Q1

(a)   ICRP stands for International Commission on Radiological Protection.

The work of the ICRP helps to prevent cancer and other diseases and effects associated with exposure to ionising radiation, and to protect the environment.

The relevant UK legislation is the Ionising Radiations Regulations 1999 (IRR99).   [10%]

(b)   The present system of radiological protection is based upon three main requirements:

1. No practice shall be adopted unless the risks associated with it are outweighed by the benefits;

2. All exposures shall be kept as low as reasonably achievable, economic and social factors being taken into account (ALARA);

3. Dose equivalents to individuals shall not exceed the limits specified by the Commission.   [15%]

(c)   The dose limits aim to prevent:

➢ Acute somatic effects (burns);

➢ Long-term somatic effects (cancers);

➢ Genetic effects.

UK legislation now sets a limit of 20 mSv/yr dose limit for employees, although in special circumstances 50 mSv/yr is permitted as long as the total exposure over five years does not exceed 100 mSv.

Dose limits are based on the assumptions that:

➢ No threshold dose exists and any dose received involves a proportionate risk of cancer;

➢ Doses received on separate occasions act cumulatively and linearly in the induction of cancer.   [25%]

(d)   There are three basic methods of protection: time, distance and shielding.

Time is an obvious way of reducing exposure and time limits are normally imposed to ensure that people cannot spend too long in an area of high radiation

Distance is a valuable factor since the dose due to a point source is inversely proportional to the square of the distance from the source.

Shielding uses materials such as lead to absorb or attenuate the radiation down to safe levels. [10%]

(e)

(i) $A = \lambda N$

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{5.27 \times 365 \times 86400} = 4.171 \times 10^{-9} \text{ s}^{-1}$$

$N = \dfrac{ML}{m}$ where M is the mass, m is the molar mass and L is Avogadro's number.

$$\therefore \quad N = \frac{10^{-6} \times 6.022 \times 10^{23}}{0.060} = 1.004 \times 10^{19}$$

$$\therefore \quad A = 4.171 \times 10^{-9} \times 1.004 \times 10^{19} = 4.186 \times 10^{10} \text{ Bq}$$ [10%]

(ii) The unshielded flux at 150 mm is given by:

$$\phi = \frac{A}{4\pi R^2}$$

Noting there are two γs per decay

$$\therefore \quad \phi = \frac{2 \times 4.186 \times 10^{10}}{4\pi \times (0.15)^2} = 2.961 \times 10^{11} \text{ m}^{-2}\text{s}^{-1}$$

The shield flux is given by

$$\phi' = \phi\exp(-\alpha d) = 2.961 \times 10^{11}\exp(-0.046 \times 150) = 2.984 \times 10^8 \text{ m}^{-2}\text{s}^{-1}$$

The average energy of Co-60 γs is

$$E = \tfrac{1}{2}(1.17 + 1.33) = 1.25 \text{ MeV}$$

Thus the dose in 1 hour is

$$D = \frac{1.6 \times 10^{-13}\, \Sigma\phi'Et}{\rho}$$

$$\therefore \quad D = \frac{1.6 \times 10^{-13} \times 3 \times 2.984 \times 10^8 \times 1.25 \times 3600}{10^3} = 6.445 \times 10^{-4} \text{ Gyhr}^{-1}$$

As the weighting factor for γ-rays is 1, the surface dose will therefore be 0.645 mSvhr^{-1}.

Although this is within the limits of the regulations for the transport of radioactive materials, it is in fact quite high and anyone handling such a source would soon reach the annual dose limit so more shielding would be desirable. [30%]

Assessor's Comments:

The least popular question attempted by 46% of candidates.

Although there were a number of good attempts, and it was clear that many candidates were not expecting a Health Physics question. The answers reflected this lack of preparation.

Many candidates were unable to distinguish the criteria underlying radiological protection required in part (b) and the methods of protection required in part (d).

Many candidates were unable to find the number of atoms in a sample given the mass of the sample and the atomic mass in atomic mass units.

There were no other consistent failings in the answers, although there were a surprising number of calculator errors.

Q2

(a)
$$\frac{dn}{dt} = -\nabla \cdot \underline{j} + (\eta - 1)\Sigma_a \phi + S$$

where $\underline{j} = -D\nabla\phi$

Source-free $\Rightarrow$ $S = 0$

Steady-state $\Rightarrow$ $\frac{dn}{dt} = 0$

Homogeneous $\Rightarrow$ D does not vary spatially

$$\therefore \quad -\nabla \cdot \underline{j} = -\nabla \cdot (-D\nabla\phi) = D\nabla^2\phi$$

Hence
$$D\nabla^2\phi + (\eta - 1)\Sigma_a \phi = 0 \quad \Rightarrow \quad \nabla^2\phi + (\eta - 1)\frac{\Sigma_a}{D}\phi = 0$$

For cylindrical geometry with rotational symmetry

$$\nabla^2 = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{\partial^2}{\partial z^2}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \phi}{\partial r}\right) + \frac{\partial^2 \phi}{\partial z^2} + (\eta - 1)\frac{\Sigma_a}{D}\phi = 0 \qquad\qquad [20\%]$$

(b) Let $B_m^2 = (\eta - 1)\frac{\Sigma_a}{D}$ and assume that $\phi(r,z) = F(r)Z(z)$

$$\therefore \quad \frac{Z}{r}\frac{\partial}{\partial r}\left(r\frac{\partial F}{\partial r}\right) + F\frac{\partial^2 Z}{\partial z^2} + B_m^2 FZ = 0$$

$$\therefore \quad \frac{1}{Fr}\frac{\partial}{\partial r}\left(r\frac{\partial F}{\partial r}\right) + \frac{1}{Z}\frac{\partial^2 Z}{\partial z^2} + B_m^2 = 0$$

This implies that
$$\frac{1}{Z}\frac{d^2 Z}{dz^2} + \beta^2 = 0$$

and
$$\frac{1}{Fr}\frac{d}{dr}\left(r\frac{dF}{dr}\right) + \alpha^2 = 0$$

with
$$\alpha^2 + \beta^2 = B_m^2$$

The equation in z is an SHM equation, so the general solution is
$$Z(z) = A\sin(\beta z) + C\cos(\beta z)$$

By symmetry arguments $\qquad\qquad\qquad A = 0$

With the boundary condition that $\phi = 0$ at $z = \pm\frac{H_0}{2}$, then

$$\beta\frac{H_0}{2} = \frac{\pi}{2} \quad \Rightarrow \quad \beta = \frac{\pi}{H_0}$$

The equation in r is a Bessel equation of zero order, so the general solution is
$$F(r) = PJ_0(\alpha r) + QY_0(\alpha r)$$

As $r \to 0$, $Y_0(\alpha r) \to -\infty$. This would give infinite flux at the centre of the reactor, which is physically impossible, so $Q = 0$.

The first zero of $J_0(x)$ occurs at $x = 2.405$, so with a boundary condition that $\phi = 0$ at $r = R_0$

$$\alpha R_0 = 2.405 \quad \Rightarrow \quad \alpha = \frac{2.405}{R_0}$$

The requirement that $\alpha^2 + \beta^2 = B_m^2$ thus means that

$$\left(\frac{2.405}{R_0}\right)^2 + \left(\frac{\pi}{H_0}\right)^2 = (\eta - 1)\frac{\Sigma_a}{D}$$

This is the *criticality condition*.　　　　　　　　　　　　　　　　　　　　　　　[40%]

(c)

(i)　　$\Sigma_a = \sum_i \Sigma_{ai}$

$$\Sigma_{ai} = N_i \sigma_{ai} = \frac{f_i M_i L}{m_i} \sigma_{ai}$$

where f_i is the isotopic abundance, M_i is the mass per unit volume, m_i is the molar mass and L is Avogadro's number.

$\sigma_a = \sigma_c + \sigma_f$

Thus　　　$\Sigma_a = \dfrac{0.188 \times 400 \times 6.022 \times 10^{23}}{0.235} \times (107 + 580) \times 10^{-28}$　　(^{235}U)

$$+ \frac{0.812 \times 400 \times 6.022 \times 10^{23}}{0.238} \times (2.75 + 0) \times 10^{-28} \quad (^{238}U)$$

$$+ 9 \qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{(everything else)}$$

$$\therefore \quad \Sigma_a = 22.47 \text{ m}^{-1}$$

Only ^{235}U is fissile

$$\therefore \quad \Sigma_f = \frac{0.188 \times 400 \times 6.022 \times 10^{23}}{0.235} \times 580 \times 10^{-28} = 11.18 \text{ m}^{-1} \qquad\qquad [15\%]$$

(ii)　　　　　　　　$\eta = \dfrac{\nu \Sigma_f}{\Sigma_a} = \dfrac{2.43 \times 11.18}{22.47} = 1.209$　　　　　　　　　　[5%]

(iii)　If extrapolation distances can be ignored, criticality is reached when

$$\left(\frac{2.405}{R}\right)^2 + \left(\frac{\pi}{H}\right)^2 = (\eta - 1)\frac{\Sigma_a}{D}$$

$$\therefore \quad \left(\frac{2.405}{0.225}\right)^2 + \left(\frac{\pi}{H}\right)^2 = 0.209 \times \frac{22.47}{0.03}$$

$$\therefore \quad 114.25 + \left(\frac{\pi}{H}\right)^2 = 156.54$$

$$\therefore \quad H = \frac{\pi}{\sqrt{156.54 - 114.25}} = 0.483 \text{ m} \qquad [10\%]$$

(iv)   The water-filled cooling jacket will act as a reflector, as water has a high scattering cross-section. The presence of a reflector reduces neutron leakage, improving neutron economy and thus reducing the critical mass. The criticality condition would therefore be met with a smaller depth of solution than that estimated in (iii).   [10%]

Assessor's Comments:

An extremely popular question attempted by all but one candidate, and done very well by many.

The bookwork derivations were generally done well, although the fact that the reactor being homogeneous implied a spatially invariant diffusion coefficient was missed by many.

Another somewhat common failing was poorly justified boundary conditions when solving the diffusion equation. Others were sloppy in distinguishing between partial and ordinary derivatives.

A lack of care/attention to detail undermined several attempts to calculate macroscopic cross-sections.

A number of candidates recognised that the water jacket would act as a reflector but incorrectly thought this would change η or B_m for the contents of the tank rather than changing the boundary condition at the edge of the tank.

Q3

(a)
$$\theta = \frac{T - T_{c1/2}}{T_{co} - T_{c1/2}} \sin\left(\frac{\pi L}{2L'}\right)$$

$$\therefore \quad \frac{T - T_{c1/2}}{T_{co} - T_{c1/2}} = \eta$$

$$T_{c1/2} = \tfrac{1}{2}(T_{co} + T_{ci}) \quad \therefore \quad T - \tfrac{1}{2}(T_{co} + T_{ci}) = \eta\left[T_{co} - \tfrac{1}{2}(T_{co} + T_{ci})\right]$$

$$\therefore \quad T = \tfrac{1}{2}\eta(T_{co} - T_{ci}) + \tfrac{1}{2}(T_{co} + T_{ci})$$

$$\therefore \quad T = \tfrac{1}{2}\left[(\eta+1)T_{co} - (\eta-1)T_{ci}\right]$$ [15%]

(b)

(i) $\dot{m}c_p\left[T_{co} - T_{ci}\right] = q$ where q is the pin power

$$\therefore \quad \dot{m}c_p = \frac{q}{\left[T_{co} - T_{ci}\right]} = \frac{180 \times 10^3}{30} = 6 \,\text{kW K}^{-1}$$ [5%]

(ii)   From the 4M16 data sheet, for Ginn's equation $Q = \dfrac{\pi \dot{m} c_p}{UA} \dfrac{L}{L'}$

Using $A = 4\pi r_o L$ $\therefore \quad Q = \dfrac{\pi \dot{m} c_p}{U 4\pi r_o L} \dfrac{L}{L'} = \dfrac{\dot{m} c_p}{U 4 r_o L'}$

Here $L' = 2.5$ m (given) and $L = 2.0$ m (half the active fuel length)

$r_i = \tfrac{1}{2} \times 9.3 = 4.65$ mm

$r_o = r_i + t_c = 4.65 + 0.6 = 5.25$ mm

From the 4M16 data sheet: $\dfrac{1}{U} = \dfrac{1}{h} + \dfrac{t_c}{\lambda_c} + \dfrac{r_o}{h_b r_i} + \dfrac{r_o}{2\lambda_f}\left(1 - \dfrac{r^2}{r_i^2}\right)$

The maximum fuel temperature will be on the centre-line of the fuel pin (at $r = 0$).

$$\therefore \quad \frac{1}{U} = \frac{1}{h} + \frac{t_c}{\lambda_c} + \frac{r_o}{h_b r_i} + \frac{r_o}{2\lambda_f}$$

$$\therefore \quad \frac{1}{U} = \frac{1}{35 \times 10^3} + \frac{6 \times 10^{-4}}{12} + \frac{5.25 \times 10^{-3}}{25 \times 10^3 \times 4.65 \times 10^{-3}} + \frac{5.25 \times 10^{-3}}{2 \times 3} = 9.987 \times 10^{-4} \,\text{m}^2\text{KW}^{-1}$$

$$\therefore \quad Q = \frac{\dot{m}c_p}{U 4 r_o L'} = 9.987 \times 10^{-4} \times \frac{6 \times 10^3}{4 \times 5.25 \times 10^{-3} \times 2.5} = 114.1$$

$$\therefore \quad \theta_{max}^2 = 1 + Q^2 = 1 + 114.1^2 \quad \Rightarrow \quad \theta_{max} = 114.1$$

$$\therefore \quad \eta_{max} = \frac{\theta_{max}}{\sin\left(\dfrac{\pi L}{2L'}\right)} = \frac{114.1}{\sin\left(\dfrac{\pi \times 2}{2 \times 2.5}\right)} = 120.0$$

$$\therefore \quad T_{max} = \tfrac{1}{2}\left[(\eta_{max} + 1)T_{co} - (\eta_{max} - 1)T_{ci}\right]$$

$$\therefore \quad T_{max} = \tfrac{1}{2}\left[(120 + 1) \times 315 - (120 - 1) \times 285\right] = 2100 \,°\text{C}$$ [40%]

(iii)   From above

$$T_{max} = \tfrac{1}{2}\left[(\eta_{max}+1)T_{co} - (\eta_{max}-1)T_{ci}\right]$$

$$\therefore \quad T_{co} = \frac{2T_{max}+(\eta_{max}-1)T_{ci}}{(\eta_{max}+1)}$$

η_{max} is not changed. Therefore if now $T_{max} = 2840$ °C

$$\therefore \quad T_{co} = \frac{2\times 2840 + (120-1)\times 285}{(120+1)} = 327.2 \text{ °C}$$

$$\therefore \quad q = \dot{m}c_p\left[T_{co}-T_{ci}\right] = 6\times 10^3\left[327.2 - 285\right] = 253.2 \text{ kW} \qquad [15\%]$$

(c) $\dfrac{1}{U} = \dfrac{1}{h} + \dfrac{t_c}{\lambda_c} + \dfrac{r_o}{h_b r_i} + \dfrac{r_o}{2\lambda_f}$ as before.

With the different fuel (only the last term is changed)

$$\frac{1}{U} = \frac{1}{35\times 10^3} + \frac{6\times 10^{-4}}{12} + \frac{5.25\times 10^{-3}}{25\times 10^3 \times 4.65\times 10^{-3}} + \frac{5.25\times 10^{-3}}{2\times 12} = 3.425\times 10^{-4} \text{ m}^2\text{KW}^{-1}$$

$$\therefore \quad Q = \frac{\dot{m}c_p}{U4r_o L'} = 3.425\times 10^{-4} \times \frac{6\times 10^3}{4\times 5.25\times 10^{-3}\times 2.5} = 39.14$$

$$\therefore \quad \theta_{max}^{\ 2} = 1+Q^2 = 1+39.14^2 \quad \Rightarrow \quad \theta_{max} = 39.15$$

$$\therefore \quad \eta_{max} = \frac{\theta_{max}}{\sin\left(\dfrac{\pi L}{2L'}\right)} = \frac{39.15}{\sin\left(\dfrac{\pi \times 2}{2\times 2.5}\right)} = 41.16$$

As before

$$T_{co} = \frac{2T_{max}+(\eta_{max}-1)T_{ci}}{(\eta_{max}+1)}$$

So if $T_{max} = 1665$ °C

$$T_{co} = \frac{2\times 1665 + (41.16-1)\times 285}{(41.16+1)} = 350.5 \text{ °C}$$

$$\therefore \quad q = \dot{m}c_p\left[T_{co}-T_{ci}\right] = 6\times 10^3\left[350.5 - 285\right] = 393.0 \text{ kW}$$

So, even though the melting point of U_3Si_2 is much lower than that of UO_2, its much higher thermal conductivity means that a substantially higher 'over-power' level is needed to cause fuel melting. This means that using U_3Si_2 fuel could enable the power density in a reactor to be increased without threatening thermal safety margins. [25%]

Assessor's Comments:

Another popular question attempted by 91% of candidates and done well by many.

A surprising number of candidates thought $T_{c1/2} = \tfrac{1}{2}(T_{co}-T_{ci})$ rather than $T_{c1/2} = \tfrac{1}{2}(T_{co}+T_{ci})$ and, worse, then fudged their proof of the result in part (a).

Several answers went astray due to sloppiness with units. Several others failed to distinguish between diameter and radius and thus calculated incorrect thermal resistances.

Other candidates showed poor attention to detail, and calculator errors were not uncommon.

Some candidates failed to show all their calculational steps making it impossible to determine where mistakes had occurred (or to award partial credit).

Q4

(a) Many fission products are unstable. Some decay by neutron emission. Unlike the neutrons emitted promptly in fission, these neutrons are emitted some time after the fission reaction that produced the relevant fission products (at a time dependent on the half-life of the fission products in question). These neutrons are in consequence known as "delayed neutrons".

Delayed neutrons have a very significant (beneficial) effect, increasing the average neutron lifetime and hence lengthening the dominant time constant governing the dynamic behaviour of the neutron population. [10%]

(b) The equilibrium relationship between n and c is (from $dc/dt = 0$)

$$c_0 = \frac{\beta}{\Lambda\lambda} n_0$$

Taking Laplace transforms (p = the transform variable)

$$p\tilde{n} - n_0 = \frac{\rho - \beta}{\Lambda}\tilde{n} + \lambda\tilde{c} \qquad (4.1)$$

$$p\tilde{c} - c_0 = \frac{\beta}{\Lambda}\tilde{n} - \lambda\tilde{c}$$

Substituting for c_0 $\therefore$ $\tilde{c}(p + \lambda) = \frac{\beta}{\Lambda}\tilde{n} + \frac{\beta}{\Lambda\lambda} n_0$

$$\therefore \quad \lambda\tilde{c} = \frac{\beta}{\Lambda}\frac{(\lambda\tilde{n} + n_0)}{(p + \lambda)}$$

Substituting in equation (4.1)

$$p\tilde{n} - n_0 = \frac{\rho - \beta}{\Lambda}\tilde{n} + \frac{\beta}{\Lambda}\frac{(\lambda\tilde{n} + n_0)}{(p + \lambda)}$$

$$\therefore \quad \tilde{n}\left[p - \frac{\rho - \beta}{\Lambda} - \frac{\beta\lambda}{\Lambda(p + \lambda)}\right] = n_0\left[1 + \frac{\beta}{\Lambda(p + \lambda)}\right]$$

$$\therefore \quad \tilde{n}\left[p - \frac{\rho}{\Lambda} + \frac{\beta}{\Lambda}\left\{1 - \frac{\lambda}{(p + \lambda)}\right\}\right] = n_0\left[1 + \frac{\beta}{\Lambda(p + \lambda)}\right]$$

$$\therefore \quad \tilde{n}\left[p - \frac{\rho}{\Lambda} + \frac{\beta}{\Lambda}\left\{\frac{p}{(p + \lambda)}\right\}\right] = n_0\left[1 + \frac{\beta}{\Lambda(p + \lambda)}\right]$$
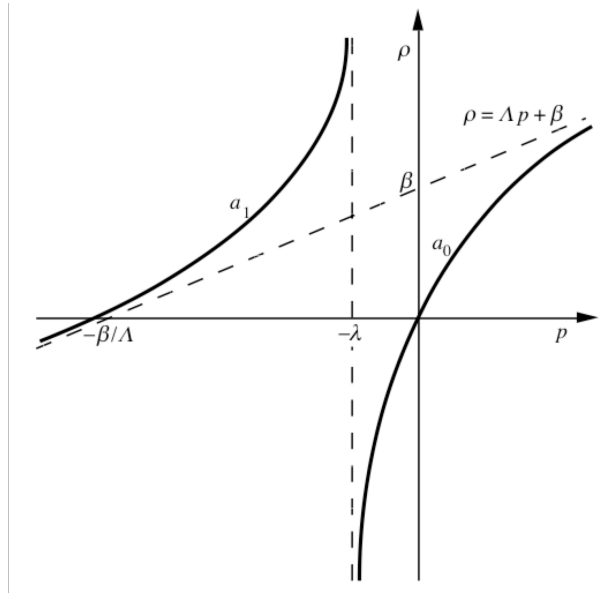
$$\therefore \quad \tilde{n} = \frac{n_0\left[1 + \frac{\beta}{\Lambda(p + \lambda)}\right]}{\left[p - \frac{\rho}{\Lambda} + \frac{\beta p}{\Lambda(p + \lambda)}\right]}$$

To take inverse transforms we need to reduce this expression to partial fraction form. This entails finding the roots of the equation formed by setting the denominator of the right-hand side to zero, i.e.

$$p - \frac{\rho}{\Lambda} + \frac{\beta p}{\Lambda(p + \lambda)} = 0$$

$$\therefore \quad \rho = p\left[\Lambda + \frac{\beta}{(p+\lambda)}\right] \qquad [40\%]$$

(c)   The relationship between ρ and p is of the form:

The dominant behaviour is associated with the root a_0. The locus for a_0 passes through the origin (which represents the critical state). Therefore for small reactivity perturbations $\Delta\rho$ from the critical state the dominant root can be estimated using a first-order Taylor series approximating the function at the origin.

From the result in (b)

$$\frac{d\rho}{dp} = \left[\Lambda + \frac{\beta}{(p+\lambda)}\right] - p\frac{\beta}{(p+\lambda)^2}$$

$$\therefore \quad \frac{d\rho}{dp} = \Lambda + \frac{\beta}{(p+\lambda)}\left[1 - \frac{p}{(p+\lambda)}\right] = \Lambda + \frac{\beta\lambda}{(p+\lambda)^2}$$

$$\therefore \quad \left.\frac{d\rho}{dp}\right|_{p=0} = \Lambda + \frac{\beta}{\lambda}$$

Given that $\rho = 0$ at $p = 0$, the first-order Taylor series gives

$$\Delta\rho = \left[\Lambda + \frac{\beta}{\lambda}\right]p$$

Thus, the inverse period corresponding to a reactivity change $\Delta\rho$ from criticality ($\rho = 0$) is

$$p = \frac{\Delta\rho}{\Lambda + \dfrac{\beta}{\lambda}} = \frac{\lambda\Delta\rho}{\Lambda\lambda + \beta}$$

and the corresponding time constant

$$T = \frac{1}{p} = \frac{\Lambda\lambda + \beta}{\lambda\Delta\rho} \qquad [30\%]$$

(d)
$$\rho = p\left[\Lambda + \frac{\beta}{(p+\lambda)}\right]$$

$$\therefore \quad \frac{\rho}{\Lambda}(p+\lambda) = p\left[(p+\lambda) + \frac{\beta}{\Lambda}\right]$$

$$\therefore \quad p^2 + p\left[\lambda + \frac{\beta-\rho}{\Lambda}\right] - \frac{\lambda\rho}{\Lambda} = 0$$

Substituting the values given

$$p^2 + p\left[0.12 + \frac{0.007 - 0.001}{0.00125}\right] - \frac{0.12 \times 0.001}{0.00125} = 0$$

$$\therefore \quad p^2 + 4.92p - 0.096 = 0$$

$$\therefore \quad p = 0.01944 \text{ or } -4.9394 \text{ s}^{-1}$$

Therefore the dominant (positive) time constant is

$$T_+ = \frac{1}{p_+} = \frac{1}{0.01944} = 51.45 \text{ s}$$

The approximate result from (c) gives

$$T = \frac{\Lambda\lambda + \beta}{\lambda\Delta\rho} = \frac{0.00125 \times 0.12 + 0.007}{0.12 \times 0.001} = 59.58 \text{ s}$$

A reasonable approximation, but an overestimate, so not a conservative result.   [20%]

Assessor's Comments:

A question attempted by 63% of candidates.

A number of candidates lost marks because of poor justification of steps in the derivations.

Others got hopelessly bogged down in algebra, particularly in part (b).

Some candidates who got bogged down on part (b) failed to move on to the rest of the question, which just needed the given result.

Other derivations went astray due to candidates' inability to distinguish ρ and p in their own handwriting.

Quite a few candidates incorrectly thought part (c) related to the prompt jump approximation.

The fact that the locus of a_0 passes through the origin in ρ-p space is an important point in deriving the result in part (c) that was not mentioned by a number of candidates who otherwise had the right idea.