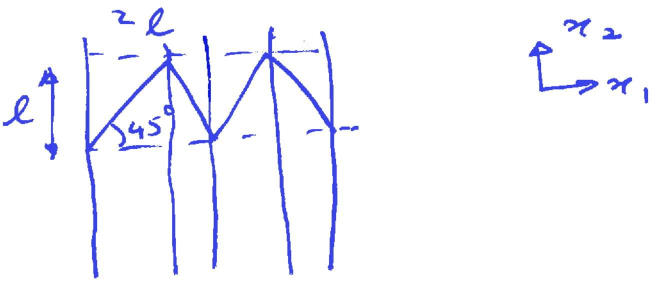
4G6 - 2016

Prof V S Deshpande

1)



(a) $$\bar{\rho} = \frac{t\ell\,(1+\sqrt{2})}{\ell^2} = (1+\sqrt{2})\frac{t}{\ell}$$

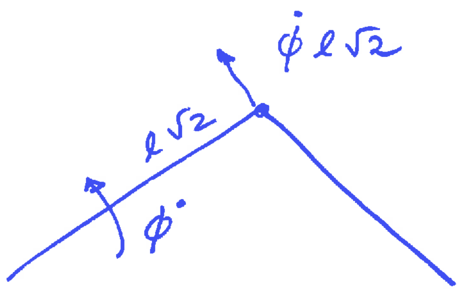
(b) Wall stress $\sigma_2 = \dfrac{\Sigma_2\,\ell}{t}$; $\varepsilon_2 = \dfrac{\sigma_2}{E_s}$

$$E_2 = \frac{\Sigma_2}{E_2} = \frac{t}{\ell}\frac{\sigma_2}{\varepsilon_2} = E_s\frac{t}{\ell}$$

$$E_2 = \frac{\bar{\rho}}{1+\sqrt{2}}E_s$$

$$\sigma_{2Y} = \sigma_{YS}\frac{\bar{\rho}}{1+\sqrt{2}} = \frac{t}{\ell}\sigma_{YS}$$

(c) (i)



resolve $\dot{\phi} l \sqrt{2}$ horizontally $\Rightarrow$ $\frac{\dot{\phi} l \sqrt{2}}{\sqrt{2}} = \dot{\phi} l$

$$\dot{\varepsilon}_{11} = \frac{\dot{\phi} l}{l} = \dot{\phi}$$

(ii) $\Sigma_1 l \, (\dot{\varepsilon}_{11} l) = 2 M_p \dot{\phi}$ $\qquad M_p = \frac{1}{4} t^2 \sigma_{TS}$

$$\Sigma_{11} = \frac{1}{2} \left(\frac{t}{l}\right)^2 \sigma_{TS} = \frac{1}{2} \frac{\bar{\rho}^2}{(1+\sqrt{2})^2} \sigma_{TS}$$

2 (a) $(T+a)v = b(T_0 - T)$

$$v = \frac{b(T_0 - T)}{T+a}$$

Power $= vT = \dfrac{bT(T_0 - T)}{T+a}$

max power @ $\dfrac{d}{dT}(vT) = 0$

$$P = vT = \frac{b(T_0 - T)}{1 + \frac{a}{T}}$$

~~$\frac{dP}{dT} = 2\left(\left(\frac{1}{a} + \frac{1}{T}\right)\right)^{2}$~~

$$\frac{dP}{dT} = 0 \Rightarrow \left(\frac{1}{a} + \frac{1}{T}\right) = \left(\frac{T_0}{T^2} - \frac{1}{T}\right)$$

$$T = \frac{-2a \pm \sqrt{4a^2 + 4aT_0}}{2}$$

$$\frac{T}{T_0} = -\frac{a}{T_0} + \sqrt{\frac{a^2}{T_0^2} + \frac{a}{T_0}}$$

$$v_{opt} = \frac{b\left[1 + \frac{a}{T_0} - \sqrt{\frac{a^2}{T_0^2} - \frac{a}{T_0}}\right]}{\sqrt{\frac{a^2}{T_0^2} + \frac{a}{T_0}}}$$

(b)

(i) Number of cross-bridges in $\frac{1}{2}$ sarcomere $= \frac{mAS}{2}$.

Allow the sarcomere to shorten by l. Since $l \gg h$, all cross-bridges have opportunity to go through 1 cycle

=>

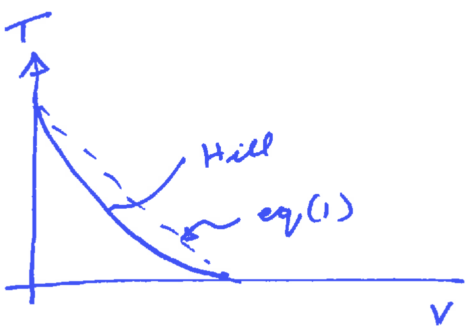
$$T l A = \int_{-\infty}^{\infty} \left[n(x) \frac{msA}{2} \right] x \lambda \, dx$$

$$T = \frac{ms\lambda}{2l} \left[\int_{-\infty}^{0} n_0 e^{\frac{bx}{v}} x \, dx + \int_{0}^{h} n_0 x \, dx \right]$$

$$= \frac{ms\lambda n_0}{2l} \left[\frac{v^2}{b^2} \left[\frac{b}{v} x e^{\frac{bx}{v}} - e^{\frac{bx}{v}} \right]_{-\infty}^{0} + \frac{h^2}{2} \right]$$

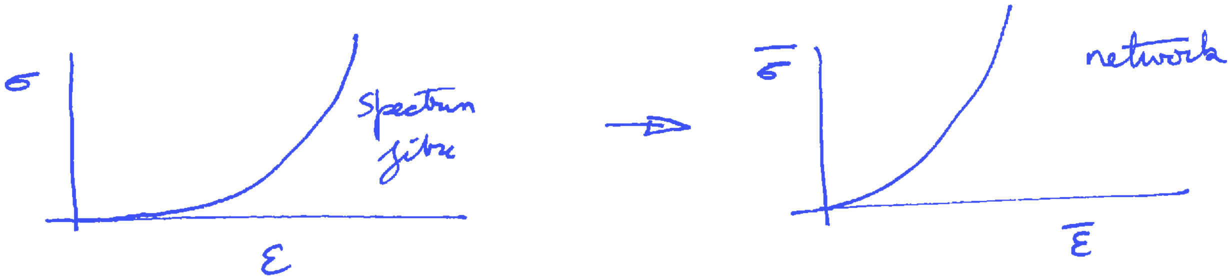
$$T = \frac{n_0 s \lambda m}{2l} \left[\frac{h^2}{2} - \frac{v^2}{b^2} \right] \quad — (1)$$

(ii)



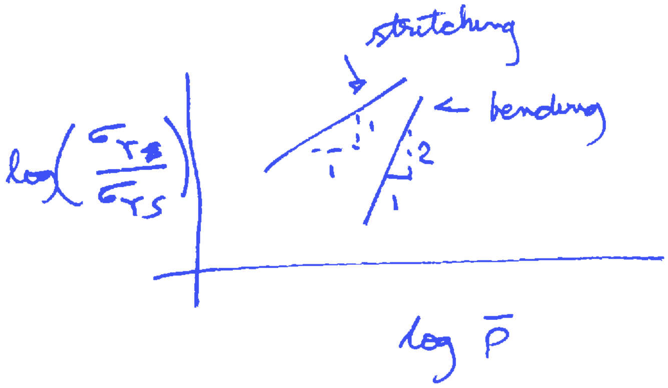
Hill equation is hyperbolic while (1) is quadratic in V. However parameters in (1) can be adjusted to give reasonable agreement with Hill.

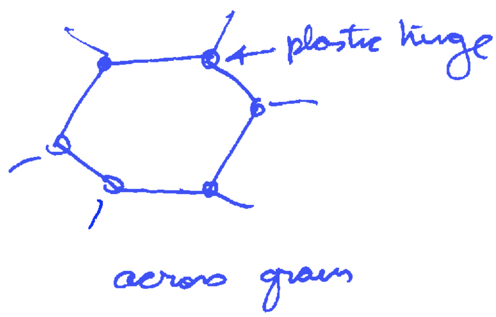
3 (a) Spectrin elastomeric fibres are arranged in a fully triangulated manner on the outer membrane of the red blood cell. Persistence length of spectrin is << triangulated cell size so spectrin behaves as a rubber with a high lock-up strain



When the fibres are pulled straight they have a high modulus & strength

(b) Wood resembles a hexagonal honeycomb structure - along the grains it is a stretching structure while across grains it is a bending structure.





(c) Proteins are transported ~~a~~ from centrosome by motor proteins such as kinesins along micro-tubules. These motors are powered by ATP.

Diffusion by ~~condn~~ contrast is a passive process driven by concentration gradients & slow for large molecules.

(d) Plants have chloroplasts for capturing light energy & for storing this energy as glucose (& starches). They consume the glucose in their mitochondria & generate ATP.

Animal cells do not have chloroplasts but do have mitochondria. Animal ~~cells~~ consume starches (& glucose) & use this to directly convert ADP to ATP in their mitochondria.

4)

(a) Thermal activation of molecules leads to a progressive loss of correlation along the length of the molecule. The ~~pe~~ persistance length ξ is the length over which correlation is lost by thermal activation.

$$kT \sim \frac{1}{2} EI \phi^2/\xi$$

↑ end-to-end rotation

$$\xi = \frac{EI\phi^2}{2kT}$$

Let $\phi \approx 1$. Stiff molecules such as micro-tubules have a persistance length which is much longer than cell size. Consequently they remain straight. Spectrin is a natural elastomer & its short persistance length causes it to coil randomly.

(b) The cellulose cell wall in plants gives it structural stiffness. Animal cells do not have this & rely upon the internal skeleton, the cytoskeleton for structural support.

(c) Myofibrils are the functional units of skeletal muscles. Each ~~of~~ myofibril is segmented into numerous individual contractile units called sarcomeres about $2.5\ \mu m$ long. The sarcomere is made up primarily of 2 types of parallel filaments designated thin & thick filaments. Viewed end-on six thin filaments are positioned around a central thick filament. Viewed along its length there are regions where thin or thick filaments are overlapping or non-overlapping. At the end of the sarcomere is a region called the Z-disc where thin filaments are anchored.

(d) Glucose transport across the cell membrane occurs by carrier mediated transport ~~by~~ via uniports. The carrier molecule alternatively exposes its binding site first on one side & then the other side of the membrane capturing & releasing the glucose. Insulin affects the binding affinity of the glucose to the carriers & thus controls the flux rate of the glucose.

Q1 Modulus of a cellular solid

Unpopular question. Most students calculated the relative density and modulus correctly but struggled on calculating the strength in the bending-dominated direction.

Q2 Huxley cross-bridge model

Generally the students had no problems with this question with some candidates making errors in calculating the optimal velocity for maximum power.

Q3 Qualitative question with regards explaining some observations including structure of cell membrane, anisotropy of wood.

A popular question that was well attempted by most students. The main omissions included students not discussing the scaling of the strength of wood in the different directions and, mixing up which molecular motors are involved in cellular transport.

Q4 Qualitative question on muscles and ion pumps

Generally well attempted other than part (a) on explaining the concept of persistence length and some students could not explain the role of insulin in glucose transport in part (d).