1. a) The maximum fuel temperature will occur at the fuel centre. The general expression for the temperature as a function of radius (r) is:

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right)+\frac{q'''}{k}=0 \qquad (1)$$

Where k is the thermal conductivity, and q''' is the volumetric heat rate.

BC for Case 1: $\left.\frac{dT}{dr}\right|_{r=r_{in}}=0,\ T(r_{out})=T_{out}$

BC for Case 2: $\left.\frac{dT}{dr}\right|_{r=0}=0,\ T(r_{out})=T_{out}$

The integration of equation (1) gives: $kr\frac{dT}{dr}+q'''\frac{r^2}{2}+C_1=0$

For solid fuel pellet (case 2), the maximum temperature T_{Max} is at $r=0$ and hence $C_1=0$ and second integration yields: $T_{max}-T_{out}=\frac{q'''}{4k}{r_{out}}^2$ or $T_{max}-T_{out}=\frac{q'}{4k}$

For the case 1, the heat flux is zero at $r=r_{in}$: $\frac{dT}{dr}=0$, thus leading to: $C_1=-\frac{1}{2}q'''r_{in}^2$

Second integration for case 1: $T_{max}-T_{out}=\int_{r_{in}}^{r_{out}}\frac{q'''}{k}\left(\frac{r}{2}+\frac{C_1}{r}\right)dr$

$T_{max}-T_{out}=\frac{q'''}{4k}\left[(r_{out}^2-r_{in}^2)+2r_{in}^2\,ln\left(\frac{r_{in}}{r_{out}}\right)\right]$

Re-arranging: $T_{max}-T_{out}=\frac{q'''r_{out}^2}{4k}\left\{\left[\left(\frac{r_{in}}{r_{out}}\right)^2-1\right]+2\left(\frac{r_{in}}{r_{out}}\right)^2 ln\left(\frac{r_{in}}{r_{out}}\right)\right\}$

The relationship between q' and q''', at: $r=r_{in}$ becomes:

$q'=\pi\left(r_{out}^2-r_{in}^2\right)q'''$ or $q'''r_{out}^2=\frac{q'}{\pi}\left[1-\left(\frac{r_{in}}{r_{out}}\right)^2\right]$

$$T_{max}-T_{out}=\frac{q'}{4\pi}\left(1-ln\left(\frac{r_{out}}{r_{in}}\right)\frac{2}{\left(\frac{r_{out}}{r_{in}}\right)^2-1}\right)$$

In case 1, the inner radius is calculated from 10% of the original fuel pellet volume, thus the inner radius of the annular pin is: $r_{in}=\sqrt{0.1}r_{out}$

In case 1, the maximum temperature is: $T_{max}=0.74\frac{q'}{4\pi k_{orig}}+T_{out}$

In case 2: $T_{max}=0.67\frac{q'}{4\pi k_{orig}}+T_{out}$

Thus, assuming that the outer temperature is the same in both cases, the greater temperature would be in case 1.

b)
In both cases H/HM will be increased. Since typical core is under-moderated, this will likely increase reactivity and discharge burnup.
Also, Be is a good moderator, and will improve moderation even further. Designers have to be careful not to over-moderate the core and end up with positive MTC.
Both options reduce fuel temperature. Therefore, reactivity, burnup and cycle length will be increased due to smaller Doppler Effect. Lower fuel temperature will also slow down swelling and fission gas release offering possibility of achieving higher burnup.
On the other hand, both options are displacing fuel. So even though the burnup is likely to increase, the cycle length may actually decrease despite gain in reactivity due to Doppler and higher H/HM. Higher H/HM and lower temperature may or may not be sufficient to compensate for the loss of fuel loading.

c) Lower fuel temperature enables higher safety margins – greater margin to melting and lower stored energy to remove in accidents. It can alternatively allow higher operating power for the same safety margins. It can also allow higher burnup due to better fuel behaviour (lower swelling and fission gas release).

d) Other options to reduce temperature:

Change fuel material from oxide (ceramic with poor conductivity) to nitride or metallic with high conductivity. May reduce melting temperature (metal), worse fuel performance, chemical reaction with cladding or water.

Increase flow rate of the coolant – higher heat transfer coefficient, lower film ΔT and therefore fuel temperature. But higher pressure drop, pumping power, fuel vibrations.

Changing fuel shape and increase its heat transfer (surface) area – cross-shaped or annular internally and externally cooled or adding fins. Could increase pumping power, more expensive manufacturing.

Increase thermal conductivity of cladding and/or gap – new cladding material or liquid metal bond in the gap. Compatibility with fuel and water, takes up space for fission gases – need larger gas plenum.

All options will require lengthy and expensive qualification and licensing process.

2. a) Assume the coolant is approximately ideal gas

$$p = \rho RT \rightarrow \rho = \frac{p}{RT}$$

If the pressure is doubled - the new density will be $\rho_{new} = 2\rho_{old}$

The change in mass flow rate can be obtained by using the fact that pumping power remains fixed. Pumping power is given by

$$W_p = \frac{\Delta p_{new}\dot{m}_{new}}{\rho_{new}} = \frac{\Delta p_{old}\dot{m}_{old}}{\rho_{old}} \quad \therefore \quad \dot{m}_{new} = \dot{m}_{old}\frac{\Delta p_{old}}{\Delta p_{new}}\frac{\rho_{new}}{\rho_{old}}$$

$\Delta p = \frac{\rho v^2}{2} f \frac{L}{D}$; while flow velocity and mass flow rate are related as: $\dot{m} = \rho v A$

Friction factor from Nuclear Databook: $f = k\,Re^{-0.2} = k\left(\frac{\rho v D}{\mu}\right)^{-0.2}$

Substituting f into the expression for Δp and noting that the geometry (L, A and D) is fixed and noting that viscosity only weakly depend on pressure:

$$\frac{\Delta p_{old}}{\Delta p_{new}} = \frac{{\rho_{old}}^{0.8}}{{\rho_{new}}^{0.8}}\frac{{v_{old}}^{1.8}}{{v_{new}}^{1.8}} = \frac{{\rho_{old}}^{0.8}}{{\rho_{new}}^{0.8}}\frac{\left(\frac{\dot{m}_{old}}{\rho_{old}}\right)^{1.8}}{\left(\frac{\dot{m}_{new}}{\rho_{new}}\right)^{1.8}} = \frac{\rho_{new}}{\rho_{old}}\left(\frac{\dot{m}_{old}}{\dot{m}_{new}}\right)^{1.8}$$

Substituting the ratio of Δp's to the first expression, we obtain:

$$\dot{m}_{new} = 1.64\,\dot{m}_{old}$$

For the same given temperature difference, we obtain the power uprate:

$$\Delta T = \frac{Q_{new}}{\dot{m}_{new}\,C_p} = \frac{Q_{old}}{\dot{m}_{old}\,C_p} \qquad \therefore\ Q_{new} = Q_{old}\frac{\dot{m}_{new}}{\dot{m}_{old}} = 1.64\,Q_{old}$$

b) The change in ΔT_{film} will be due to a change in heat transfer coefficient which can be estimated from Dittus-Boelter correlation from CUED Thermo data book:

$$Nu = 0.023 Re_D^{0.8} Pr^{0.4}$$

$Re = \frac{\rho v D}{\mu}$ and will be affected by the change in coolant density and velocity.

$Pr = \frac{\mu C_p}{\lambda}$ is a weak function of gas pressure and its change can be neglected.

Thus,

$$\frac{Nu_{new}}{Nu_{old}} = \frac{h_{new}}{h_{old}} = \frac{{\rho_{new}}^{0.8}}{{\rho_{old}}^{0.8}}\frac{{v_{new}}^{0.8}}{{v_{old}}^{0.8}} = \frac{{\rho_{new}}^{0.8}}{{\rho_{old}}^{0.8}}\frac{\left(\frac{\dot{m}_{new}}{\rho_{new}}\right)^{0.8}}{\left(\frac{\dot{m}_{old}}{\rho_{old}}\right)^{0.8}} = \left(\frac{\dot{m}_{new}}{\dot{m}_{old}}\right)^{0.8} = (1.64)^{0.8} = 1.49$$

From the new heat balance, we can estimate the new film ΔT

$$q''_{new} = h_{new}\,\Delta T_{new}$$

$$\frac{q''_{new}}{q''_{old}} = \frac{h_{new}\,\Delta T_{new}}{h_{old}\,\Delta T_{old}} \qquad \therefore \qquad \frac{\Delta T_{new}}{\Delta T_{old}} = \frac{h_{old}\,q''_{new}}{h_{new}\,q''_{old}} = \frac{1.64}{1.49} = 1.1$$

c) High pressure drop implies high viscosity and/or flow rate. Typically, high viscosity coolants would also have higher heat capacity. High heat capacity and flow rate will allow reducing the coolant temperature rise across the core. Low ΔT across the core would lead to higher temperature of heat addition to the power cycle heat exchanger and therefore higher power conversion efficiency, although probably at the expense of higher pumping power requirements due to the high core Δp.

A core may have high Δp also because of small coolant flow area. This may be required to design compact, high power density cores (e.g. fast reactors or submarines).

d) Low Δp core would have an advantage of more easily achievable natural circulation under normal conditions (e.g. in ESBWR) or in accidents to allow passive decay heat removal. In some cases, coolant flow velocity may need to be kept deliberately slow, for example to avoid corrosion/erosion of structural materials (lead-cooled reactors) or prevent entrainment of water droplets into steam flow (some advanced BWR designs). Low Δp also means less pumping requirements. Although, gas cooled reactors have a low pressure drop but high coolant velocities, which leads to high pumping power. In LWR LOCA, having low Δp core means it is easier to reflood.

3.

a) The tank is part of passive containment cooling system (PCCS). Its primary objective is to reduce the containment temperature and pressure following LOCA so that they don't exceed the design limits. It can also provide the ultimate heat sink in other accident conditions. The steel containment vessel serves as a heat transfer surface that conducts the heat from the inside of the containment and transfers it to the atmosphere for 72 hours. The hot steam inside the containment is condensed on the walls of the containment and flows under gravity into RWIST and from there back into the core, where decay heat evaporates the water turning it into steam which escapes back into the containment. The water storage tank is drained under gravity and sprayed on the containment vessel on the outside which cools the containment walls. When the tank is emptied, the decay heat of the core is sufficiently reduced for natural circulation of air along the outside surface of the containment to be sufficient to remove the heat and keep the containment pressure within the design limits.

b)

Mass of primary water: M = V * ρ(300 C, 150 bar) = 300 * 725.6 = 217680

Stored energy in primary system (from steam tables) = [u(300 C, 150 bar) - u(25C, 1bar)]*M = (1317.6 – 104.8) kJ/kg * 217680 kg = 264 GJ

Integral of decay heat over 72 h assuming conservatively infinite irradiation time =

= 3000 $\int_0^{72h} 0.066\, t^{-0.2}\, dt$ = 5302 GJ

The total energy to be removed from the containment = 5566 GJ

This has to be absorbed by sensible and latent heat of the PCCS water. Assume the LOCA protection systems work and core reflood is successful so that there is no fuel damage and hydrogen release.

Energy to be absorbed = m [C_p (100 C – 25 C) + h_{fg}] = 5566 GJ

m = $5566*10^9$ / [4200*75 + $2257.4*10^3$] = 2164 ton

c) Energy sources - decay heat, delayed fission, stored energy in primary and secondary coolant, exothermic chemical reactions (Zr + steam, hydrogen, CO). Heat sinks – containment walls, dedicated residual heat removal system or containment cooling system circulating water through containment sprays and/or core and through external heat exchanger, heat removal through steam generators, containment venting through filters. Inside containment there are temporary heat sinks: IRWST, ice condensers, heat capacity of structures and equipment within the containment.

d) Decay heat equation from Nuclear Databook: $P_d(t) = 0.066P_0[t^{-0.2} - (t_0 + t)^{-0.2}]$

For 2-batch refuelling scheme and assuming that the batches have equal power share, the decay power of the core will be the sum of decay power produced by each batch. However, at the end of equilibrium cycle, the first batch was irradiated for 18 months, while the second batch only for twice that time, i.e. 3 years. Therefore, the decay heat of 2-batch core is:

$$P_d(t) = 0.066P_{0,1}\left[t^{-0.2} - \left(t_{0,1} + t\right)^{-0.2}\right] + 0.066P_{0,2}\left[t^{-0.2} - \left(t_{0,2} + t\right)^{-0.2}\right]$$

$$= 0.066 \cdot \frac{3000}{2}[3600^{-0.2} - (365 \cdot 3 \cdot 24 \cdot 3600 + 3600)^{-0.2}] +$$

$$0.066 \cdot \frac{3000}{2}[3600^{-0.2} - (365 \cdot 1.5 \cdot 24 \cdot 3600 + 3600)^{-0.2}] = \mathbf{33.1\ MW_{th}}$$

For continuous refuelling case, the batches are infinitely small and each was loaded into the core at t_0 which would vary between 0 (the most recently loaded assembly) to T_0 = 3y (the oldest assembly in the core). The fraction of power (contribution to total) of each differential batch is $P_0 \frac{dt_0}{T_0}\ 0.066\ [t^{-0.2} - (t_0 + t)^{-0.2}]$. Therefore, total decay power 1h after shutdown is:

$$P_d(1h) = \frac{P_0}{T_0}\int_0^{3y} 0.066\ [t^{-0.2} - (t_0 + t)^{-0.2}]\, dt_0 = \mathbf{32.2\ MW_{th}}$$

4.

a) Negative reactivity feedbacks help reduce power peaking and make the power distribution more uniform. Increase in local power will result in an increase in local temperatures. If temperature reactivity feedback is negative, neutron balance in regions with high power (and thus temperature) will be perturbed reducing the local reactivity and consequently power so that the power distribution in a core with strong temperature feedbacks will be more uniform. The power distribution will also become progressively more uniform with the core heat up.

Similar logic can be applied also to Xe feedback as Xe concentration is related to local neutron flux and thus power. Therefore, in a fresh Xe-free core, the power distribution will be less uniform than in a slightly depleted core in which Xe has reached its equilibrium concentration. In such a case, high power regions will have higher Xe concentration and thus lower local reactivity than low power regions.

b) Negative reactivity coefficients will result in a large reactivity decrement upon core heat up. In order to keep the core critical for a reasonable cycle length, the initial fissile inventory needs to be higher than necessary for criticality. For the same initial fuel enrichment, the core with strongly negative reactivity coefficients will have lower reactivity at HFP condition than the same core with less negative coefficients. This reactivity difference will have to be compensated by either higher fuel enrichment or more frequent refuelling or shorter operating cycle with clear economic penalty.

In addition, strongly negative coefficients (or not sufficiently negative) may result in more cumbersome and thus more expensive safety case and/or introduction of additional reactivity control mechanisms (eg in CANDU reactors) to compensate for undesirable magnitude or sign of reactivity feedback coefficients.

c)

- Change fuel-to-moderator volume ratio (or H/HM) will shift the neutron spectrum so that relative impact of absorption and moderation will be altered.
- Change coolant temperature by changing coolant ΔT core, flow rate or T-inlet. This will result in different coolant density and thus H/HM.
- Adding boron to water will increase the absorption effect while leave the moderation effect unchanged.
- Changing fuel composition/enrichment. Upon change in moderator density due to expansion, the fuel fissile and fertile nuclides will either increase or decrease absorption and/or fission.

- Using burnable poisons with strong absorption in a particular region of neutron energy spectrum. For example, Gd-157 isotope is a very strong thermal neutrons absorber. Therefore, upon moderator thermal expansion, the spectrum hardening will result in less absorptions in Gd and therefore positive contribution to MTC.
- Changing the core or reflector size, this will change the core leakage. As the moderator density reduces, the core becomes more transparent to neutrons and leakage will increase. Deliberately "leaky" core will have more negative MTC.

d)

FTC: HZP to HFP, no Xe, no change in moderator temperature

FTC = $\Delta\rho/\Delta T$ = (2000 – 4000)/(900 – 300) = –3.33 pcm/C

Xe reactivity worth: HFP, no Xe to HFP, Xe-eq

$\Delta\rho_{Xe}$ =6000 – 4000 = –2000 pcm

MTC: CZP to HZP (change in both fuel and moderator temperature)

$\Delta\rho$ = FTC*ΔT_{fuel} + MTC*ΔT_{mod} = –2000 pcm

$\Delta\rho$ = –3.33*(300 – 80) + MTC*(300 – 80) = –2000 pcm

MTC = (–2000 + 733)/220 = – 5.7 pcm/C

Q1 Temperature distribution in nuclear fuel

A popular question attempted by all candidates. This was surprising because it required somewhat lengthy derivation, which some of the candidates were nevertheless able to successfully work out. The second part of the question was descriptive and required understanding of heat transfer fundamentals and implications of temperature in a broader reactor design scope of issues. This part was answered with variable degree of success.

Q2 Coolant flow in reactor design

This was the least popular question despite being conceptually simple. It tested basic understanding of reactor coolant flow and its effects on heat transfer, pumping power required and broader implications to reactor core design. Most candidates were able to identify the relevant phenomena and the relations between them. However, no one was able to construct the “full story” which led to the correct answer.

Q3 Decay heat removal in accidents

This was another popular question attempted by all candidates. It required familiarity and understanding of operating principles of passive decay heat removal systems in modern reactor designs. Most candidates could identify and describe these features. However, many found it difficult to perform a simple energy balance between the sources and sinks of heat in a Loss of Coolant Accident, creating a distribution of marks. Another common difficulty was calculating decay heat for a core with continuous refuelling.

Q4 Effects of reactivity feedback coefficients

This was the easiest and least labour-intensive question. It is surprising that it was not more popular. It was mostly descriptive and required only a simple calculation which most candidates performed correctly. Description of reactivity feedbacks and their importance to reactor design required deeper understanding of reactor engineering and its relation to reactor physics. This was achieved to a varied degree of success by the candidates and created a distribution of marks.