**Question 1.**

a) Interval analysis

| **Interval** | | **Sum FCP** | | **Heat Load** | | **Total Heat Flows** | |
|---|---|---|---|---|---|---|---|
| **Tmin (C)** | Tmax (C) | Heat Sources (kW/C) | Heat Sinks (kW/C) | Heat Sources kW | Heat Sinks (kW) | Within Interval (kW) | Cumulative |
| | | | | | | | |
| 110 | 140 | 1 | 0 | -30 | 0 | -30 | -30 |
| 90 | 110 | 1 | 2 | -20 | 40 | 20 | -10 |
| 85 | 90 | 1 | 3 | -5 | 15 | 10 | 0 |
| 70 | 85 | 2 | 3 | -30 | 45 | 15 | 15 |
| 40 | 70 | 1 | 0 | -30 | 0 | -30 | -15 |

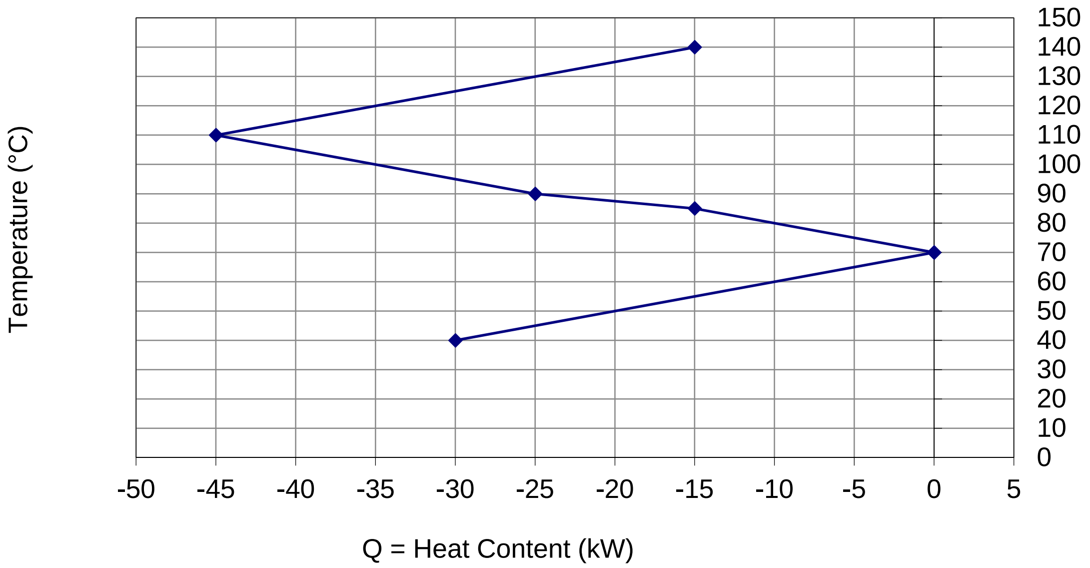
The largest deficit in cumulative heat flow occurs between the 70-85 and 40 to 70 interval. Adding 15 kW to the energy cascade at the top of the cascade makes it feasible. Therefore the Pinch occurs at 70 °C and 15 KW of external heat are needed. The energy cascade looks like

| **Interval** | | **Total Heat Flows** | |
|---|---|---|---|
| **Tmin** | Tmax | Within Interval | Cumulative |
| | | | -15 |
| 110 | 140 | -30 | -45 |
| 90 | 110 | 20 | -25 |
| 85 | 90 | 10 | -15 |
| 70 | 85 | 15 | 0 |
| 40 | 70 | -30 | -30 |

And 30 kW of cooling are needed.

**[6 marks]**

The Grand composite curve is shown below.



**[1 mark]**

i) The external heat needs to be provided between 70 and 85 °C, the heat capacity is this interval is unity (net heat sink). Therefore the minimum exergy of the heat which must be supplied is

$$E_Q = \int \left(1 - \frac{T_0}{T}\right) d\dot{Q}$$

$$= \int_{70}^{85} \left(1 - \frac{T_0}{T}\right) d\dot{Q}$$

$$= \int_{70}^{85} \left(1 - \frac{T_0}{T}\right) 1.dT$$

$$= (85 - 70) - 298 \times \ln\left(\frac{85 + 273}{70 + 273}\right) = 2.24\ kW$$

**[2 marks]**

ii) The interval between 70 and 85 °C can be used to as a heat sink for a heat engine. The work that could be produced by this engine would be,

$$dW = \eta dQ_h = \eta(dQ_c + dW)$$

$$dW = \frac{\eta}{1 - \eta} dQ_c$$

**[1 mark]**

Using the most efficient Carnot Engine.

$$dW = \frac{1 - T/T_h}{T/T_h} dQ_c$$

$$W = \int \left(\frac{T_h}{T} - 1\right) dQ_c$$

$$W = \int_{70+273}^{85+273} \left(\frac{T_h}{T} - 1\right).1.dT$$

$$W = (70 - 85) + 1000 \times \ln\left(\frac{85 + 273}{70 + 273}\right)$$

$$W = 27.8\,kW$$

**[2 marks]**

c) It is wrong to assert that the heat rejected from a power station is "waste" heat. Its temperature in the ideal case is so low, that its exergy value is very small. The factory owner wants heat above a certain temperature, not at the environment temperature.

One measure of the value of the heat is the exergy of the heat (Calculated previously). This represents the opportunity cost of not using this heat to generate power.

One measure therefore might be something like Electricity price * Exergy. However, this is still a little unfair to the power station, since the opportunity cost to the power station differs from the exergy of the heat it releases. The powerstation is not a perfect Carnot Cycle AND it may deliver heat at a higher temperature than required. There is sufficient information available to determine the loss of revenue from the power station.

The LP turbine in the power station, takes the steam from 1 bar and 300°C (h = 3000 kJ/kg from chart) and drops the pressure isentropically to 0.02 bar (h = 2400 kJ/kg from chart). Therefore turbine work is

$$W_t = 3000 - 2400 = 600\frac{kJ}{kg}of\ steam$$

If not used in the turbine, the steam goes from 1 bar and 300°C (h = 3000 kJ/kg) to wet saturated at 1 bar (h = 908.5 from saturated tables). The heat supplied is

$$Q = 3000 - 419 = 2581\frac{kJ}{kg}of\ steam$$

Therefore the steam required is $\frac{15}{2581} = 5.83 \times 10^{-3}$ kg/s giving a lost work of $\frac{15}{2581} * 600 = 3.49\ kW$. With an electricity price of £0.2 per kwh this means a lost revenue of $7.28 * 0.2 =$£0.70 per hour, or £0.46 per kWh of heat delivered.

This is actually quite a bit of lost work, much more than the exergy value of the heat required by the user. Partly this arises from the fact that the heat is supplied at 100 °C (with some even hotter at 300 to 100 °C), when at most it needs to be 85 °C (though in practice you need some temperature difference to give heat transfer). The lost revenue per kwh hour is also higher than the natural gas price, so it would appear not to make economic sense to do combined heat and power. This counter intuitive result comes in part from the fact that the ratio of the exergy value of methane to that of electricity is not the same as the ratio of the price of methane to price of electricity. The power station has a huge amount of capital invested in it, which must be recovered, and the price of electricity is not the same as the cost of electricity. From an energy and CO2 perspective it would still make sense to do the combined heat and power because the power station loses 3.49 kW of work, but 15 kW of natural gas heating is avoided. The typical efficiency of a natural gas CCGT is more than 50 % so the 15 kW of natural gas could be used in another station to produce > 7.5 kw of electricity. Overall there will have been a gain in the electricity produced for the same fuel burn. Sometimes economics does not align with sustainability. The gas price given is so low that there is an incentive to waste methane. Burning more methane generates more revenue than reducing the methane burned and increasing efficiency. As noted in lecture, the price of natural capital (natural gas) often bears no resemblance to its "value".

**[4 marks for discussion+4 for supporting calcs]**

*Examiners Comments: This question was attempted by almost all candidates. Most got full marks on part (a) and were able to work out the heat duties. Sketching of the grand composite curves was a bit hit and miss in (b) and not all candidates were able to use the composite curves to work out at what temperatures heat was added. Some candidates simply put the heat in at the highest temperature. Part (c) was relatively well answered with most candidates able to get about half the marks, by either writing a reasonable discussion or doing some calculations; full marks were answered by those who did both.*

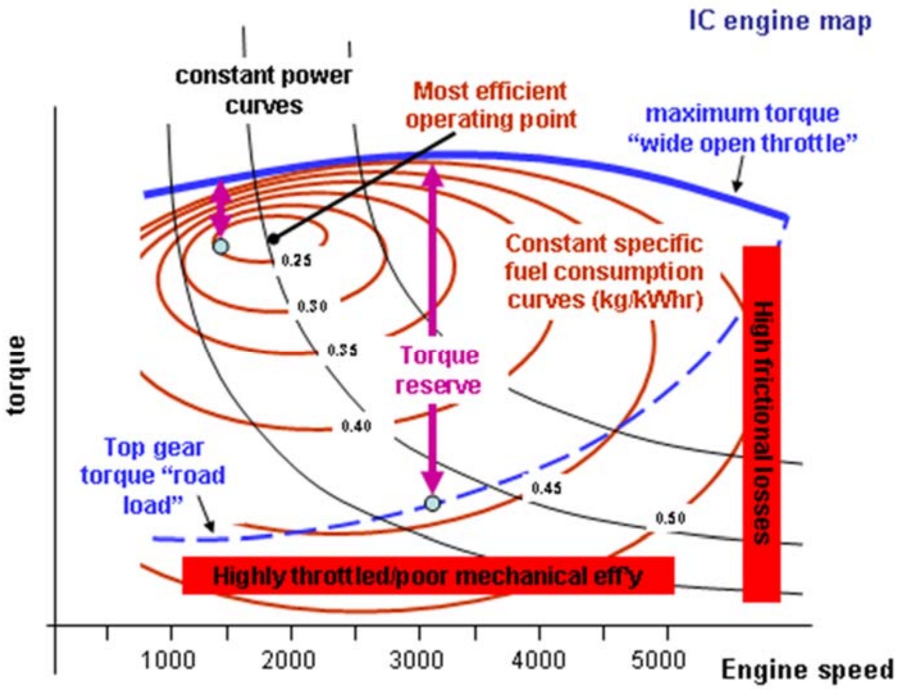
**Question 2**

**a)** Most cars use petrol engines, whilst larger vehicles will use a diesel engine. Petrol engines are constrained in size by the need to ensure that the fuel does not ignite spontaneously before the flame from the spark can sweep the cylinder. Spontaneous ignition causes knocking and will damage the engine. This size is determined by the induction time for the fuel air mix, and the speed of the flame front. In addition, petrol engines have an additional drawback, which is the need to fix the air to fuel to make sure the fuel can burn. Air is drawn into the engine by the down stroke, fixing the volume of air in the cylinder. Thus, to change the power output, when the amount of fuel changes, the only way to change the amount of air is to throttle the intake. Thus an engine is always throttled (causing irreversibility) when the engine is not running at full load. For the diesel engine, it is possible to simply reduce the amount of fuel added. The diesel engine does not suffer from knocking this as fuel is consumed throughout the cylinder homogenously as it evaporates from small droplets. Thus, diesel engines can have much larger cylinders and compression ratio. In the marine industry very large engines are used, with very high compression ratios and very low surface area to volume ratios (which limits heat losses) allowing efficiencies of up to 60 %. Smaller engines are not able to reach such high efficiencies, but can still be considerably better than the previous generation of engines. Another factor in high efficiency of the large diesel engines is that they are not required to produce sudden bursts of power, as required in a car engine. This need for a "torque reserve" severely limits the efficiency of car and truck engines. The operating map for a typical engine is shown below.

Large diesel engines can achieve a large power with lower speed as the work per stroke is higher. Car engines which are constrained by mass, achieve high powers by running the engine faster but having less work per stroke. The large compression ratios are only possible in the diesel engines, as petrol engines will experience knocking. Slower engines have lower frictional losses, and higher compression ratios increase the thermodynamic efficiency.

The maximum torque would correspond to the case where the throttle is fully open (or in a diesel no more fuel can be added because there is insufficient oxygen). The contours of sfc show there is an optimum operating point, close to the maximum torque line. The only way to provide the torque reserve in a traditional petrol engines vehicle is to gear the drive train so that when cruising the engine operates at low torque. This mean that when power is required the engine can rapidly increase torque by simply adding fuel (and reducing the throttling). A car with a large torque reserve is will feel very "powerful" and is pleasant to drive. Maintaining the torque reserve means that engines are therefore operating far below their optimum load. If the torque reserve can be removed, the efficiency of even small engines could increase dramatically.

**[8 marks]**



**b)** A downsized engine whose operating point under normal cruising conditions is close to the optimum for fuel consumption, will operate very efficiently, but will not be able to respond to sudden demands for power. The power of both petrol and diesel engines is largely determined by how much oxygen is present in the cylinder before combustion is initiated. Thus, to provide extra power, more air must be fed to the cylinders. Superchargers and turbo chargers are devices which compress the air going into the cylinder on the intake stroke, so that rather than being sucked in at sub atmospheric pressure, the air is at a higher pressure. For a fixed volume cylinder this means more oxygen in the cylinder, more fuel can be injected and more power extracted. A turbocharged small engine could therefore have similar characteristics to a larger conventional engine, but with the added advantage that when the turbocharging is inactive the engine is "small" and better matched to cruising. The only disadvantage is the delay in activing at the compression device, which can lead to turbo lag. The torque reserve is therefore provided by the engine being able to suddenly become larger.

**[7 marks]**

**C)** Fully electric vehicles use no fossil fuels directly, being power by stored electricity in batteries. The current limitations are battery technology (which limits range, e.g. to 1oo miles) **and** the carbon intensity of the electricity. Hybrid drive train use batteries and motors to provide the torque reserve, allowing the engine to be sized for cruise. This leads to a higher efficiency, but also extra cost and complexity as two power systems are needed. The hybrid uses electricity generated from the engine to recharge a relatively small set of batteries, so the car is still fully fuelled by petrol. This does however that the weight of batteries needed is much smaller than the fully electric EV.

***[5 marks]***

*Examiners Comments: Generally, well answered. Most gave a reasonable discussion of all the points requested. Almost all were able to reproduce the relevant engine performance maps.*

**Question 3**

a) i) Allocation problems arise when a system produces more than one product and the burdens associated with each product individually have to be allocated. Allocation by substitution takes one of the co-products and asks what environmental burden can be avoided, by using the co-product to stop someone else in the market place producing the same co-product. This requires either a homogenous market, in which the average value burden of the co-product is known uniquely, and that the co-product is actually currently a product. LCA does not take into account the feed-back caused by an activity from the market, so where the co-product itself takes a very large market share and distorts the average the analysis can be somewhat questionable. Using averages can be avoided if the co-product is actually used to prevent an activity directly. In this case the answer is the same as expanding the system boundary to include the system which is using the co-product. Alternatively, a reference system which is currently making the co-product can be subtracted from the system of interest.

**[2 marks for describing the allocation method, 2 for discussion of the limitations and reference systems]**

ii) Marginal allocation determines how much extra environmental burden there is per extra unit of production of a co-product. This either requires a physical model of the burdens to exist or the ability to vary independently the proportions of co-products so that a measurement can be made. In some cases, e.g. biofuels, there is limited scope to do this as the proportions of co-products are fixed, e.g. by biology. The waste heat is produced in proportion with the amount of B so there is no opportunity to vary B and the waste heat independently

**[2 marks]**

b) The total environmental burden to produce $n_a$ and $n_b$ items in a total production run of 100 units is

$$E = 0.1 n_a n_b + n_a + 2 \times n_b + 100$$

Where $n_a + n_b = 100$

If allocation by substitution is used, the $0.5 n_b$ GJ of waste heat can be used else where, and a credit applied to the overall burden.

$$E = 0.1 n_a n_b + n_a + 2 \times n_b + 100 - 0.5\, n_b$$

$$E = 0.1 n_a n_b + n_a + 1.5 \times n_b + 100$$

**[2 marks for writing down the burden]**

i)

***[Examiners note: This question can be interpreted in several ways, all of which are acceptable. Strictly speaking, marginal allocation would require the amounts of A and B to be varied independently, i.e. the constraint that the total of A and B must add up to 100 should be ignored. Alternatively, the change in burden when 1 B is produced (so that 99 A are produced) can be calculated, and the burden of B taken to be this – the burden of one unit of A, if A were produced alone (i.e. using a reference system of factory that only produces A). This section was marked***

***generously, and most marks were lost not because of the allocation but because of student's failure to correctly add up the burdens]***

Method 1: ignore the constraint and look at marginal increase in burden when factory produces A, 1 unit of B. Could differentiate, but as this is discrete system best to consider the two cases

Base case: 100 Units of A : $E = 0.1 \times 100 \times 0 + 100 + 1.5 \times 0 + 100 = 200\ GJ$

Base case: 100 Units of A+1 Unit of B : $E = 0.1 \times 100 \times 1 + 100 + 1.5 \times 1 + 100 = 211.5\ GJ$

Therefore the burden of embodied energy of B is $211.5 - 200 = 11.5GJ$

**[3 marks]**

Alternatively:

Method 2: Constrain production run

Base case: 100 Units of A : $E = 0.1 \times 100 \times 0 + 100 + 1.5 \times 0 + 100 = 200\ GJ$

Making one unit of B: 99 Units of A + 1 of B: $E = 0.1 \times 99 \times 1 + 99 + 1.5 \times 1 + 100 = 210.4\ GJ$

The marginal burden is 10.4 GJ, but this is for producing 1 unit of B and one less unit of A. Using the base case as the reference system, gives the burden of A as 2 GJ. Therefore, the burden for the unit of B is 10.4 +2 = 12.4 GJ.

**[3 marks]**

ii) Reference case of producing 100 units of A gives E = 200 GJ per 100 units of A.

50 units of A and 50 of B: E = $E = 0.1 \times 50 \times 50 + 50 + 1.5 \times 50 + 100 = 475\ GJ$

Therefore the burden per unit of B is $(475 - 200/2)/50 = 7.5\ GJ$

**[3 marks]**

c) The issue here is the use of averages and whether or not the factory does actually use zero primary energy. It depends on what the electricity form the panels displaces. There are a number of different (and equally valid view points)

The factory is connected to the grid so the electricity it uses is not fully renewable. Instead you should use the average grid burden for the energy delivered. Presumably this is the primary energy input burden given earlier. If the energy delivered by the solar panels displaces the average grid electricity then it will be ok to say the factory uses no net primary energy as its input.

If the factory actually used the solar panels to supply its electricity, and did not draw power from the grid, then you could perhaps argue that there is no net energy burden by the factory. However, at the system level this might not be valid, since if solar electricity could displace the other more polluting forms of electricity on the grid there would be an opportunity cost in using the electricity in the factory.

If there is no variation in time, i.e. the electricity burden on the grid does not vary with time and the solar panels have no time dependence, then both the cases above would average to give no-net burden.

However, the mix of energy sources on the grid change with time of day and also time of year. Similarly, the solar panels will only produce electricity at certain times of day and will be more effective in summer than winter. The average grid mix tends to become more carbon intensive at times of high demand, which in the UK corresponds to winter in the evening. In summer, on a nice windy day, the grid mix will be less carbon intensive and the energy produced by the solar panels will be displacing less carbon. Thus, it depends on when the factory uses electricity, and when the solar panels generate electricity. Lifecycle analysis uses averages and an average can be sometimes misleading.

Electricity cannot be stored, and the grid is balanced in real time. Thus, the analysis can be complicated even further by considering the marginal power plant on the grid, rather than using the instantaneous average. When 1 GJ of solar power is fed on to the grid, elsewhere, another powerplant will have to reduce its output by an equivalent amount. Which power station actually changes its output is complicated, and is often based on complex contracts. If the system was based on spot price, the most expensive would turn off first, and the most expensive in winter would probably be a CCGT (at the moment coal is cheap). The nuclear has to run, and wind is never displaced. A similar argument would apply to the electricity used by the factory.

Generally however, the UK peaks in winter and is less carbon intensive in summer, so you might expect the solar panels to give less benefit than the average analysis suggests. If you take the marginal view, the solar will never be displacing the low carbon nuclear and wind sources on the grid, so this would increase its benefit.

LCA analysis tends to use averages, which is one of its flaws. The true picture is more complicated and there is no single right answer.

**[6 marks]**

*Examiners comments: Most were able to give describe allocation methods in part (a). Mostly the students lost marks here for not giving a fully complete answer, e.g. failing to fully discuss when the methods are appropriate. Very few noted that heat and product are produced in direct proportion so you cannot use marginal allocation. Some noted (quite correctly) that there is an ambiguity in the question regarding whether you can use marginal allocation for produce B, when the system is fully constrained with a total production run of 100 units (and were give marks for this insight). This meant there were several possible, and acceptable answers for part (b ii), and marks were awarded for all sensible answers. In fact, this ambiguity didn't seem to cause any issues, and most marks were lost by arithmetic errors or other more basic issues, e.g. for example, not doing allocation at all, or failing to properly apply a credit for the heat. Most marks for b(ii) were lost by arithmetic errors or a failure to use any reference system. In part (c) many candidates failed to notes electricity is not storable, so the time at which it is produced and used is important. Just because on average the plant used not fossil fuels doesn't mean it didn't use any fossil fuel throughout the year. The answer should depend on grid mix, the marginal plan, and with solar power what you do at night etc..*