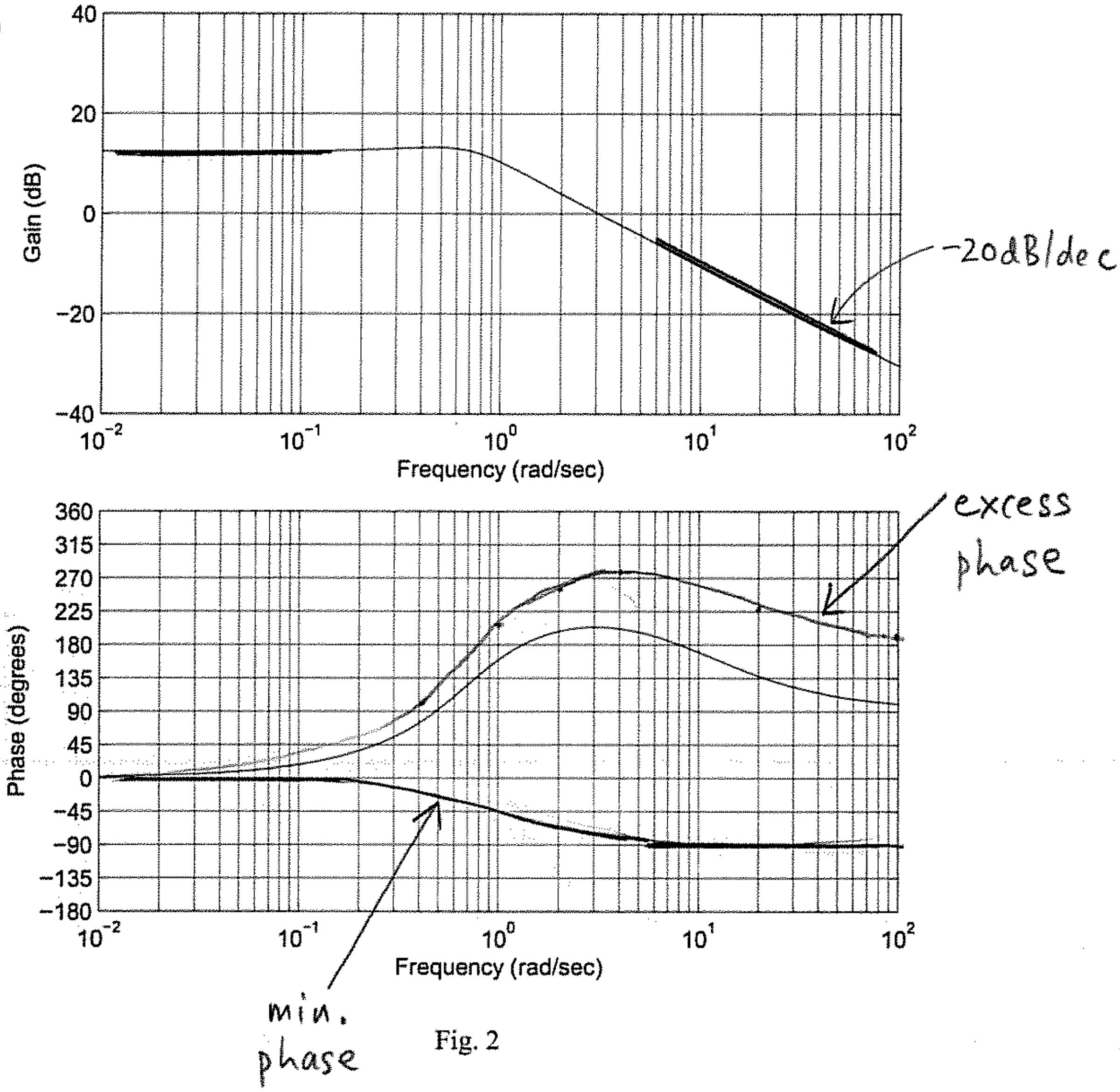

Fig. 2

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2(a)(i) exact computer plot

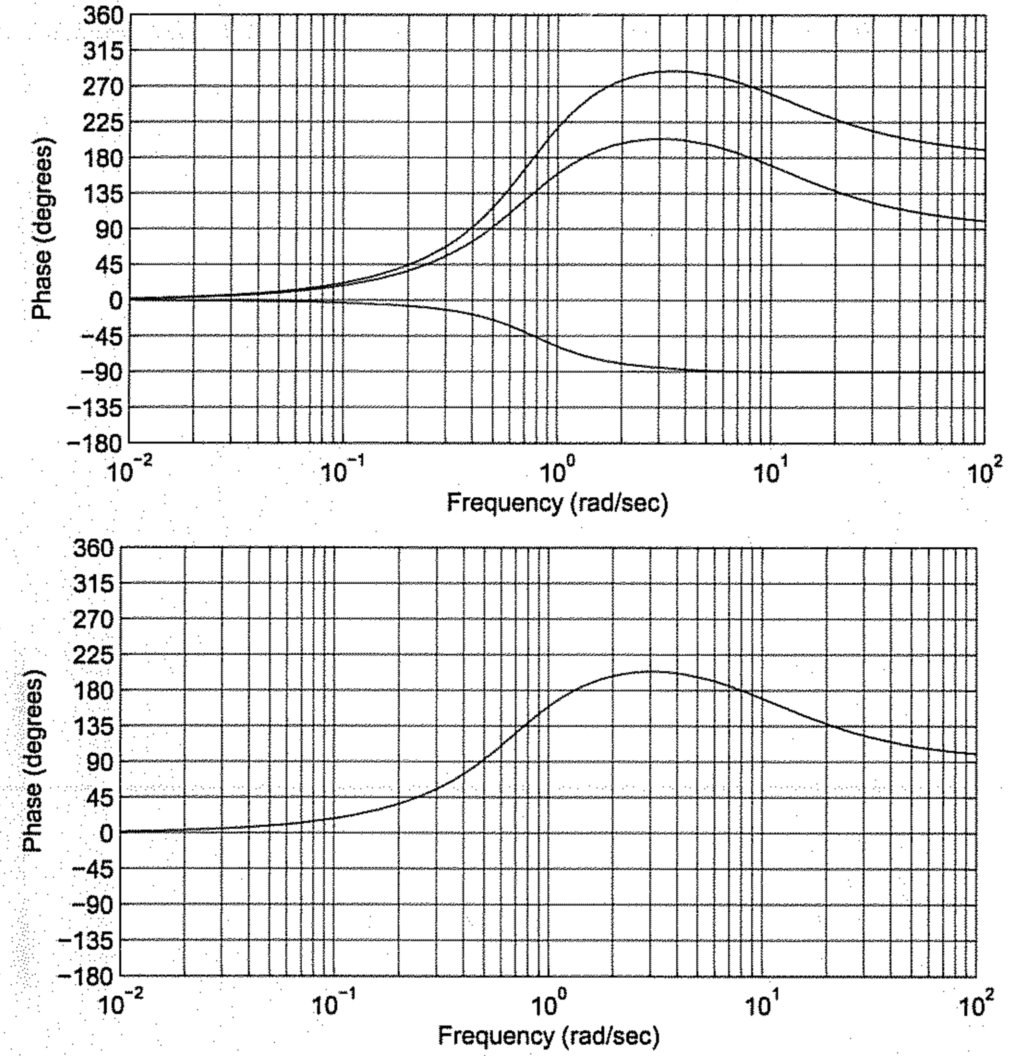


Fig. 1

2(a)(ii) An almost pole-zero cancellation in the right half plane has very little effect on the Bode diagram. E.g.

$$\frac{s - z}{s - z + \varepsilon}$$

for $z > 0$ and ε small has close to unity gain and zero phase at all frequencies.

(iii) The excess phase goes above $+180^\circ$ which requires 2 RHP poles. Excess at high frequencies tends to 180° which suggest 1 RHP zero. Crosses 180° and 270° at 0.7 and 7 rad/sec approx. Suggests an all-pass factor:

$$\frac{10 - s}{10 + s}\left(\frac{s + 0.6}{s - 0.6}\right)^2$$

[Actual transfer function:

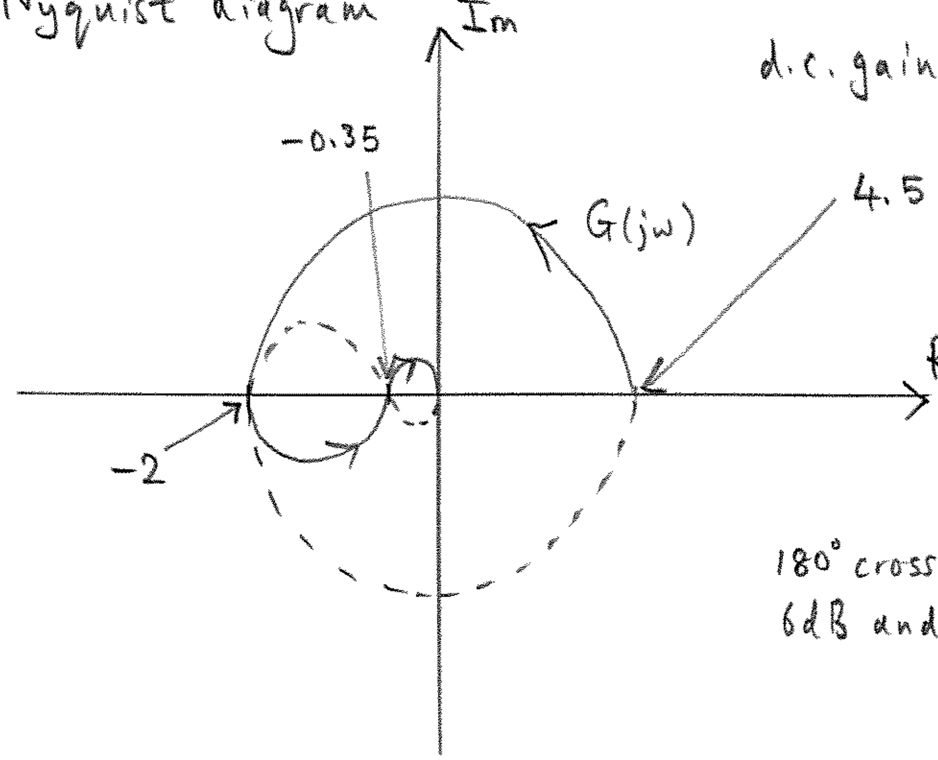
$$3\,\frac{(-0.1s + 1)(s + 0.7)}{(0.1s + 1)(s^2 - s + 0.5)}$$

so true all-pass:

$$\frac{10 - s}{10 + s}\left(\frac{s^2 + s + 0.5}{s^2 - s + 0.5}\right)$$]

(iv) Need crossover above 0.6 and below 10 rad/sec approximately.

(b)(i) Nyquist diagram



d.c. gain $\simeq$ 13dB

180° crossings are at 6dB and -9dB approx.

2 RHP poles of $G(s)$. Nyquist stability criterion requires 2 anti-clockwise encirclements for closed-loop stability.

Holds

$$\Leftrightarrow \quad -2 < -\frac{1}{k} < -0.35$$

$$\Leftrightarrow \quad 0.5 < k < 2.86$$

(ii) Crossover of 3 rad/sec seems already suitable. Phase margin insufficient. Need at least 22° more. Put in more than this since we will introduce a lag afterwards to meet B. Choose

$$k(s) = \left(\frac{s+a}{s+b}\right)\left(\alpha\,\frac{s+w_c/\alpha}{s+w_c\alpha}\right) = k_2(s)\,k_1(s)$$

$w_c = 3$ try $\alpha = 2 \Rightarrow \phi_{max} = 37°$ (plenty)

Need $\frac{a}{b} \cdot \frac{1}{\alpha} = \frac{10}{4.5}$. Choose $a = 0.3$ (decade below crossover) and $b = 0.06$.

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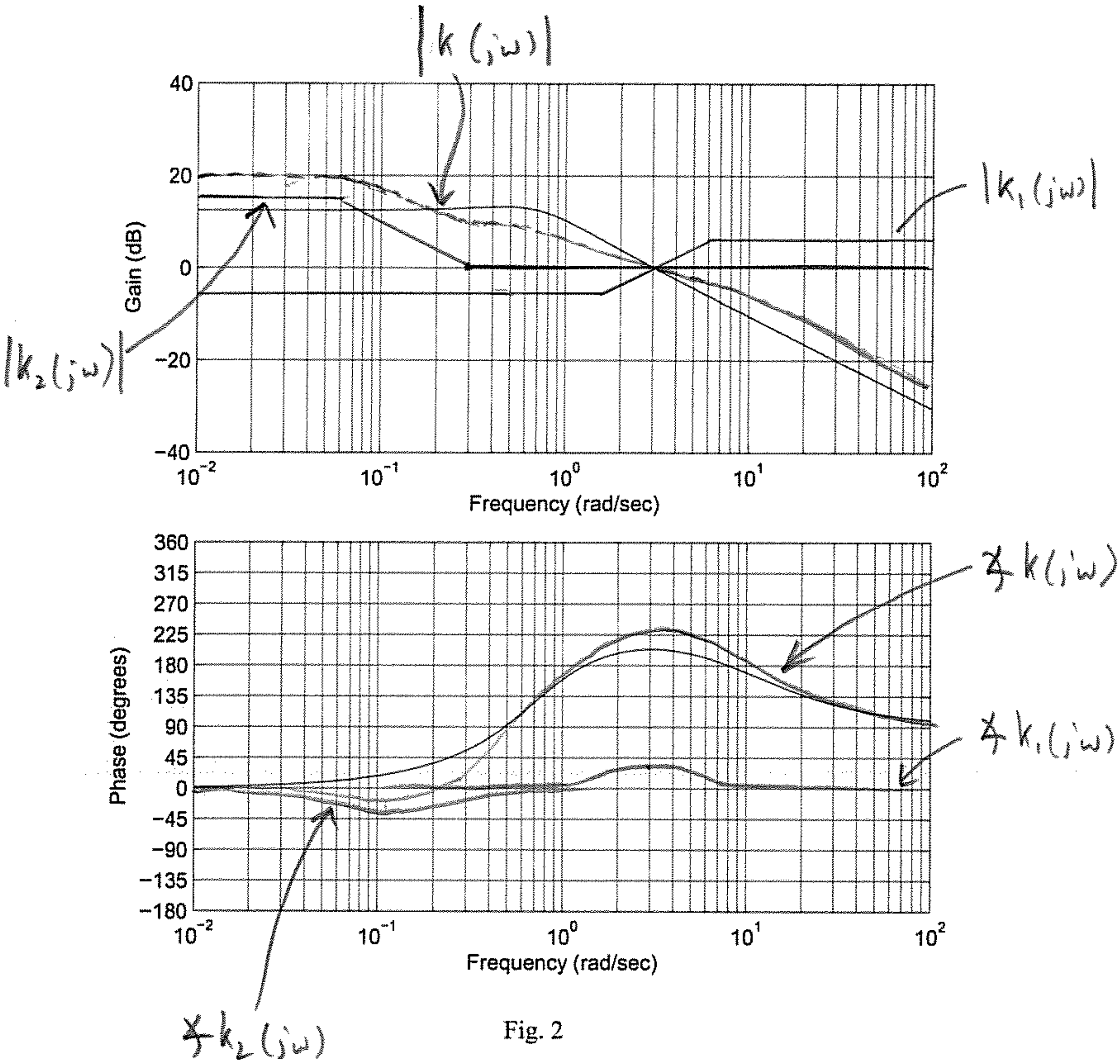

Fig. 2

Above just misses spec B at 0.1 rad/sec.
Need another iteration to increase a to say 0.5.

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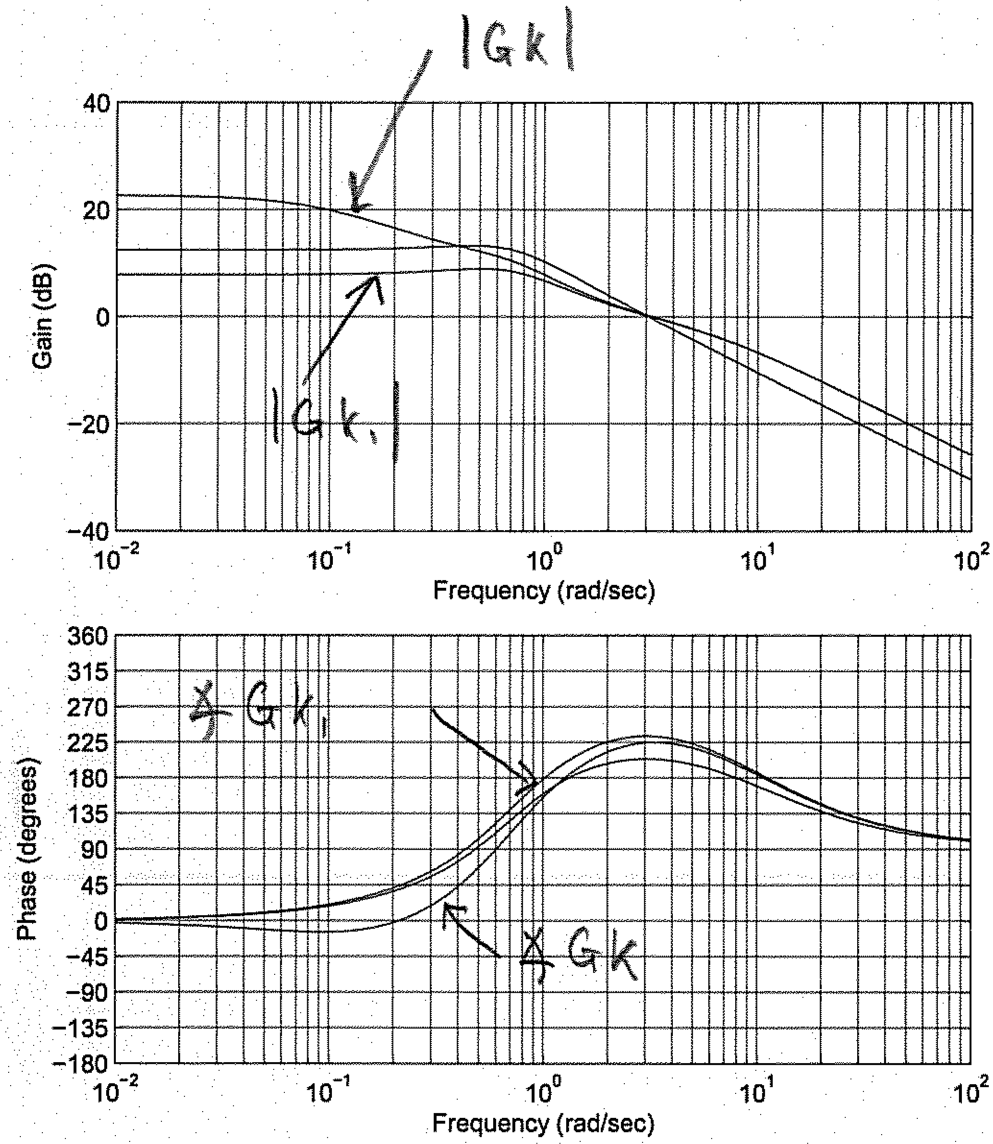


Fig. 1

Computer plot with

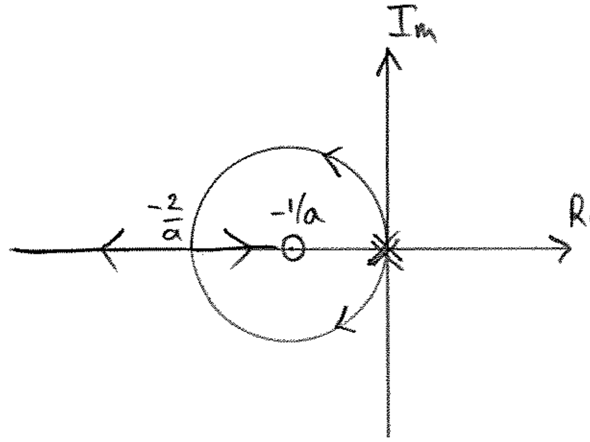
$\omega_c = 3$

$\alpha = 1.7$

$a = 0.55$

$b = 0.1$

3(a)

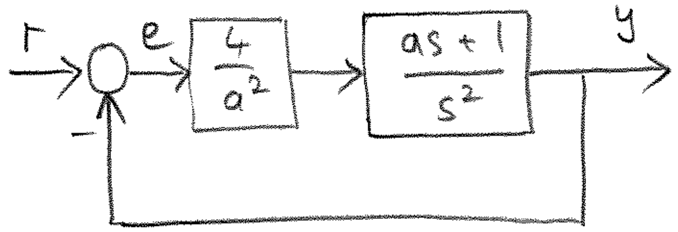


Breakaway pts.
$= 0, -\frac{2}{a}$

(b) Closed loop poles: $0 = s^2 + k(as+1)$
$= s^2 + kas + k$

$$(ka)^2 = 4k \Rightarrow k = \frac{4}{a^2}$$

(c)



$$\frac{\bar{e}(s)}{\bar{r}(s)} = S(s) = \frac{1}{1 + \frac{4}{a^2}\frac{as+1}{s^2}}$$

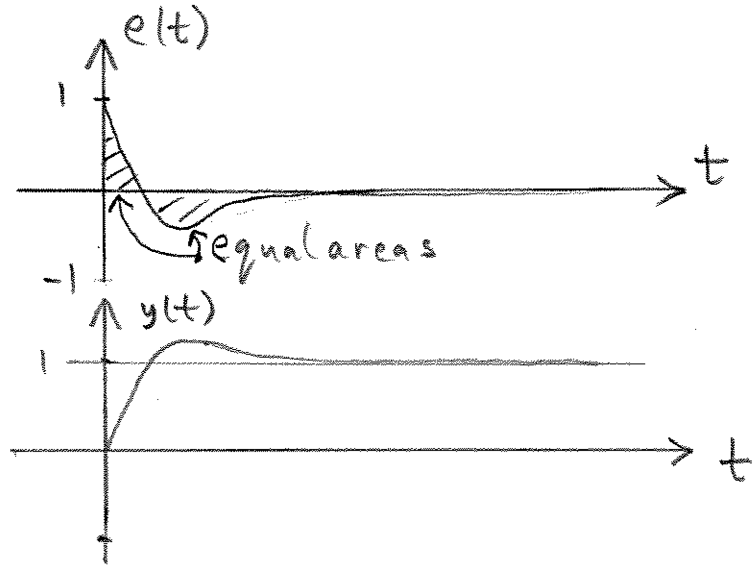
$$= \frac{a^2 s^2}{a^2 s^2 + 4as + 4} = \frac{a^2 s^2}{(as+2)^2}$$

(d) $$\bar{e}(s) = \frac{a^2 s^2}{(as+2)^2} \cdot \frac{1}{s} = \frac{s}{(s+2/a)^2}$$

$$\bar{e}(s) = \int_0^\infty e(t)\, e^{-st}\, dt$$

$$\bar{e}(0) = 0 \Rightarrow \int_0^\infty e(t)\, dt = 0$$

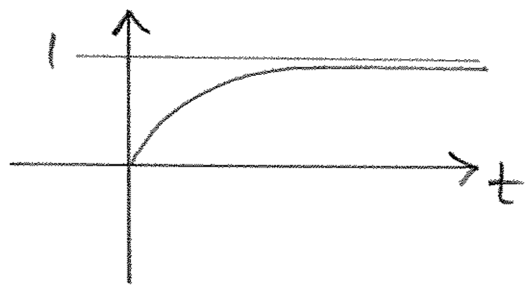
(e)



y(t) exhibits overshoot, which would be unsatisfactory as the step response to have the lift overshoot its destination floor each time

(f) $$\bar{y}(s) = \frac{1}{s(as+1)} = \frac{1}{s} - \frac{a}{as+1} = \frac{1}{s} - \frac{1}{s+1/a}$$

$$\Rightarrow \quad y(t) = 1 - e^{-t/a}$$

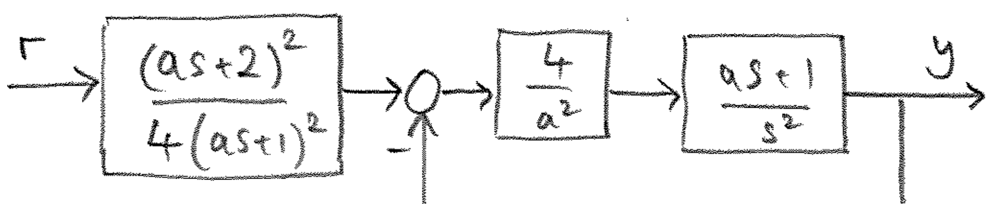


nice step response with no overshoot

This transfer function is achievable since it rolls off at least as fast as G(s) and retains the RHP zeros (there are none!)

(g) For the design in Part (c): $$\frac{\bar{y}(s)}{\bar{r}(s)} = \frac{4(as+1)}{(as+2)^2}$$

Hence we need a pre-filter $$= \frac{(as+2)^2}{4(as+1)} \cdot \frac{1}{as+1} = H(s)$$



Examiner's Comments

Q1. Vectored thrust aircraft. Sensitivity and conformal mapping.
Most candidates made good attempts at this question. None of the question parts caused undue difficulties for the majority of candidates.

Q2. Bode gain/phase and compensator design.
A rather unpopular question but with most of the few attempts being very good.

Q3. Lift control system. Two-degree-of-freedom design.
A popular question attempted by all candidates. A common mistake was to fail to recognised that a two-degree-of-freedom design was needed in the final two parts.